Finite Element Analysis of Vibration Behavior of Micro-Rotors Utilizing a Developed Strain Gradient-Based Beam Element

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ABSTRACT: In this paper, a three-dimensional finite element model is developed based on the strain gradient theory to investigate the vibration characteristics of micro-rotors. The model is not only capable of dealing with small-size effects, but also the flexibility of bearings, internal damping and mass eccentricity in the system. The expressions related to the strain energy of the shaft of the micro-rotor are derived on the basis of strain gradient theory together with the kinetic energy of the system considering mass eccentricity in the disk and rotary inertia and gyroscopic effects of the rotating shaft-disk system. By using the extended Hamilton's principle to obtain weak forms of governing equations and approximating displacement components by special interpolation functions which can be used to model a strain gradient based micro-beam, equations of the motion are discretized into a finite element form. The natural frequencies, critical speeds and the threshold of instability rotational speed of the micro-rotor are obtained by transforming discretized equations of motion into state space form. The response of the micro rotor under excitation of the mass eccentricity of the disk in forced vibrations is also presented. Numerical results show profound effects of higher order material constant on vibration characteristics of the micro-rotor.

1- Introduction

Micro-rotating machinery with compact energy sources and high power density has been developed in the recent decade to run portable electronic devices. To achieve efficiency targets, these miniaturized turbo-machinery must spin at extremely high rotational speeds which can reach up to one million revolutions per minute [1]. At this relatively high spinning rate, the rotordynamic behavior of these systems plays an important role on the stage of design. Since existing rotordynamic models are based on the classical continuum theory and this theory is incapable of appropriately predicting the mechanical behavior of the small-scale structures [2], non-classical continuum theories such as strain gradient theory have been proposed for vibration analysis of micro-rotors by Asghari and Hashemi [3].

Although analytical expressions for natural frequencies of micro-rotors are obtained by using Galerkin's method, the effects related to the flexibility of the bearings, internal damping and system response have not been discussed due to the limitations of the method used. As a result, in this paper by using finite element method, these vibration characteristics together with the small-size effect of micro-rotors are studied on the basis of strain gradient theory.

2- Problem Statement and Governing Equations

A micro-rotor, shown in Fig. 1 consists of a flexible and slender micro shaft and a rigid eccentric disk is mounted on two flexible bearings at both ends. The frame *X-Y-Z* is an inertial or fixed coordinate system, while frame *x-y-z* is a coordinate system which rotates about longitudinal axes at angular speed *Ω*.

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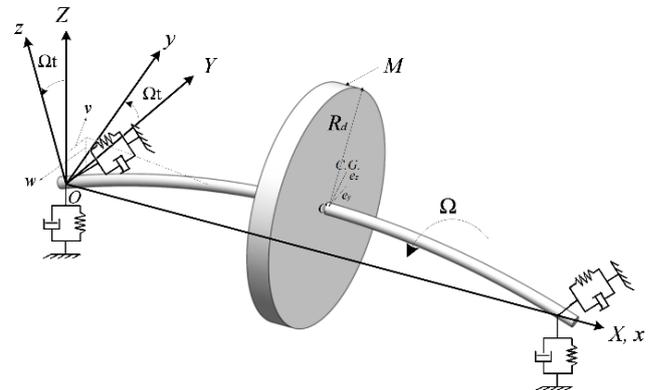
Figure 1. A micro-rotor mounted on flexible bearings and rotating with constant speed *Ω*

Using strain gradient theory with Euler-Bernoulli beam model, the total strain energy can be stated as:

$$U = \frac{1}{2}\int_0^L \left\{ \Lambda_1 I_s \left((\frac{\partial^3 v}{\partial x^3})^2 + (\frac{\partial^3 w}{\partial x^3})^2 \right) + \left((\Lambda_2 A_s + EI_s)\left((\frac{\partial^2 v}{\partial x^2})^2 + (\frac{\partial^2 w}{\partial x^2})^2 \right) \right) \right\} dx ,$$ (1)

After writing the total kinetic energy, as well as external work acting on the micro-rotor, Hamilton's principle can be considered as:

$$\int_{t_1}^{t_2} \left(\delta T - \delta U + \delta W \right) dt = 0,$$ (2)

Performing mathematical operations in accordance with the calculus of variations, we arrive at the governing equations for the lateral motion of a micro-rotor.

3- Solution Method

To solve the governing equation so as to find vibration characteristics of micro-rotor, finite element method will be used. To do so, weak form formulation is employed and displacement components are estimated at the node by interpolation function $\Theta_i(s)$ proposed by Kahrobaiyan et al. [7] as:

$$v(s,t) = \sum_{i=1}^{6} \Theta_i(s) q_{yi}(t),$$
$$w(s,t) = \sum_{i=1}^{6} \Theta_i(s) q_{zi}(t), \tag{3}$$

Following the procedure, the governing equations of the motion of the micro-rotor are discretized as

$$[M]\{\ddot{q}(t)\} + [C]\{\dot{q}(t)\} + [K]\{q(t)\} = \{f(t)\}, \tag{4}$$

To find resonant frequencies and information regarding the stability of the micro-system, Eq. (6) can be transformed into state space form as

$$\begin{bmatrix} [M] & 0 \\ 0 & [I] \end{bmatrix}\{\dot{Q}\} + \begin{bmatrix} [C] & [K] \\ -[I] & 0 \end{bmatrix}\{Q\} = \{0\}, \tag{5}$$

