



Study of Slip Effect on Electro-osmotic Micromixer Performance Based on Entropy Index

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ABSTRACT: In this article electrokinetic mixing through heterogeneous microchannels has been studied and the effects of slip coefficient, zeta-potential, Debye-Huckel parameter and Reynolds number on mixing efficiency have been investigated. The microchannels have homogenous surface properties except for zeta-potential. In order to study the electro-osmotic mixing, the Navier-Stokes, Nernst-Planck, electric potential and concentration equations have been solved numerically. In order to evaluate the mixing efficiency, entropy of concentration has been used as a quantitative index. The results show that the behavior of electro-osmotic micromixers is highly depended to amount and distribution of wall zeta-potential. Furthermore, mixing efficiency increases with reduction of slip coefficient and Debye-Huckel and Reynolds number parameters in most cases. It is seen that slip coefficient can decrease or increase mixing efficiency dependent on the Reynolds number. Furthermore the accuracy of Helmholtz-Smoluchowski approximate model is also investigated and it is found that in high wall zeta-potential cases or low values of Debye-Huckel parameter, results of this model have significant error compared to Nernst-Planck model. It is also found that the mixing performance increases when as the charge pattern of micromixer is more asymmetric so that the certain mixing value can be obtained in shorter length which is importance in micromixers design.

Review History:

Received: 18 February 2016

Revised: 19 July 2016

Accepted: 23 October 2016

Available Online: 9 November 2016

Keywords:

Mixing

Heterogeneous zeta-potential

Mixing entropy

Electro-osmotic flow

Helmholtz-Smoluchowski model

1- Introduction

It is known that electrical field can make the flow move inside the narrow channels by applying force to the aggregated ions adjacent to the wall [1]. To determine such flows; named as electro-osmotic flows, it is required to solve electrical potential and electric charges transfer equations rather than Navier-stokes equations. Numerically solving these equations involve significant calculations especially for heterogeneous microchannels [2], hence, some simple modeling such as Helmholtz-Smoluchowski model is also suggested to solve these flows [3].

The main purpose of this study is to investigate the mixing using electro-osmotic flows. In order to evaluate the mixing performance, we used the entropy index related to mixing with a suitable weight function.

2- Governing Equations

The governing equations are as follows

- Poisson-Boltzmann equation

$$\nabla^2(\psi + \phi) = \frac{K^2}{2}(n^+ - n^-) \quad (1)$$

$K=H/\lambda$ is non-dimensional Debye-Huckel parameter and λ is the characteristic thickness of electric double layer [4].

- Nernst-Planck equations

In order to obtain ionic concentrations Nernst-Planck equations need to be solved:

$$\bar{\nabla} \cdot (\bar{V} \bar{n}^i) = \frac{1}{Re Sc^i} \left\{ \nabla^2 n^i + \bar{\nabla} \cdot [n^i (\bar{\nabla} \psi + A \bar{\nabla} \phi)] \right\} \quad (2)$$

The above equation is written for negative and positive ions (i.e. $n^i = n^+, n^-$). The non-dimensional parameter $A = E_{ref} H / (K_b T / ze)$ is the ratio of external induced voltage to basic voltage [4].

- Navier-stokes equations

Under steady conditions with constant physical properties, the electro-osmotic flow is governed by:

$$\bar{\nabla} \cdot (\bar{V} \bar{V}) = -\bar{\nabla} P + \frac{1}{Re} \nabla^2 \bar{V} - B \rho_e (\bar{\nabla} \psi + A \bar{\nabla} \phi) \quad (3)$$

$B = n_0 K_b T / \rho U_{ref}^2$ is a non-dimensional parameter which presents the ratio of ionic pressure to dynamic pressure. The hydrophobicity effect, the slip condition in non-dimensional form is considered as $u = \beta(\partial u / \partial n)$ which β is slip coefficient [4].

- Helmholtz-Smoluchowski model

This model considers the electrical body force acting only in region near the wall, where the highest net charge exists. The effect of this body force is to move the liquid parallel to the wall and the value of this tangential velocity; known as Helmholtz-Smoluchowski slip velocity, is estimated as [5]:

$$u_s = -\frac{\epsilon E_{ext} \zeta}{\mu} \quad (4)$$

By implementation this condition, the electro-osmotic velocity field is achieved without solving potential and electrical charges fields so that Navier-Stokes equations reduce to:

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$$\bar{\nabla}(\bar{V}\bar{V}) = -\bar{\nabla}P + \frac{1}{Re}\nabla^2\bar{V} \quad (5)$$

3- Mixing Entropy Index

Here, the index of Shannon entropy is modified so that the mass flux is taking into accounts as below [6]:

$$S_{mix} = \frac{-\int_A \rho u C \ln(C) dy}{\int_A \rho u dy} \quad (6)$$

Using weighted entropy index at each cross-section of microchannel, the mixing efficiency for that section can be defined as:

$$\epsilon_s = \frac{S_{mix} - S_{inlet}}{S_{\infty} - S_{inlet}} \quad (7)$$

4- Conventional Micromixer

The microchannel investigated in this article and its surface heterogeneities is designed for the purpose of a micromixer. The microchannel has the height of H and length $L=6H$ with a middle section of length $2H$ designed as mixing section. This section has heterogeneous zeta-potential mainly consist of positive and negative patches. In the inlet and outlet sections, the homogenous zeta-potential is considered. One of the three distinct configuration discussed in this article, has been schematically in Fig. 1. The direction of electric field is from left to right.

