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Thermo-Mechanical Stress Analysis in a Rotating Radially Graded FG-Disc with Non-Uniform Thickness

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ABSTRACT: In this paper, the set of governing equations of temperature and thermo-mechanical stresses analyses in a rotating radially graded FG-disc with non-uniform thickness are derived. All mechanical and thermal properties of the material including elastic modulus, Poisson's ratio, density and thermal conductivity and expansion coefficients are considered to be graded radially according to a power law function; The volume fraction changes in radial direction between two desired values. In thermal analysis, convention heat transfer through two sides of the disc are considered and thermal boundary conditions are considered as constant temperature at inner edge and convention heat transfer at outer one. In order to increase the accuracy, variation of convective heat transfer coefficient in radial direction and its dependency on the rotating speed are considered. Considering complexities of equations, differential quadrature method (DQM) is used as strong approach and both thermal and mechanical equations are solved numerically. Effect of various parameters such as rotating speed, variation of thickness and power law index on the distribution of temperature, stress and deflection of the disc are investigated. Finally, based on the Tamura-Tomota-Ozawa model (TTO), yield strength of the disc is derived and its elastic and plastic parts are detected.

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1- Introduction

Thermo-mechanical stress analysis of rotating disks is one of the most important engineering problems which can be used in many industrial applications such as centrifugal compressors, steam and gas turbines, aerospace devices and flywheels.

Using a semi-analytical method, Bayat et al. [1] studied mechanical and thermal stresses in a rotating disk with variable thickness due to radially symmetry loads. Hassani et al. [2] obtained elastic solutions for thermo-mechanical analysis of functionally graded rotating disks by semi-exact methods of Liao's homotopy analysis method, Adomian's decomposition method and variational iteration method. By applying finite element method, Shahzamanian et al. [3] analyzed thermoelastic contact problem of functionally graded rotating brake disk with the heat source due to contact friction that Coulomb contact friction was assumed as the heat source. Nie and Batra [4] demonstrated stress analysis and material tailoring in isotropic linear thermoelastic rotating disks whose thickness, mass density, thermal expansion coefficient, and shear modulus were variable in the radial direction. Using finite element method, Sharma et al. [5] focused on the analysis of thermo-elastic stresses, displacements and strains in a thin circular functionally graded material disk subjected to thermal loads in which temperature profiles were modeled by the solution of heat conduction equation. Dynamics of rotating disks with stationary heat source was presented by Wauer and Schweizer [6] whereas thermo-elastic simulations were performed applying the fully coupled thermo-mechanical theory and were implemented by a finite element discretization. An analytical solution was presented for steady thermal stresses in rotating functionally graded hollow circular disks with constant angular velocity about its central axis by Peng and Li [7]. Kalali et al. [8] presented an exact solution for elasto-plastic stress analysis in rotating FG discs.

2- Governing Equations

As depicted in Fig. 1, a rotating disc with radial dependent thickness and properties is considered. The volume fraction of ceramic varies in radial direction as

$$f_c = f_{cb} + \left(f_{ca} - f_{cb}\right)\left(\frac{r-b}{a-b}\right)^p \quad (1)$$

where f_{cb} and f_{ca} are the values of the volume fraction of ceramic at inner and outer radii of the disc and p is the power law index. Using Eq. (1), any property of the disc can be written as

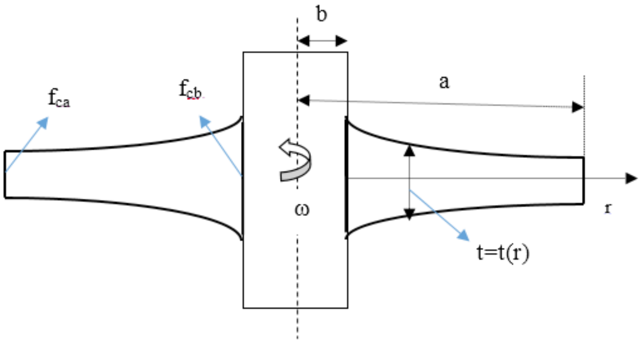

Figure 1. The geometry of the rotating FG disc with variable thickness

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$$P = P_m + \left(P_c - P_m\right)\left[f_{cb} + \left(f_{ca} - f_{cb}\right)\left(\frac{r-b}{a-b}\right)^p\right] \tag{2}$$

In which subscripts m and c indicate values of the corresponding property in metal and ceramic, respectively.

