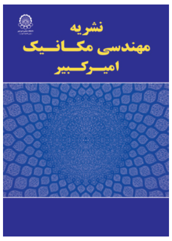

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Whirling and Stability Analysis of FG-rotors with Variable Diameter Subjected to Axial Load and Torsional Torque

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ABSTRACT: In this paper, whirling and stability analyses of a rotor made of functionally graded materials are investigated. The rotor is modeled based on Timoshenko beam theory and gyroscopic effects are considered. Diameter and mechanical properties of the rotor are considered to be variable in longitudinal direction and the rotor is considered to be subjected to axial load and torsional torque. In order to generalization of the modeling of bearings, each of them is replaced with four springs; two translational and two rotational acting on two perpendicular directions. Using Newton's second law, the set of governing equations and external boundary conditions are derived and solved numerically using differential quadrature method (DQM). Convergence and accuracy of the presented solution are confirmed and effect of various parameters including power index, angular velocity of spin, value and sign of applied axial load and torsional torque on the forward and backward frequencies and stability of the rotor are investigated. Numerical results show that all forward and backward frequencies and therefore critical speeds decrease by increase in power law, applying axial pressure load and torsional torque and increase by applying axial tension force.

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1- Introduction

Natural frequencies, critical speeds and mode shapes of rotating shafts are important aspects in design of rotors. In order to achieve a more uniform stress distribution and decreases the total weight of rotors, rotors with non-uniform diameters are more interesting in comparison with those with constant diameter. Also, as thermal loads can be supported more easily by ceramics rather than metals, FG-rotors can be considered as a new option for high temperature conditions.

Some practical applications of rotor dynamics can be listed as rotating shafts, turbines and aerospace devices and many papers are presented regarding different aspects of rotor dynamics. Grybos [1] investigated the effect of shear deformation and rotary inertia of a rotor on its critical speeds. Choi et al. [2] presented the consistent derivation of a set of governing differential equations describing the vibration in two orthogonal planes and the torsional vibration of a straight rotor with dissimilar lateral principal moments of inertia, subjected to a constant compressive axial load. Free vibration analysis of a rotating shaft under a constant torsional torque was investigated by Jun and Kim [3]. They modeled rotor as a Timoshenko beam and gyroscopic effect and torque applied at each part of the shaft were considered. Behzad and Bastami [4] investigated the effect of shaft rotation on its natural frequency. They studied natural frequencies by considering the gyroscopic effect, axial force originated from centrifugal force and Poisson effect. Vibration analysis of an in-extensional simply supported rotating shaft with nonlinear curvature and inertia was presented by Hosseini and Khadem [5]. In their research rotary inertia and gyroscopic effects were considered, but shear deformation was neglected. Afshari et al. [6] used differential quadrature element method and presented a numerical solution for whirling analysis of a multi-step rotor with desired number of interior supports. Using DQM, a numerical solution for longitudinal-torsional and two plane transvers vibration analysis of composite rotors was presented by Irani et al. [7]. Torabi et al. [8] used transfer matrix method and presented an exact solution for whirling analysis of multi-step rotors carrying the desired number of discs. Torabi and Afshari [9] derived basic functions for whirling analysis of Timoshenko rotor and presented an exact closed form solution for whirling analysis of axial-loaded rotors.

In this paper, DQM is hired and whirling and stability analysis of a longitudinally graded Timoshenko rotor subjected to an axial load and a torsional torque are investigated for general boundary conditions. Diameter of the rotor is considered to vary in longitudinal direction according to an arbitrary function.

