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Semi-Analytic Solution of Nanofluid and Magnetic Field Effects on Heat Transfer from a Porous Wall

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ABSTRACT: The present paper studies effect of nanofluids on natural convection in an assumed non-Darcy, porous medium on vertical wall embedded with MHD field. The wall temperature and free stream temperature are assumed constant. Base fluid is water that contains Al_2O_3, CuO and Cu nanoparticles. It is assumed that the flow is laminar and in no slip and thermal equilibrium condition. Partial differential equations are transformed to ordinary differential equations that are solved by Differential Transform Method (DTM). The current semi-analytic solution is compared with 4th order Runge-Kutta results and a good agreement is achieved. The obtained results show that adding nanoparticles to water, increases the heat transfer amount that is also increased with nanoparticle concentration. On the other hand, increasing the magnetic field decreases the heat transfer coefficient.

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1- Introduction

It is shown that adding solid nanoparticles to pure fluids increases effective thermal conductivity of the mixture. Many researchers have been attracted to study the nanofluid heat transfer in different geometries. Rosca et al. [1] studied a non-Darcy mixed convection from a horizontal plate embedded in a nanofluid saturated porous media.

In this paper, the porous medium is assumed to be isotropic, homogeneous and non-Darcy flow and all thermodynamic properties are considered to be constant. The effect of various parameters such as volume fraction φ, mass flux parameter f_w, Grashof number and magnetic parameters on the fluid flow and heat transfer parameters are evaluated by investigating the nondimensional velocity and temperature curves. Finally, the Nusselt number is obtained for different conditions.

2- Governing equations

According to Fig. 1, wall surface temperature (T_w) is constant and the plate is located in the vicinity of a fluid with ambient temperature (T_∞).

The boundary layer equations near the flat vertical surface according to Kishan et al. study [2] are as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u+\frac{c\sqrt{K}}{\upsilon}u^2=\frac{K}{\mu}\left(\frac{\partial p}{\partial x}+\rho g\right)-\frac{\sigma B_0^2 K}{\mu}u \tag{2}$$

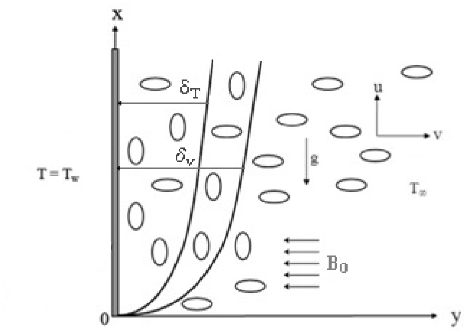

Figure 1. Schematic view of the present problem

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{3}$$

Boundary conditions include:

$$y=0: v=v_w, T=T_w \tag{4}$$

$$y\to\infty: u=0, T=T_\infty \tag{5}$$

Where in σ is the electrical conductivity, B_0 the coefficient of magnetic field, μ the fluid viscosity, υ the dynamic viscosity of the fluid, K the permeability, g the gravity acceleration, c the experimental constant, α the thermal diffusivity coefficient, and $\upsilon_w = Ex^{-1/2}$ in which E is the real constant.

2- 1- Thermo-physical properties of nanofluids

Density, dynamic viscosity and heat capacity at constant pressure for nanofluid are obtained as follows [3]:

$$\rho_{nf}=\rho_f(1-\varphi)+\rho_s\varphi \tag{6}$$

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$$\mu_{nf} = \frac{\mu_f}{(1-\varphi)^{2.5}} \tag{7}$$

$$(\rho C_p)_{nf} = (\rho C_p)_f (1-\varphi) + (\rho C_p)_s \varphi \tag{8}$$

Finally, effective thermal conductivity is obtained by the Patel et al. model [3]:

$$\frac{k_{eff}}{k_f} = \left[1 + \frac{k_s A_s}{k_f A_f} + c k_s Pe \frac{A_s}{k_f A_f}\right] \tag{9}$$

in which

$$\frac{A_s}{A_f} = \frac{d_f}{d_s}\frac{\varphi}{1-\varphi} \tag{10}$$

$$Pe = \frac{u_s d_s}{\alpha_f} \tag{11}$$

C is constant and equals to 25000 [3], d_f is diameter of the base fluid molecules which is considered 0.278 nm, d_s is diameter of the nanoparticles, K_b Boltzmann constant and u_s is Brownian motion velocity of nanoparticles which is obtained as follows [3]:

$$u_s = \frac{2 k_b T}{\pi \mu_f d_s^2} \tag{12}$$

2-2-Nondimensional equations

The following variables are used for converting the governing equations into dimensionless forms [4]:

$$\eta = \frac{y}{x} Ra_x^{\frac{1}{2}} \qquad u = \frac{\alpha}{x} Ra_x f'(\eta)$$

$$v = -\frac{\alpha}{2x} Ra_x^{\frac{1}{2}} (f - \eta f') \qquad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty} \tag{13}$$

$$Ha^2 = 1 + \frac{\sigma B_0^2 K}{\rho \upsilon} \tag{14}$$

then the governing equations can be written as follows:

$$f''\left[1 + Ha^2 (1-\varphi)^{2.5}\right] + 2Grff'' = \left[(1-\varphi) + \frac{\rho_s \beta_s}{\rho_f \beta_f}\varphi\right](1-\varphi)^{2.5}\theta' \tag{15}$$

$$\theta'' + \left[\frac{\left(\frac{1-\varphi}{2}\right) + \frac{\rho_s c_{p_s}}{2\rho_f c_{p_f}}\varphi}{\left[1 + \frac{k_s A_s}{k_f A_f} + c k_s Pe \frac{A_S}{k_f A_f}\right]}\right] f\theta' = 0 \tag{16}$$