The governing equation and boundary conditions for temperature distribution can be written as [9]

$$\begin{aligned} &\frac{d^2T}{dr^2} + \frac{1}{kA}\frac{d\left(kA\right)}{dr}\frac{dT}{dr} - \frac{h}{kA}\frac{dA_2}{dr}T = -\frac{h}{kA}\frac{dA_2}{dr}T_\infty \\ &r = b: \qquad T = T_b \\ &r = a: \quad -k\frac{dT}{dr} = h\left(T - T_\infty\right) \end{aligned} \tag{3}$$

where $A=2\pi rt$, $A_2=2\pi(r^2-b^2)$ and k is thermal conductivity coefficient. Also, h is thermal convection coefficient calculated as [10]

$$h = 0.0193k\,\nu^{-0.8}\omega^{0.8}r^{0.6} \tag{4}$$

in which ν is kinematic viscosity coefficient.

For stress analysis, governing equation and boundary conditions can be sated as ($\zeta=r/a$)

$$\begin{aligned} &\frac{d^2f}{d\zeta^2} + a_1\left(\zeta\right)\frac{df}{d\zeta} + a_2\left(\zeta\right)f = a_3\left(\zeta\right) + a_4\left(\zeta\right) \\ &r = b \quad u = 0 \\ &r = a \quad \sigma_r = 0 \end{aligned} \tag{5}$$

where

$$f = \frac{F}{aE_m} \qquad F = r\sigma_r \tag{6}$$

Also, radial displacement can be calculated as

$$u = \frac{r}{E}\left(\frac{dF}{dr} + \frac{1}{t}\frac{dt}{dr}F + \rho r^2\omega^2 - \upsilon\frac{F}{r}\right) + \alpha\,\Delta T \tag{7}$$

and the yield stress of the material can be written as

$$\sigma_y = \sigma_{ym}\left[\left(1 - f_c\right) + \frac{q + E_m}{q + E_c}\frac{E_c}{E_m}f_c\right] \tag{8}$$

in which σ_{ym} is the yield stress of the metal and q=4.5 GPa [11].

3- Results and Discussion

In this section, numerical results are presented for a disc made of Ti and TiB$_2$ whose properties are listed in Table 1.

In order to study the effect of power-law index, consider a disc with the following properties:

$$\varphi = \frac{b}{a} = 0.25 \qquad \gamma = \frac{a}{t_b} = 20 \qquad t^* = \frac{t}{t_b} = 1 - 0.75\left(\zeta - \varphi\right)$$

$$\Omega = \omega a\sqrt{\frac{\rho_m}{E_m}} = 0.001 \qquad \beta = \alpha_m T_\infty = 0.0075 \qquad T_b^* = \frac{T_b}{T_\infty} = 0.5$$

$$h_0 = 2\left(\frac{E_m a^2}{\rho_m v^2}\right)^{0.4} = 2\times10^6 \qquad f_{ca} = 0.25 \qquad f_{cb} = 0.75$$

Table 1. Properties of metal and ceramic.

	Ti	TiB$_2$
E, GPa	116	565
ρ, kg/m^3	4506	4520
υ	0.32	0.108
α, 10^{-6} K^{-1}	8.6	6.4
k, W/m.K	21.9	96
σ_y, MPa	450	-

In Figs. 2 to 6 distribution of temperature rise, radial and circumferential components of stress, von-Mises stress and the radial component of displacement are depicted for various values of the power law index (p).

Fig. 2 shows that power law index has a weak effect on temperature distribution and the increase in the power law index leads to a little decrease in temperature rise. Figs. 3 to 5 show that the rise in the value of the power law index leads to the decrease in radial, circumferential components of stress and von-Mises stress. Fig. 5 shows that with the increase in power-law index, the designer is able to eliminate plastic regions and keep the whole disc in elastic zone; but, it should be noted that as Fig. 6 shows, the rise in the value of power-law index, increases the value of radial displacement and deformation in disc which limits the value of the increase in power law index. In other words, an optimization can be done to find the best value of the power law index.

In order to study the effect of variation of thickness, a disc with the following properties is considered:

$$\varphi = 0.5 \qquad \gamma = 10 \qquad p = 1$$

$$\Omega = 0.01 \qquad \beta = 0.005 \qquad T_b^* = 0.6$$

$$h_0 = 1\times10^6 \qquad f_{ca} = 0.25 \qquad f_{cb} = 0.5$$

$$t^* = \exp\left[-\alpha\left(\zeta - \varphi\right)\right]$$

Actually, with the increase in α, the disc becomes thinner in outer regions.

For various values of α, the variation of von-Mises stress is depicted in Fig. 7. This figure shows that with the increase in α, there is less possibility of creating plastic regions in the disc. In other words, in comparison with a disc of uniform thickness, non-uniform discs have better characteristics in both stress and deformation analyses. Also, using the discs with non-uniform thickness decreases the total weight of the system which can be considered as another merit of discs with a non-uniform thickness.