2- Governing Equations

As depicted in Fig. 1, an FG-rotor of length L, diameter d, rotating at constant angular velocity Ω is considered. The rotor is imposed to constant axial force P and torsional torque T and its properties varies in longitudinal direction in a power law function as

$$E(z) = E_m + (E_c - E_m)\left(\frac{z}{L}\right)^p$$
$$\rho(z) = \rho_m + (\rho_c - \rho_m)\left(\frac{z}{L}\right)^p \tag{1}$$

or in an exponential form as

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$$E(z)=E_m \exp\left[\ln\left(\frac{E_c}{E_m}\right)\left(\frac{z}{L}\right)^p\right]$$
$$\rho(z)=\rho_m \exp\left[\ln\left(\frac{\rho_c}{\rho_m}\right)\left(\frac{z}{L}\right)^p\right] \tag{2}$$

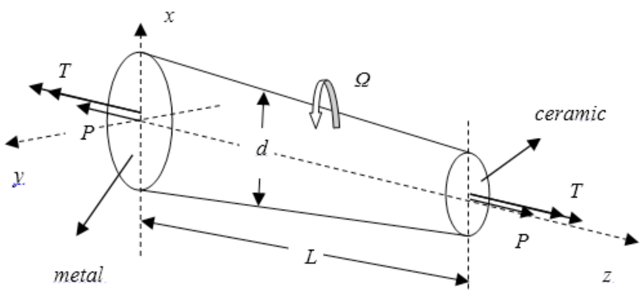

Figure 1. FG-rotor with variable diameter.

where E and ρ are modulus of elasticity and density, respectively and subscripts m and c indicate to properties of metal and ceramic, respectively.
Using following dimensionless parameters:

$$\zeta=\frac{z}{L} \quad \mu_E=\frac{E_c}{E_m} \quad \mu_\rho=\frac{\rho_c}{\rho_m} \quad E^*(\zeta)=\frac{E}{E_m} \quad \rho^*(\zeta)=\frac{\rho}{\rho_m}, \tag{3}$$

Eq. (1) can be written as

$$E^*(\zeta)=1+(\mu_E-1)\zeta^p \quad \rho^*(\zeta)=1+(\mu_\rho-1)\zeta^p \tag{4}$$

And Eq. (2) can be stated as follows:

$$E^*(\zeta)=\frac{E}{E_m}=\exp\left[\ln(\mu_E)\zeta^p\right]$$
$$\rho^*(\zeta)=\frac{\rho}{\rho_m}=\exp\left[\ln(\mu_\rho)\zeta^p\right] \tag{5}$$

The set of governing equation for whirling analysis of the rotor can be written as

$$\frac{\partial}{\partial z}\left[kGA\left(\frac{\partial u_x}{\partial z}-\varphi_y\right)\right]+P\frac{\partial^2 u_x}{\partial z^2}-\rho A\frac{\partial^2 u_x}{\partial t^2}=0 \tag{6-a}$$

$$\frac{\partial}{\partial z}\left[kGA\left(\frac{\partial u_y}{\partial z}+\varphi_x\right)\right]+P\frac{\partial^2 u_y}{\partial z^2}-\rho A\frac{\partial^2 u_y}{\partial t^2}=0 \tag{6-b}$$

$$\frac{\partial}{\partial z}\left(EI\frac{\partial \varphi_x}{\partial z}\right)+T\frac{\partial \varphi_y}{\partial z}-kGA\left(\frac{\partial u_y}{\partial z}+\varphi_x\right)$$
$$-2\rho I\Omega\frac{\partial \varphi_y}{\partial t}-\rho I\frac{\partial^2 \varphi_x}{\partial t^2}=0 \tag{6-c}$$

$$\frac{\partial}{\partial z}\left(EI\frac{\partial \varphi_y}{\partial z}\right)-T\frac{\partial \varphi_x}{\partial z}+kGA\left(\frac{\partial u_x}{\partial z}-\varphi_y\right)$$
$$+2\rho I\Omega\frac{\partial \varphi_x}{\partial t}-\rho I\frac{\partial^2 \varphi_y}{\partial t^2}=0 \tag{6-d}$$

in which $u_x(z,t)$, $u_y(z,t)$, $\varphi_x(z,t)$ and $\varphi_y(z,t)$ are components of displacement and rotation in x and y directions, respectively. A and I are cross-section area and moment of inertia about the x or y axes, respectively and k is shear correction factor. Also, boundary conditions can be considered as

