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Free Vibration of Carbon Nanotube and Boron Nitride Nanotube Double-bonded Modified Couple Stress Theory Timoshenko Micro Beams Under Various Physical Fields

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ABSTRACT: In this article, free vibration analysis of double-bonded Timoshenko micro beams rested in an elastic foundation under various physical fields is investigated based on modified couple stress theory. Properties and distribution of carbon and boron nitride nanotubes are used based on experimental and analytical equations, which They have a lower error and do not use in other studies. The governing equations of motions are derived based on Hamilton's principle. The effects of various parameters such as electric field, magnetic field, material length scale parameter and elastic foundation modulus on the natural frequency of the micro structures are studied. The results of this work show that different physical fields on the microbeams have more influence on the dimensionless natural frequencies, so the effect of the CNTs on the natural frequencies for micro beams is more than the BNNTs. Moreover, if the double-bonded micro beams are considered simultaneously as CNT and BNNT, the increase of natural frequency is less than when two micro-beams become only as CNT or BNNT. Also, it is shown that the effect of elastic foundation is more important than the electric and magnetic fields as well as material length scale parameter on the natural frequency of microbeams.

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1- Introduction

In last decades, by great progresses in science and engineering, there are more needs to optimize structures based on nanotechnology than ever. One of the major objectives in this field is increasing the facilities of spacecraft and decreasing the weight of space systems. Making light, strong and spacecrafts resistant to space radiation has become possible with this material. Nano sensors, with high-performance materials improved and highly efficient systems are only a sample of nanotechnology applications. In this paper, free vibration analysis of double-bonded Timoshenko micro beams based on modified couple stress theory (MCST) under various physical fields is investigated.

2- Methodology and Solution Method

A two-phase double-bonded micro beams of length L, width b, and thickness h is shown in Fig. 1.

In this work, the Timoshenko micro beams theory are used. According to this theory, the axial and transverse displacements are given as follows [3]:

$$u_1(x,t)=z\,\phi(x,t)\,,\,u_3(x,t)=w\,(x,t) \tag{1}$$

Based on MCST, the strain energy is a function of both strain and curvature tensors. Then, the first variation of the strain energy in a deformed isotropic linear elastic body occupying a volume ∀ can be written as follows:

$$\delta U=b\int_0^L\int_{-\frac{h}{2}}^{\frac{h}{2}}\begin{pmatrix}\sigma_{11}\delta\varepsilon_{11}+\sigma_{13}\delta\varepsilon_{13}+\sigma_{31}\delta\varepsilon_{31}\\+m_{12}\delta\chi_{12}+m_{21}\delta\chi_{21}\end{pmatrix}dzdx \tag{2}$$

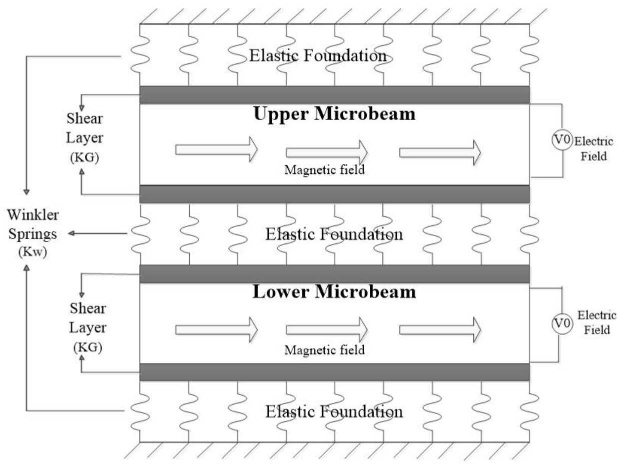

Figure 1. Geometry of a double-bonded Timoshenko micro beams reinforced by CNTs and BNNTs

where σ, ε, m and χ are the stress, strain, deviatory part of couple stress, and the symmetric curvature tensor, respectively. Also, the first variation of the Kinetic energy on the time interval is described as follows:

$$\delta T=b\int_0^L\int_{-\frac{h}{2}}^{\frac{h}{2}}\rho\left(z\frac{\partial\phi}{\partial t}\delta(z\frac{\partial\phi}{\partial t})+\frac{\partial w}{\partial t}\delta(\frac{\partial w}{\partial t})\right)dzdx \tag{3}$$

