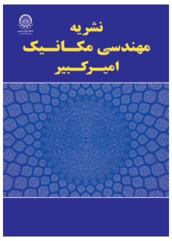

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Free Vibration Analysis of a Functionally Graded Cylindrical Nanoshell Surrounded by Elastic Foundation Based on the Modified Couple Stress Theory

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ABSTRACT: In this article, free vibration analysis of functionally graded cylindrical nanoshell on the basis of the modified couple stress theory is investigated. The nanoshell is embedded in an elastic Pasternak medium, which is obtained by adding a shear layer to the Winkler model. In addition, the boundary conditions at two ends of cylindrical nanoshell are simply supported. It is assumed that the functionally graded cylindrical nanoshell, is made of aluminum and ceramic, follows the volume fraction definition and law of mixtures, and its properties change as a power function through its thickness. Governing equations and boundary conditions are obtained by applying the Hamilton's principle and are based on first-order shear deformation. Navier solution is used for predicting the natural frequencies of functionally graded cylindrical nanoshell. Finally, the effect of parameters such as material length scale, circumferential wave number, the length to radius ratio, shear correction factor, power low index and elastic foundation coefficients of Winkler and Pasternak on natural frequency of functionally graded cylindrical nanoshell are identified. The results show, there is a very good agreement between the results of molecular dynamics simulations by previous researchers with the results of this study.

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1- Introduction

Functionally graded materials (FGMs) are composite materials which have many advantages and superior properties. The applications of functionally graded materials can be very broad. The FG materials can be used in activators [1] optical sensors [2] and other engineering fields. It is noted that , nonlocal theory of Eringen is one of the best and most well-known continuum mechanics theories that includes small scale effects with good accuracy in nano/micro scale devices but the results show that modified couple stress theory matches experimental results better than Eringen's nonlocal elasticity and classical theories [4]. Due to the increasing use of FG structures and the practical importance of the free vibration of the FG cylindrical nanoshells surrounded by elastic foundation using modified couple stress theory and also lack of information in the open literature in this regard, this problem is studied in the present work. The main purpose of the present work is to propose an analytical model to study the free vibration behavior of simply supported FG cylindrical nanoshell surrounded by elastic foundation using modified couple stress theory and solving by using the Navier procedure. The results show that material length scale parameter, FG power index (*N*), spring constant, shear correction factor, length of nanoshell and circumferential wave number play an important role in the natural frequency of the cylindrical nanoshell.

2- Solution Procedure

In order to solve the governing equations of MEE nanoshell with simply supported boundary condition, the Navier procedure is used assuming the substitutions as follows:

$$\begin{Bmatrix} u(x,\theta,t) \\ v(x,\theta,t) \\ w(x,\theta,t) \\ \psi_x(x,\theta,t) \\ \psi_\theta(x,\theta,t) \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\cos(\frac{m\pi}{L}x)\cos(n\theta)e^{i\omega t} \\ V_{mn}\sin(\frac{m\pi}{L}x)\sin(n\theta)e^{i\omega t} \\ W_{mn}\sin(\frac{m\pi}{L}x)\cos(n\theta)e^{i\omega t} \\ \Psi_{x\,mn}\cos(\frac{m\pi}{L}x)\cos(n\theta)e^{i\omega t} \\ \Psi_{\theta mn}\sin(\frac{m\pi}{L}x)\sin(n\theta)e^{i\omega t} \end{Bmatrix} \tag{1}$$

Where n and m are the circumferential and axial wave numbers, respectively. Now, by substituting Eq. (1) into governing equations, the motion equations are written as a matrix form

$$\{[k]-\omega^2[M]\}\{d\}=0 \tag{2}$$

Where $\{d\}$ is the displacement amplitude vector and ω is the natural frequency of FG cylindrical nanoshell. In Eq. (2) $[M]$ is the mass matrix and also $[k]$ is the stiffness matrix. Finally, by setting the determinant of the coefficient matrix equal to zero, we can find the natural frequencies of FG cylindrical nanoshell.

3- Results

According to the latest studies conducted by the writers, no experiments have been done according to the FG cylindrical nanoshell model. So, for validation of results, the model presented in this study can be validated with the results received from molecular dynamic (MD) simulation

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[5] by setting N=0. The results in Table 1 show that using the modified couple stress theory by considering the value of a material length scale parameter as $R/3$, results have a maximum error of 1.55% compared to the MD simulation results. So the material length-scale parameters value of modified couple stress theory is selected as $l = R/3$ to acquire the best concordance with the results of MD simulation.