$$z=0\quad\begin{cases} kGA\left(\frac{\partial u_x}{\partial z}-\varphi_y\right)+P\frac{\partial u_x}{\partial z}-\bar{K}_{lx}^{(L)}u_x=0\\ kGA\left(\frac{\partial u_y}{\partial z}+\varphi_x\right)+P\frac{\partial u_y}{\partial z}-\bar{K}_{ly}^{(L)}u_y=0\\ EI_x\frac{\partial \varphi_x}{\partial z}+T\varphi_y-\bar{K}_{rx}^{(L)}\varphi_x=0\\ EI_y\frac{\partial \varphi_y}{\partial z}-T\varphi_x-\bar{K}_{ry}^{(L)}\varphi_y=0\end{cases}$$
$$z=L\quad\begin{cases} kGA\left(\frac{\partial u_x}{\partial z}-\varphi_y\right)+P\frac{\partial u_x}{\partial z}+\bar{K}_{lx}^{(R)}u_x=0\\ kGA\left(\frac{\partial u_y}{\partial z}+\varphi_x\right)+P\frac{\partial u_y}{\partial z}+\bar{K}_{ly}^{(R)}u_y=0\\ EI_x\frac{\partial \varphi_x}{\partial z}+T\varphi_y+\bar{K}_{rx}^{(R)}\varphi_x=0\\ EI_y\frac{\partial \varphi_y}{\partial z}-T\varphi_x+\bar{K}_{ry}^{(R)}\varphi_y=0\end{cases} \tag{7}$$

where K shows the stiffness's of the bearings; subscript "l" is used to translational springs and subscript "r" is used for rotational ones. Also superscript "L" is used to left boundary (z=0) and superscript "R" is used for right one (z=L).
Using the method of separation of variables as

$$\{u_x \quad u_y \quad \varphi_x \quad \varphi_y\}=\{LU(z) \quad LV(z) \quad \Theta(z) \quad \Psi(z)\} \tag{8}$$

in which ω is angular natural frequency of vibration, and also using following dimensionless parameters:

$$A^*=\frac{A}{A_0} \qquad I^*=\frac{I}{I_0}$$
$$P^*=\frac{PL^2}{E_mI_0} \qquad T^*=\frac{TL}{E_mI_0}$$
$$r^2=\frac{I_0}{A_0L^2} \qquad s^2=\frac{E_mI_0}{kG_mA_0L^2}$$
$$\gamma^2=\frac{\rho_mA_0L^4\Omega^2}{E_mI_0} \qquad \lambda^2=\frac{\rho_mA_0L^4\omega^2}{E_mI_0} \tag{9}$$
$$\begin{Bmatrix}K_{lx}^{(L)}\\K_{lx}^{(R)}\end{Bmatrix}=\frac{L}{kA_0G_m}\begin{Bmatrix}\bar{K}_{lx}^{(L)}\\\bar{K}_{lx}^{(R)}\end{Bmatrix} \qquad \begin{Bmatrix}K_{ly}^{(L)}\\K_{ly}^{(R)}\end{Bmatrix}=\frac{L}{kA_0G_m}\begin{Bmatrix}\bar{K}_{ly}^{(L)}\\\bar{K}_{ly}^{(R)}\end{Bmatrix}$$
$$\begin{Bmatrix}K_{rx}^{(L)}\\K_{rx}^{(R)}\end{Bmatrix}=\frac{L}{E_mI_0}\begin{Bmatrix}\bar{K}_{rx}^{(L)}\\\bar{K}_{rx}^{(R)}\end{Bmatrix} \qquad \begin{Bmatrix}K_{ry}^{(L)}\\K_{ry}^{(R)}\end{Bmatrix}=\frac{L}{E_mI_0}\begin{Bmatrix}\bar{K}_{ry}^{(L)}\\\bar{K}_{ry}^{(R)}\end{Bmatrix}$$

the set of governing equations (6) can be written in the following dimensionless form:

$$E^*A^*(U''-\Psi')+(E^*A^*)'(U'-\Psi)$$
$$+s^2P^*U''-\lambda^2s^2\rho^*A^*U=0 \tag{10-a}$$

$$E^*A^*\left(V''+\Theta'\right)+\left(E^*A^*\right)'\left(V'+\Theta\right)$$
$$+s^2P^*V''-\lambda^2s^2\rho^*A^*V=0 \quad (10\text{-b})$$