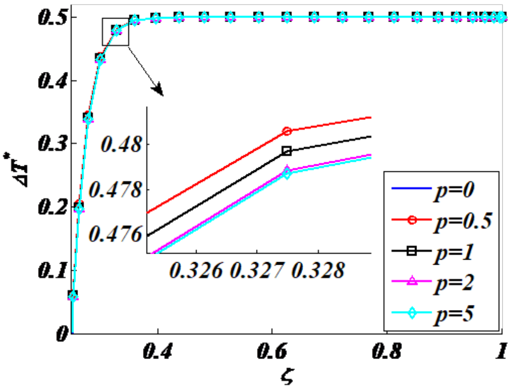

Figure 2. Distribution of temperature rise.

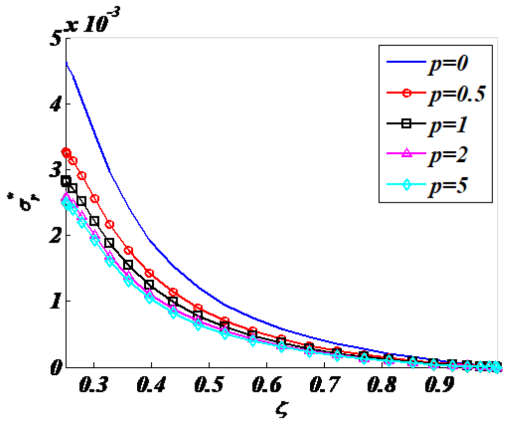

Figure 3. Distribution of radial stress

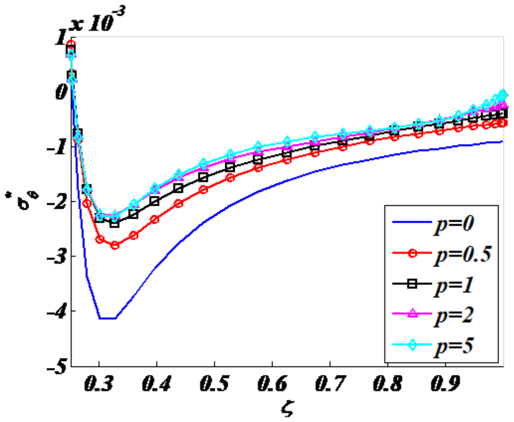

Figure 4. Distribution of circumferential stress

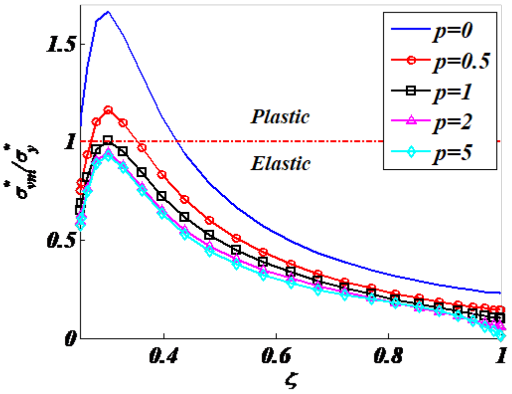

Figure 5. Distribution of von-Mises stress

4- Conclusions

Using differential quadrature method, a numerical solution for thermo-mechanical stress analysis of rotating FG disc with the non-uniform disc was presented. Numerical examples showed that the increase in the value of power-law index decreases the possibility of the existence of plastic regions and increases radial deformation. In other words, an optimization can be done to find the best value for the power law index.

Also, it was shown by numerical examples that in comparison with a disc of uniform thickness, non-uniform ones give better characteristics in stress and deformation analyses and decrease the total weight of the disc.

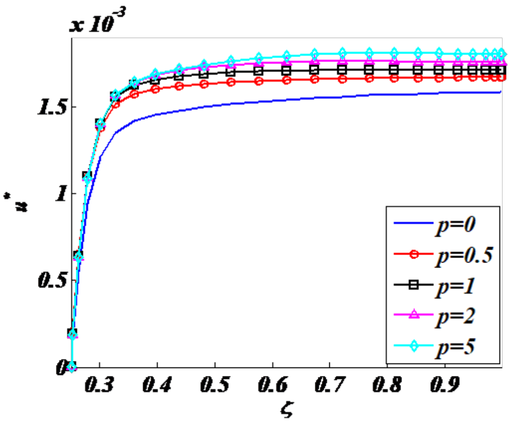

Figure 6. Distribution of radial displacement

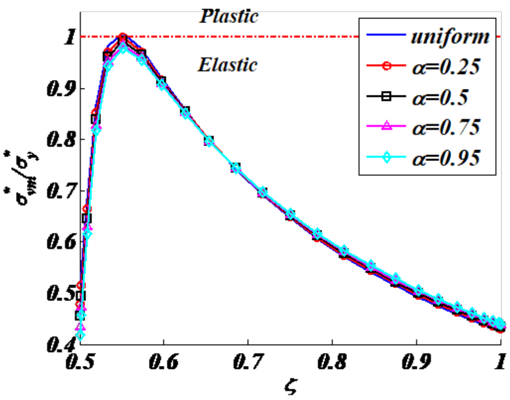

Figure 7. Distribution of von-Mises stress

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