By using the strain density and kinetic equations, the equation of motion of double-bonded micro beams reinforces by CNTs and BNNTs in presence of different physical fields are obtained as follows:

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$$\begin{aligned}&-((EI)_{eq}+2S_1(EQI)_{eq}+2S_2(EPI)_{eq}+\frac{1}{4}(a_2^{'}A)_{eq})\frac{\partial^2\phi_1}{\partial x^2}\\&+\left(\frac{1}{2}k(\mu A)_{eq}\right)\left(\phi_1+\frac{\partial w_1}{\partial x}\right)+\left(\frac{1}{4}(a_2^{'}A)_{eq}\right)\frac{\partial^3 w_1}{\partial x^3}\\&+(\rho I)_{eq}\frac{\partial^2\phi_1}{\partial t^2}=0\end{aligned}\tag{4-a}$$

$$\begin{aligned}&\left(-\frac{1}{4}(a_2^{'}A)_{eq}\right)\frac{\partial^3\phi_1}{\partial x^3}-\left(\frac{1}{2}k(\mu A)_{eq}\right)\frac{\partial\phi_1}{\partial x}+\left(\frac{1}{4}(a_2^{'}A)_{eq}\right)\frac{\partial^4 w_1}{\partial x^4}\\&-\left(\frac{1}{2}k(\mu A)_{eq}\right)\frac{\partial^2 w_1}{\partial x^2}+k_w(2w_1-w_2)+k_G\left(\frac{\partial^2 w_2}{\partial x^2}-2\frac{\partial^2 w_1}{\partial x^2}\right)\\&+(\rho A)_{eq}\frac{\partial^2 w_1}{\partial t^2}=0\end{aligned}\tag{4-b}$$

$$\begin{aligned}&-((EI)_{eq}+2S_3(EQI)_{eq}+2S_4(EPI)_{eq}+\frac{1}{4}(a_2^{'}A)_{eq})\frac{\partial^2\phi_2}{\partial x^2}\\&+\left(\frac{1}{2}k(\mu A)_{eq}\right)\left(\phi_2+\frac{\partial w_2}{\partial x}\right)+\left(\frac{1}{4}(a_2^{'}A)_{eq}\right)\frac{\partial^3 w_2}{\partial x^3}\\&+(\rho I)_{eq}\frac{\partial^2\phi_2}{\partial t^2}=0\end{aligned}\tag{4-c}$$

$$\begin{aligned}&\left(-\frac{1}{4}(a_2^{'}A)_{eq}\right)\frac{\partial^3\phi_2}{\partial x^3}-\left(\frac{1}{2}k(\mu A)_{eq}\right)\frac{\partial\phi_2}{\partial x}+\left(\frac{1}{4}(a_2^{'}A)_{eq}\right)\frac{\partial^4 w_2}{\partial x^4}\\&-\left(\frac{1}{2}k(\mu A)_{eq}\right)\frac{\partial^2 w_2}{\partial x^2}+k_w(2w_2-w_1)+k_G\left(\frac{\partial^2 w_1}{\partial x^2}-2\frac{\partial^2 w_2}{\partial x^2}\right)\\&+(\rho A)_{eq}\frac{\partial^2 w_2}{\partial t^2}=0\end{aligned}\tag{4-d}$$

By defining the governing equations of motions, the Navier's solution procedure is employed to obtain the analytical solution. For this purpose, the displacement functions are expressed as a product of undetermined coefficients and known trigonometric functions in a way that satisfies the governing equations. The following expansions of the displacement field are assumed [2]:

$$\begin{cases}(w_1,w_2)=\sum_{m=1}^{\infty}(B_m,D_m)\sin(\frac{m\pi x}{L})e^{i\omega t}\\(\phi_1,\phi_2)=\sum_{m=1}^{\infty}(A_m,C_m)\cos(\frac{m\pi x}{L})e^{i\omega t}\end{cases}\tag{5}$$

3- Results and Discussion

In this work, it is assumed that the micro beams are reinforced by CNTs and BNNTs that their properties are used based on experimental and analytical equations [1]:

$$\begin{aligned}E&=307.26\times T^{0.84}\upsilon^{1.54}\varsigma^3+2.577\times T^{0.64}\upsilon^{1.36}\varsigma^2\\&+7.786\times T^{0.61}\upsilon^{1.22}\varsigma+1.325\times T^{-0.28}\upsilon^{1.18}\end{aligned}\tag{6}$$

Also, according to Table 1 the electric and magnetic fields are applied on the both of micro beams that the effects of both fields are compared in this work.

