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Free and Forced Whirling Analyses of Rotors with Multiple Unbalanced Discs Under Axial Force

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ABSTRACT: In this paper the set of equation for free and forced whirling analyses of rotors with any number of discs is derived. By considering gyroscopic effects, the rotor is modeled based on the Timoshenko beam theory and discs are considered as concentrated elements having both translational and rotational inertias. At the position of each disc, the rotor is imposed to distributed and concentrated axial forces which vary versus time. Also, transverse load composed of unbalanced masses and total weight of the system is considered. For a simply supported rotor, the free whirling analysis is investigated using Galerkin method and using Galerkin and Newmark-beta methods, the forced whirling analysis is studied numerically. Forward and backward frequencies and Campbell diagrams are presented in free whirling analysis and variation of deflection, bending moment and shear force in any point of the rotor are depicted versus time in forced whirling analysis. The most advantages of the presented paper are consideration of time-dependency of rotating speed in forced whirling analysis and its applicability for rotors with any number of mounted discs.

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1- Introduction

The rotor dynamics is concerned with the study of dynamic and stability characteristics of the rotating machineries and plays an important role in improving safety and the performance of the entire systems that they are part of. Grybos [1] studied the effect of shear deformations on the critical speeds of rotors. Jei and Lee [2] derived critical speeds of an un-symmetric rotor with rigid dices. Free and forced whirling analysis of viscoelastic rotors was investigated by Sturla and Argento [3]. Jun and Kim [4] studied the effect of torsional torque on whirling of rotors. Afshari et al. [5] used Differential Quadrature Element Method (DQEM) and studied the whirling analysis of multi-span multi-step Timoshenko rotor. Using Differential Quadrature Method (DQM), Irani et al. [6] presented a numerical solution for longitudinal-torsional and two-plane transverse vibration of composite rotors. Torabi et al. [7] presented an exact solution for whirling analysis of multi-step rotors carrying concentrated elements.

In this study, by using the Galerkin and Newmark-beta methods, free and forced whirling analyses of rotors with any number of discs under time variable axial forces are presented.

2- Governing Equations

As depicted in Fig. 1, a uniform rotor of length L and diameter d rotating at angular velocity Ω with various numbers of discs is considered.

The rotational speed increases from zero to its nominal value (Ω_0) in time t_0 as

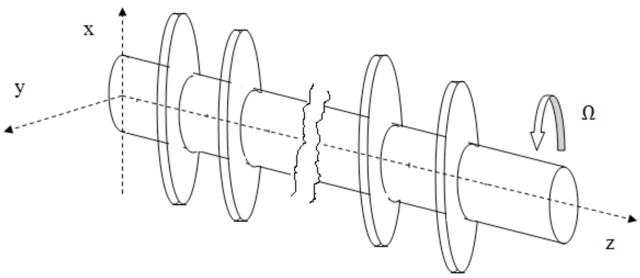

Figure 1. Rotor and concentrated discs

$$\frac{\Omega}{\Omega_0} = \begin{cases} 2\frac{t}{t_0} - \left(\frac{t}{t_0}\right)^2 & t \le t_0 \\ 1 & t \ge t_0 \end{cases} \tag{1}$$

Also, all discs are imposed on a distributed time dependent axial force ($q(t)$) and a concentrated one ($F(t)$).

Using Newton's second law, the set of governing equations can be written as

$$kGA\left(\frac{\partial^2 u_x}{\partial z^2} - \frac{\partial \varphi_y}{\partial z}\right) - \left[R_0 + \sum_{i=1}^{N} P_i(t) H(z - z_i)\right]\frac{\partial^2 u_x}{\partial z^2}$$
$$-\sum_{i=1}^{N} P_i(t)\delta(z - z_i)\frac{\partial u_x}{\partial z} + f_x(x,t)$$
$$= \left[\rho A + \sum_{i=1}^{N} M_i \delta(z - z_i)\right]\frac{\partial^2 u_x}{\partial t^2}$$

$$kGA\left(\frac{\partial^2 u_y}{\partial z^2} + \frac{\partial \varphi_x}{\partial z}\right) - \left[R_0 + \sum_{i=1}^{N} P_i(t) H(z - z_i)\right]\frac{\partial^2 u_y}{\partial z^2}$$