$$s^2\left[E^*I^*\Theta''+\left(E^*I^*\right)'\Theta'+T^*\Psi'\right]$$
$$-E^*A^*\left(V'+\Theta\right)-2\gamma\lambda s^2r^2\rho^*I^*\Psi-\lambda^2s^2r^2\rho^*I^*\Theta=0 \quad (10\text{-c})$$

$$s^2\left[E^*I^*\Psi''+\left(E^*I^*\right)'\Psi'-T^*\Theta'\right]$$
$$+E^*A^*\left(U'-\Psi\right)+2\gamma\lambda s^2r^2\rho^*I^*\Theta-\lambda^2s^2r^2\rho^*I^*\Psi=0 \quad (10\text{-d})$$

and dimensionless form of boundary conditions (7) can be written as

$$\zeta=0\quad\begin{cases}\left(1+s^2P^*\right)U'-\Psi-K_{lx}^{(L)}U=0\\ \left(1+s^2P^*\right)V'+\Theta-K_{ly}^{(L)}V=0\\ \Theta'+T^*\Psi-K_{rx}^{(L)}\Theta=0\\ \Psi'-T^*\Theta-K_{ry}^{(L)}\Psi=0\end{cases}$$

$$\zeta=1\quad\begin{cases}\left(\mu_E\mu_d^2+s^2P^*\right)U'-\mu_E\mu_d^2\Psi+K_{lx}^{(R)}U=0\\ \left(\mu_E\mu_d^2+s^2P^*\right)V'+\mu_E\mu_d^2\Theta+K_{ly}^{(R)}V=0\\ \mu_E\mu_d^4\Theta'+T^*\Psi+K_{rx}^{(R)}\Theta=0\\ \mu_E\mu_d^4\Psi'-T^*\Theta+K_{ry}^{(R)}\Psi=0\end{cases} \quad (11)$$

in which prime indicates to derivative with respect to ζ and μ_d is the ratio of the diameter of the rotor at ζ=1 to its value at ζ=0.

3- Numerical Results

In this section numerical results are presented for a rotor made of Al and Al_2O_3 with the properties presented in Table 1. Also following dimensionless values are used:

Table 1. Properties of metal and ceramic.

		E (GPa)	*ρ* (Kg/m3)
Metal	Al	70	2702
Ceramic	Al_2O_3	380	3800

Also following dimensionless values are used:

$$r=0.03\quad s=0.05$$
$$K_{lx}^{(L)}=K_{ly}^{(L)}=1000\quad K_{lx}^{(R)}=K_{ly}^{(R)}=2000$$
$$K_{rx}^{(L)}=K_{ry}^{(L)}=50\quad K_{rx}^{(R)}=K_{ry}^{(R)}=100$$

Consider a rotor the properties of which vary exponentially and its diameter changes as $d=d_0\exp(-0.5\zeta)$. The rotor is subjected to P^*=3 and T^*=2. Fig. 2 shows effect of power law index (p) on the Campbell diagram of the first mode. This figure shows that as value of the power law index increases, both forward and backward frequencies and critical speeds of the rotor decrease.

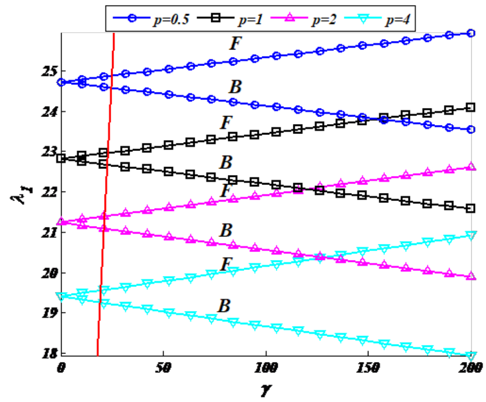

Figure 2. Effect of power law index on the Campbell diagram.

4- Conclusions

Using DQM, a numerical solution for whirling and stability analyses of an axially loaded FG Timoshenko rotor under torsional torque with general boundary conditions were investigated. Numerical examples demonstrated that increase in value of power law index decreases all frequencies.

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