Fig. 2. shows the effects of CNTs and BNNTS on the natural frequencies of Timoshenko micro beams. it is observed that if

Table 1. Applying the physical fields on the Timoshenko micro beams reinforced by CNTs and BNNTs

	S_1	S_2	S_3	S_4
Both of upper and lower micro beams in presence of electric field	1	0	1	0
Upper and lower in presence of electric and magnetic field respectively	1	0	0	1
Upper and lower in presence of magnetic and electric field respectively	0	1	1	0
Both of upper and lower micro beams in presence of magnetic field	0	1	0	1

the micro beams are considered by CNT with the presence of magnetic fields, increasing the natural frequencies are more than when the micro beams are considered by the BNNTs with the presence of electric field. Moreover, if the double-bonded micro beams are reinforced simultaneously by CNT and BNNT under the electric and magnetic fields, the increasing of natural frequency is less than when two micro beams reinforced only by CNTs or BNNTs. The effect of elastic foundation on the free vibration behavior of double-bonded Timoshenko micro beams reinforced by CNTs and BNNTs is depicted in Fig. 3. It can be seen that the maximum natural frequency occurred in presence of Winkler and Pasternak foundation. If the Pasternak shear modulus does not existence, natural frequencies reduce drastically.

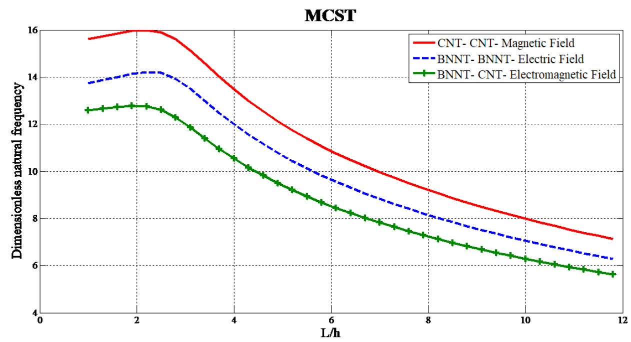

Figure 2. The effect of nanotubes properties and different fields on the natural frequencies

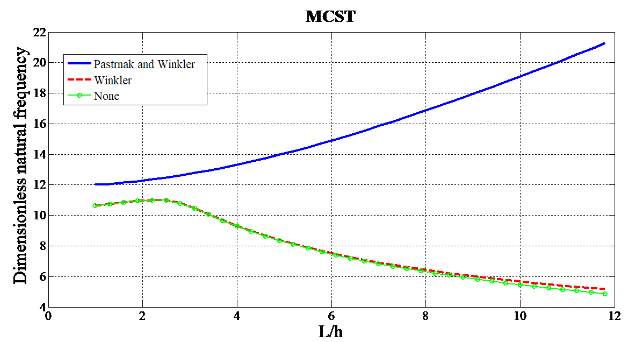

Figure 3. Effect of elastic coefficient on the natural frequencies

4- Conclusions

In this article, the free vibration analysis of double-bonded Timoshenko micro beams in an elastic medium was studied based MCST. The obtained results can be summarized as follows:

1. Applying the electric and magnetic fields on the micro beams had a huge effect on increasing rigidity of micro structures and thus natural frequency increases.
2. The effect of CNTs on the natural frequency of micro beams is more than when they are reinforced by only BNNTs or reinforced by CNTs and BBNTS in the presence of electric and magnetic fields.
3. The effect of elastic foundation is more important than the electric and magnetic fields as well as material length scale parameter on the natural frequencies of double-bonded Timoshenko micro beams.

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