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$$
\begin{aligned}
&-\sum_{i=1}^{N} P_i(t)\,\delta(z-z_i)\frac{\partial u_y}{\partial z}+f_y(x,t)\\
&=\left[\rho A+\sum_{i=1}^{N} M_i\,\delta(z-z_i)\right]\frac{\partial^2 u_y}{\partial t^2}\\
&EI\frac{\partial^2 \varphi_y}{\partial z^2}+kGA\left(\frac{\partial u_x}{\partial z}-\varphi_y\right)-\sum_{i=1}^{N}\left(F_i R_i \sin\varphi_i\right)\delta(z-z_i)=\\
&\left[\rho I+\sum_{i=1}^{N} I_i\,\delta(z-z_i)\right]\frac{\partial^2 \varphi_y}{\partial t^2}-2\left[\rho I+\sum_{i=1}^{N} I_i\,\delta(z-z_i)\right]\Omega\frac{\partial \varphi_x}{\partial t}
\end{aligned}
\tag{2}
$$

in which u_x, u_y, φ_x and φ_y are components of displacement and rotation, respectively. ρ, E and G are density, modulus of elasticity and shear modulus, respectively and A and I indicate area and the moment of inertia, respectively. P is the total axial force and f_x and f_y are transverse loads which are created by gravity and unbalance masses as

$$
\begin{aligned}
&f(x,t)=-\rho A g-\sum_{i=1}^{n} M_i\, g\,\delta(z-z_i)\\
&+\Omega^2\sum_{i=1}^{N} m_i^p e_i\begin{bmatrix}\cos(\Omega t+\theta_i)\\ +j\sin(\Omega t+\theta_i)\end{bmatrix}\delta(z-z_i)
\end{aligned}
\tag{3}
$$

Using Eq. (3) and the following complex variables

$$
u=u_x+ju_y \quad \varphi=\varphi_x+j\varphi_y \quad f=f_x+jf_y \tag{4}
$$

the set of governing equations can be written as

$$
\begin{aligned}
&-\left[\rho A+\sum_{i=1}^{N} M_i\,\delta(z-z_i)\right]\frac{\partial^2 u}{\partial t^2}+kGA\left(\frac{\partial^2 u}{\partial z^2}+j\frac{\partial \varphi}{\partial z}\right)\\
&-\left[R_0+\sum_{i=1}^{N} P_i(t)\,H(z-z_i)\right]\frac{\partial^2 u}{\partial z^2}-\sum_{i=1}^{N} P_i(t)\,\delta(z-z_i)\cdot\\
&=\rho A g-j\Omega^2\sum_{i=1}^{N} m_i^p e_i\sin(\Omega t+\theta_i)\,\delta(z-z_i)\\
&+\sum_{i=1}^{N}\left[M_i g-m_i^p e_i\Omega^2\cos(\Omega t+\theta_i)\right]\delta(z-z_i)\\
&-\left[\rho I+\sum_{i=1}^{N} I_i\,\delta(z-z_i)\right]\frac{\partial^2 \varphi}{\partial t^2}+2j\left[\rho I+\sum_{i=1}^{N} I_i\,\delta(z-z_i\right.\\
&+EI\frac{\partial^2 \varphi}{\partial z^2}-kGA\left(\varphi-j\frac{\partial u}{\partial z}\right)=-\sum_{i=1}^{N} F_i R_i e^{-j\varphi_i}\,\delta(z-z_i)
\end{aligned}
\tag{5}
$$