Table 1. Comparison of natural frequencies (THz) of non-classical nanoshell with results of MD simulation in different $L/2R$ ratios.

$L/2R$	MD simulation [5]	Modified couple stress theory [l=R/3]	Error (%)
8.3	0.5299	0.5308	0.17
10.1	0.3618	0.3634	0.44
13.7	0.1936	0.1901	1.55
17.3	0.11.3	.0.11.5	0.18
20.9	0.0724	0.0729	0.69
24.5	0.0519	0.0511	0.15
28.1	0.0425	0.0422	0.70

From the results of Table 2 it can be concluded that by increasing FG power index, the natural frequency increases. This is because, by increasing the FG power index, the stiffness of nanoshell tends to increase and eventually causes an increase in nondimensional natural frequency. Also, from the results of Table 2, it is observed that, by increasing the material length scale parameter, the natural frequency increases.

Table 3 shows, by increasing the hardness values of elastic foundation, the natural frequency tends to increase. Also it is clear, the effect of hardness value of Pasternak elastic foundation on natural frequency of FG cylindrical nanoshell is more than the hardness value of Winkler elastic foundation. Table 4 shows by increasing the shear correction factor the natural frequency tends to increase. From Table 4 it is observed that, in classical theory, the shear correction factor has minimal effect on natural frequency in comparison with modified couple stress theory.

Table 2. Variations of dimensionless natural frequency of FG nanoshell surrounded by elastic foundation with change in FG power index and material length scale parameter.

	Classical ($l = 0$)	$l = R/4$	$l = R/3$	$l = R/2$
Metal	0.2811	0.29081	0.29724	0.3119
N=20.	0.3825	0.39557	0.40428	0.4242
N=2	0.5871	0.60729	0.62068	0.6512
Ceramic	0.6861	0.70975	0.72550	0.7614

Table 3. Variations of dimensionless natural frequency of FG nanoshell surrounded by elastic foundation with changes in elastic foundation hardness values, circumferential wave number and material length scale parameter.

	$l = 0$ $k_w = 0$ $k_p = 10^{13}$	$l = 0$ $k_w = 10^{13}$ $k_p = 0$	$l = R/3$ $k_w = 0$ $k_p = 10^{13}$	$l = R/3$ $k_w = 10^{13}$ $k_p = 0$
n=1	0.529049	0.049063	0.559808	0.052037
n=2	1.022134	0.029717	1.129045	0.428181
n=3	1.522988	0.066770	1.777270	1.155771
n=4	2.025849	0.126049	2.527951	2.119794
n=5	3.033577	0.202959	3.400754	3.219520
n=6	3.537864	0.296999	4.409878	4.308180
n=7	4.042286	0.407991	5.565107	5.301731

Table 4. Variations of dimensionless natural frequency of FG nanoshell surrounded by elastic foundation with changes in shear correction (SC) factor and material length scale parameter.

SC Factor (K_s)	Classical ($l = 0$)	$l = R/4$	$l = R/3$	$l = R/2$
5/1000	0.522788	0.52804	0.52856	0.52993
5/100	0.528429	0.53073	0.53139	0.53287
5/10	0.528990	0.54047	0.54507	0.55214
5/6	0.529045	0.54712	0.55916	0.58670
1	0.529046	0.54736	0.55980	0.58892

4- Conclusions

This paper presented an analysis of size-dependent vibration of FG cylindrical nanoshell. In addition, the medium of surrounding elastic has been explained as the model of Winkler and Pasternak characterized by the spring. Modified couple stress theory was used to consider the size-dependent effect. Using FSDT and Hamilton's principle, the governing equations of simply supported FG cylindrical nanoshell were derived. The results show that the material length scale parameter, shear correction factor, FG power index (N), circumferential (n) wave number, constant of spring and length play an important role on the natural frequency of FG cylindrical nanoshell. The importance results of this article can be expressed as follows:

1. The results show an increase in the length and FG power index (N), culminating in an increase in stiffness and thus, an increase in the natural frequency of cylindrical nanoshell.
2. The results show that the shear correction factor in size-dependent cylindrical nanoshell has an important role in natural frequency.
3. By increasing the spring constant the natural frequency tends to increase but the effect of hardness value of Pasternak elastic foundation on natural frequency of FG cylindrical nanoshell is more than hardness value of Winkler elastic foundation.

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