Moreover, the response of the micro-rotor under mass eccentricity can be obtained as:

$$\{q(t)\} = \left(-\Omega^2[M] + i\,\Omega[C] + [K]\right)^{-1}\{F\}\,e^{i\Omega t}. \tag{6}$$

4- Results and Discussion

The finite element codes according to the procedure described in this section are written in MATLAB software to find vibration characteristics of the micro-rotor. We assume a specific ratio among higher-order material constants as,

$$\{a_1, a_2, a_3, a_4, a_5\} = \{0.0393, 0.1323, -0.0338, 0.0320, 0.3302\}\Lambda_1$$

based on the result which can be obtained for the aluminum. Moreover, a dimensionless higher-order material constant is defined as $\eta = \Lambda_1/EI_s$. It should be noted that in the case of $\eta = 0$, the results will be reduced to those of the classical continuum theory.

The model is validated for convergence, as well as for accuracy with existing results in specific cases. In this regard, some of the natural frequencies of a micro-rotating shaft are compared with reference [5] and presented in Table 1.

Moreover, the variations of the first natural frequency of a micro-rotor for various values of η as a function of bearing stiffness κ is illustrated in Fig. 2. In addition, the deflection of the micro-rotor center as system response due to mass eccentricity in the disk for different values of η is also shown in Fig. 3.

Table 1. Comparison of resonant frequencies of the micro-spinning shaft by Hashemi and Asghari study [5]

Ω^*		0	10	20	30	
ω_1^*	Ref.	1.1226	1.1855	1.2518	1.3214	(FW)
			1.0629	1.0066	0.9536	(BW)
	Present study	1.1224	1.1847	1.2504	1.3192	(FW)
			1.0633	1.0075	0.9549	(BW)
ω_2^*	Ref.	4.4754	4.7226	4.9828	5.2557	(FW)
			4.2411	4.0196	3.8109	(BW)
	Present study	4.4727	4.7181	4.9762	5.2469	(FW)
			4.2401	4.0201	3.8128	(BW)
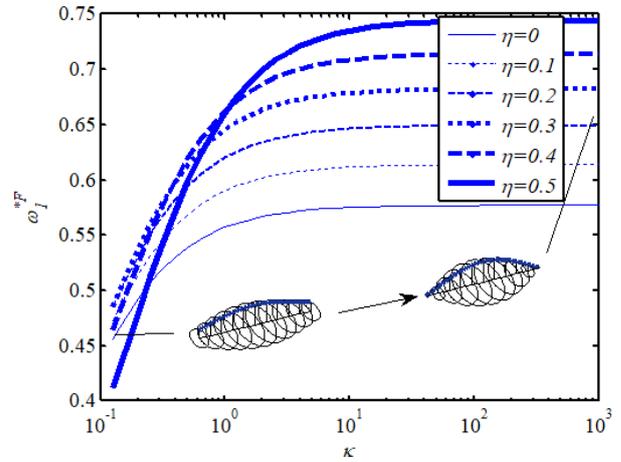
Figure 2. Variations of the first natural frequency of a micro-rotor versus bearing stiffness for various η.
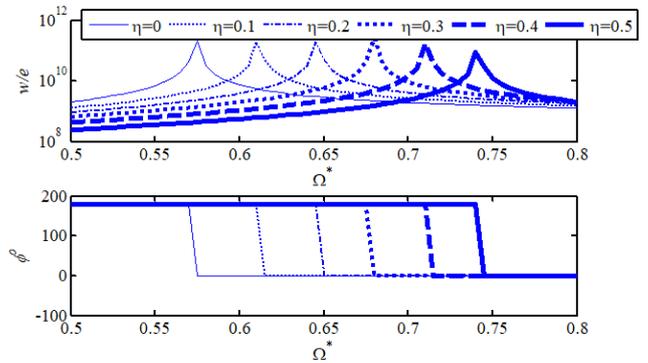
Figure 3. Bode plot of system response for different η

5- Conclusions

A three-dimensional finite element model is developed based on the strain gradient theory to investigate the vibration characteristics of micro-rotors. After validating the proposed method, numerical results from vibration analysis of the micro-rotor show that:

1. By increasing the higher order material constant, the natural frequencies and the critical speeds of the micro-rotor increase.
2. Regardless of the higher order material constant, odd and even natural frequencies of the micro-rotor decrease,

respectively by increasing mass and mass moment of inertia of the disk.

3. The resonant frequencies of the micro-rotor in forwarding whirling motion increase and in backward whirling motion decrease by increasing rotational speed.

4. By increasing the stiffness of the bearings, the resonant frequencies of the micro-rotor rise and the mode shapes deform to simply supported beam. This variation intensifies with an increase of higher order material constant.

5. Internal damping in the micro-rotor causes instability in the micro-rotor at a specific rotational speed known as the threshold of instability. By increasing higher order material constant, the threshold of instability rotational speed goes up.

6. Under mass eccentricity of the disk, the rotational speed, in which maximum and minimum amplitude of vibration occurs, increases with the increase of higher order material constant.

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