3- Solution Procedure

For a simply supported rotor, the following boundary conditions should be considered:

$$
z=0,L \quad u=0,\frac{\partial \varphi}{\partial z}=0 \tag{6}
$$

which can be satisfied using the following definitions:

$$
\begin{aligned}
u(z,t)&=\sum_{n=1}^{\infty} a_n(t)\sin\left(\frac{n\pi z}{L}\right)\\
\varphi(z,t)&=\sum_{n=1}^{\infty} b_n(t)\cos\left[\frac{(n-1)\pi z}{L}\right]
\end{aligned}
\tag{7}
$$

Inserting Eq. (7) into Eq. (5) and using the Galerkin method, the set of governing equations can be written as

$$
[M]\{\ddot{X}(t)\}+[G]\{\dot{X}(t)\}+[K(t)]\{X(t)\}=\{F(t)\} \tag{8}
$$

Eq. (8) can be solved using Newmark-beta method [8] and for free vibration analysis, the following relation can be used:

$$
\left|\omega^2[M]+\omega[C]+[K]\right|=0 \tag{9}
$$

4- Results and Discussion

In order to investigate the effect of axial forces on Campbell diagrams, a rotor of length L=4 m and diameter d=30 mm which carries a single disc of diameter d_o=60 mm and thickness t=15 mm is employed. The disc is located at $z=L/3$ and is imposed on a distributed axial force of intensity q. For the first two modes, the effect of q on the Campbell diagrams is depicted in Figs. 2 and 3. These figures show that both the decrease and increase in frequencies can be found. Actually, external axial forces create tension or compression in different parts of the rotor and therefore its stiffness increases in some parts and decreases in other parts which leads to a rise in some frequencies and the decrease in other ones.

Consider a rotor of length L=5 m, diameter d=60 mm with three unbalanced discs of the properties presented in Table 1. The rotating speeds increases in two seconds from zero to its nominal value (Ω_0=100 rpm). Figs. 4 to 7 show the variation of components of displacement and bending moment in the middle length of the rotor. These figures show that

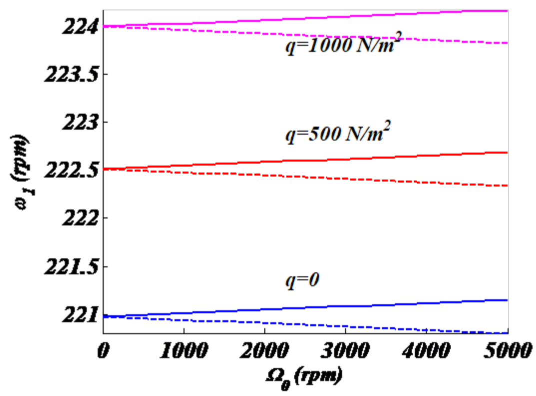

Figure 2. Campbell diagram for the first mode

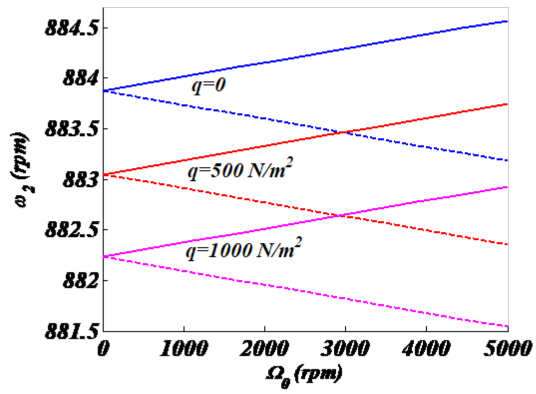

Figure 3. Campbell diagram for the second mode.