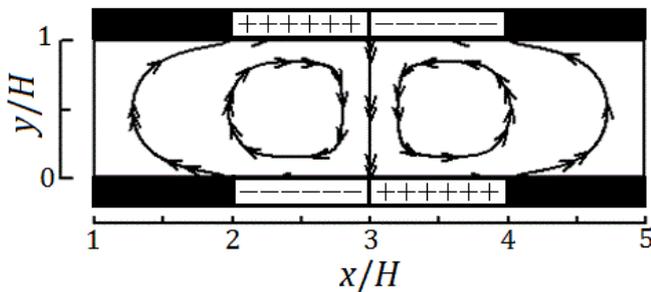


Figure 1. Schematic of electro-osmotic streamline for a prescribed configuration of patches in middle section

5- Numerical Method

The set of governing equations are highly coupled and hence a precise iterative procedure is performed to solve the flow and concentration fields. For this purpose, proper initial distributions for electric field (both external and internal) and velocity field are considered.

Then Nernst–Planck equations are solved to obtain the distribution of electric charges. In this stage, the first estimation for electric body force can be evaluated. After this step, Navier-Stokes, Nernst–Planck and Poisson-Boltzmann equations are solved repeatedly until the proper convergence achieved. Having the velocity field, the concentration equation will be solved in order to investigate the mixing performance.

6- Results and Discussion

The result presented here, are obtained with two methods namely, using Helmholtz-Smoluchowski model and Nernst–

Planck model. The comparison of these models for $K=41$ (which corresponds to a narrow EDL) is shown in Fig. 2. ζ_p is the zeta-potential value in homogenous section and ζ_m is the amount of zeta-potential in middle section. It is seen that for narrow EDL, Helmholtz-Smoluchowski model predicts the mixing efficiency similar to Nernst–Planck.

Fig. 3 shows the mixing efficiency variations for the microchannel with hydrophobic walls ($\beta=0.025$). It is seen in this figure that presence of slip at any Reynolds number, reduces the mixing efficiency compared to the case with no-slip.

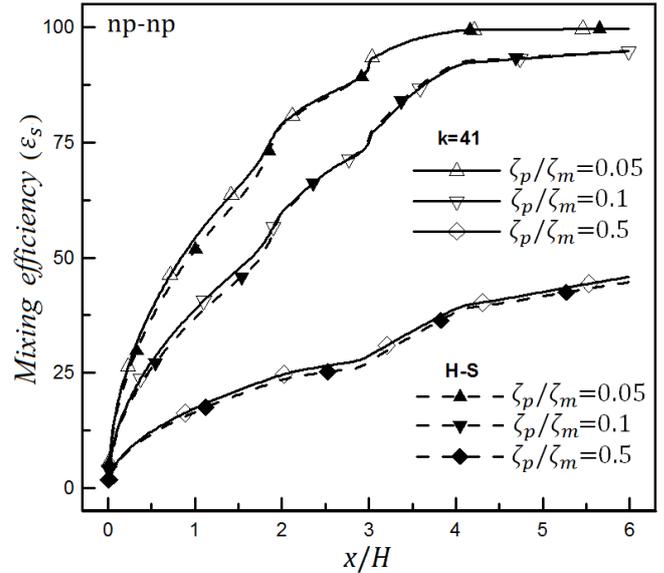


Figure 2. Mixing efficiency value of Nernst–Planck model and H-S model for $K=41$

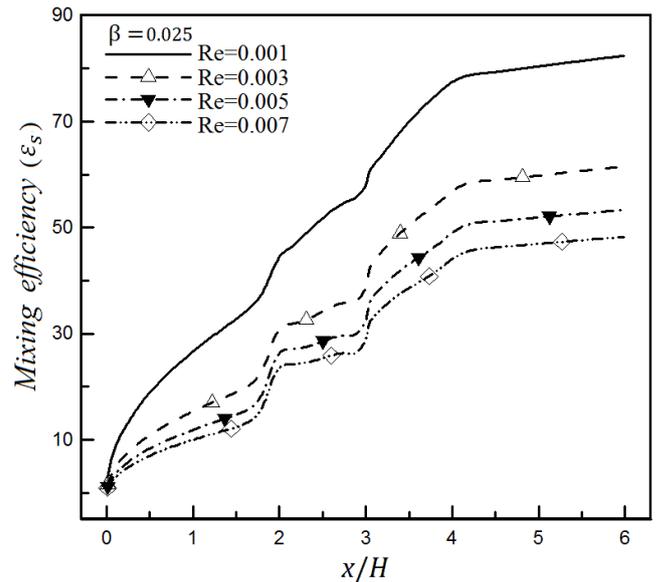


Figure 3. Mixing efficiency changes per specific slip coefficient (0.025) and different Reynolds number for (np-pn) pattern

7- Conclusions

In this article mixing of electro-osmotic flows analyzed by introducing a suitable index based on entropy concept. Both H-S approximate model and Nernst–Planck precise model are examined to evaluate the electrokinetic mixing. It is shown that for thick EDL (say $K < 20$) and also for high zeta-potential, the H-S model overestimated the results compared to Nernst–Planck solution. It is also found that higher mixing is achieved when the charge pattern is more asymmetric. Another finding is that presence of slip enhances the mixing efficiency especially for homogenous microchannel. Indeed, heterogeneous microchannels prepare almost perfect mixing even without slip effects.

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Please cite this article using:

A. R. Farahinia, J. Jamaati, H. Niazmand, Study of Slip Effect on Electro-osmotic Micromixer Performance Based on Entropy Index, *Amirkabir J. Mech. Eng.*, 49(3) (2017) 535-548.
DOI: 10.22060/mej.2016.776