Where Gr is Grashof number [4]:

$$Gr = \frac{c\sqrt{K} K g \beta_T (T_w - T_\infty)}{\upsilon^2} \tag{17}$$

New boundary conditions are:

$$f(0) = f_w \qquad \theta(0) = 1 \tag{18}$$

$$f'(\infty) = 0 \qquad \theta(\infty) = 0 \tag{19}$$

f_w, mass flow rate parameter is obtained as follow [4]:

$$f_w = -\frac{2E}{\sqrt{\alpha K g \beta_T (T_w - T_\infty)}} \tag{20}$$

Dimensionless heat transfer coefficient can be written as follows:

$$\frac{Nu}{Ra_x^{1/2}} = -\frac{k_{nf}}{k_f}\theta'(0) \tag{21}$$

2- 2- Semi-analytical solution with DTM

The Taylor series expansion for function *x(t)* in the *D* interval is as follows [5]:

$$x(t) = \sum_{k=0}^{\infty} \frac{(t-t_i)^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=t_i} \qquad \forall t \in D \tag{22}$$

Now, nondimensional equations in the previous section will be analyzed in the DTM. To do this:

$$f(\eta) \to F(k) \qquad \theta(\eta) \to T(k) \tag{23}$$

Boundary conditions are also considered to be as follows:

$$\begin{aligned} &F(0) = f_w \qquad F(1) = \alpha \\ &T(0) = 1 \qquad T(1) = \beta \end{aligned} \tag{24}$$

3- Results and Discussion

Fig. 2 shows Nusselt number for Cu-water nanofluid at different *Ha* and concentrations. According to the figure, Nusselt number increases with concentration and decreases with *Ha* due to volumetric Lorentz forces.

Fig. 3 depicts the Nusselt number with *Ha* for different nanofluids. It can be seen that the maximum Nusselt number belongs to the Cu-water nanofluid at the simulated conditions. For Cu-water nanofluid, changing the *Ha* from 0 to 3, decreases the $Nu.Ra^{-0.5}$ from 0.21 to 0.068.

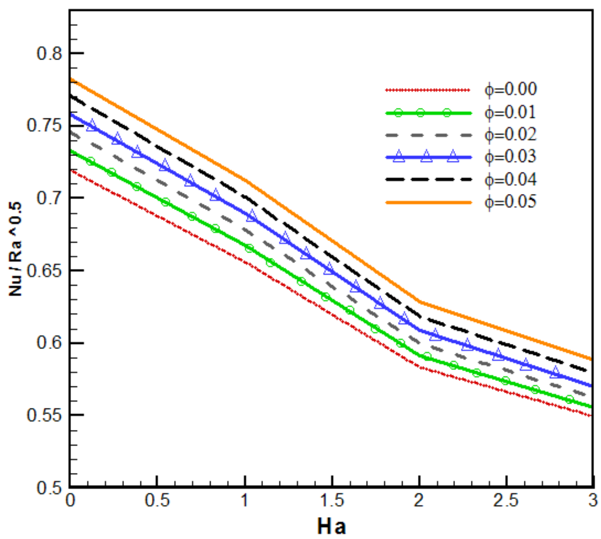

Figure 2. Nusselt number for Cu-water at different *Ha* and concentrations

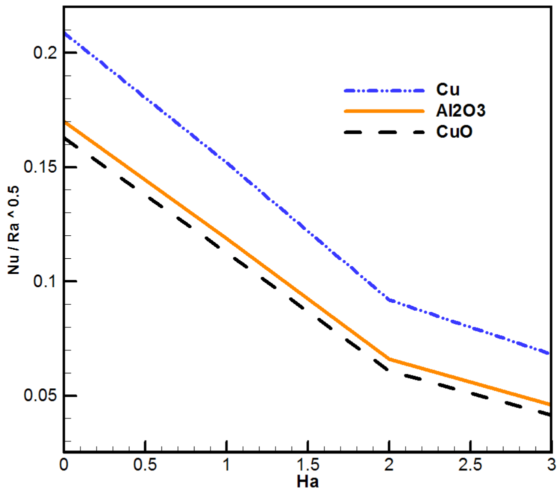

Figure 3. Nusselt number versus *Ha* for different nanofluids

4- Conclusion

Comparing the present method (DTM) results with the fourth order Runge-Kutta method shows that DTM is an efficient method to solve the current problem. Using this method, heat transfer and fluid flow characteristics of the nanofluid flow for different Ha, concentration and mass flowrate parameters are obtained and discussed.

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