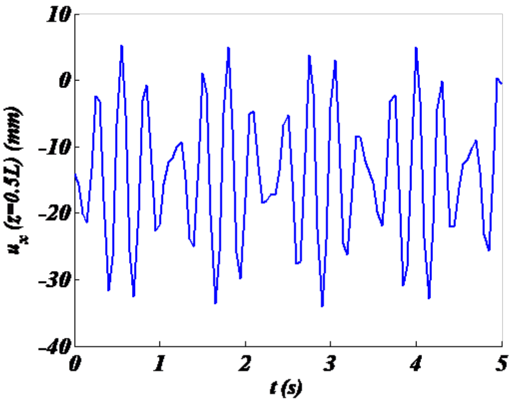

Figure 4. The *x* component of displacement.

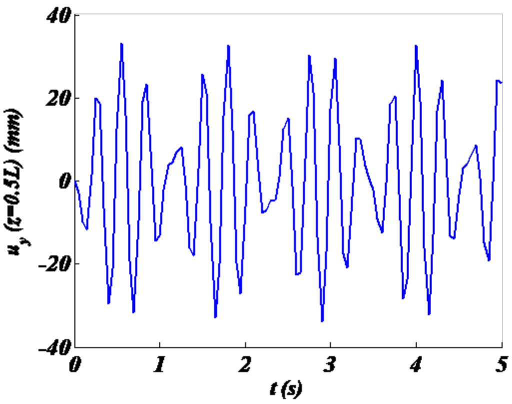

Figure 5. The *y* component of displacement.

Table 1. Properties of the unbalanced discs

Position	[0.25*L* 0.25*L* 0.25*L*]
Outer diameter, mm	[80 90 85]
Thickness, mm	[10 15 15]
Unbalancing, g.mm	[1000 3000 500]
Initial position of unbalanced masses with axis x, Deg.	[45 80 120]
Distributed forces, kN/m2	[20 40 30]×sin(20t)
Concentrated forces, kN	[10 5 20]×sin(20t)
Radial position of concentrated forces, mm	[35 40 40]
Circumferential position of concentrated forces, Deg.	[0 100 220]

displacement and bending moment vary with time between their maximum and minimum values; these values can be used directly to study fatigue in the rotor.

5- Conclusions

Using Galerkin and Newmark-beta methods, free and forced whirling analyses of rotors with any number of discs under time variable axial forces were presented. The numerical results showed that Campbell diagram of the rotor was strongly affected by axial forces and both the decrease and increase in frequencies can occur. Also, it was concluded that bending moment vary periodically with time between its lower and upper bounds; these values are very useful and necessary to study fatigue of the rotor.

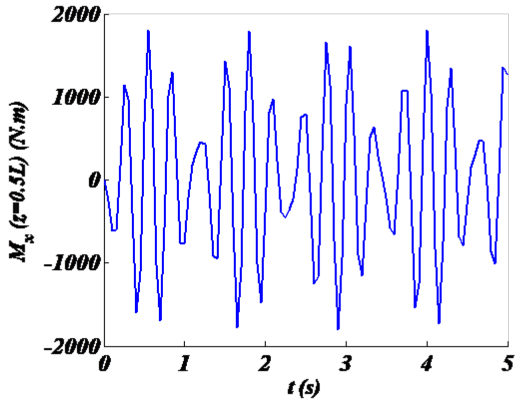

Figure 6. The *x* component of bending moment.

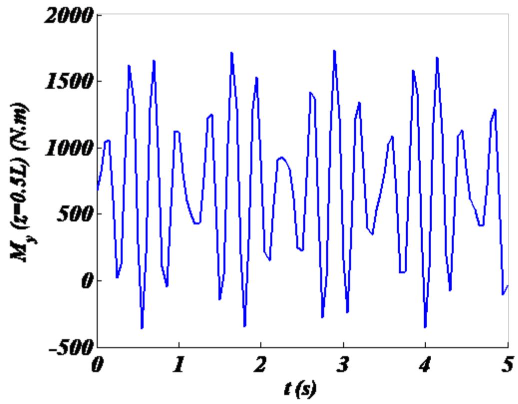

Figure 7. The *y* component of bending moment.

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