Amirkabir Journal of Mechanical Engineering

Amirkabir J. Mech. Eng., 50(1) (2018) 51-54
DOI: 10.22060/mej.2016.876

Stability Analyses and Dynamic Response of Fluid Conveyed Thin-Walled Piezoelectric Cylinder Under Harmonic Excitation

A. Shooshtari*, V. Atabakhshian

Engineering Faculty, Bu-Ali Sina University, Hamedan, Iran

ABSTRACT: In this paper, the vibration and instability analyses of a thin-walled smart cylinder subjected to the combined electro-thermo-mechanical loadings as well as internal fluid flow are investigated based on piezoelasticity theory and nonlinear Donnell's shell theory. The cylinder material is considered to be made of piezo-ceramics as PZT4 to have a better resistance to the fluids. The fluid flow is assumed to be incompressible, inviscid, irrotational and isentropic where its mathematically modeling is performed based on a potential scalar function. The higher order governing equations of motion are directly obtained by minimizing the energy of the system, using Lagrange equation of motions and modal expansion analysis. The obtained governing equations are then solved via the state space problem as well as fourth order numerical integration to obtain the nonlinear electro thermodynamical response of the system. In the numerical results section, the effects of various parameters such as mean flow velocity, aspect ratio, temperature change and excitation frequency on the natural and damping frequencies, electro-thermo-dynamical response and energy spectrum of the system is studied in detail. It is hoped that the results of this study play an important role to design new instability alert sensors for fluid conveying pipes.

Review History:

Received: 13 June 2016
Revised: 26 October 2016
Accepted: 27 November 2016
Available Online: 15 December 2016

Keywords:

Smart materials
Nonlinear vibration
Fluid flow
Cylindrical shell
Harmonic excitation

1- Introduction

In recent years, piezoelectric materials have attracted more attention due to their interesting applications to smart structures such as sensors and actuators. This is due to the coupling effects between electrical and mechanical fields of piezoelectric materials. Hitherto, a lot of applications of piezoelectric materials have been reported, including transducers that convert electric energy into mechanical energy and vice versa, frequency control filters, sensors, and controllers. On the other hand, dynamic behavior of the structures in contact with the fluid flows (e.g. plates, shells, airfoils and etc.) have been attracted more attention in recent years due to their wide applications in many engineering fields such as in chemistry, physics, medicine, military, aerospace, atomic power plants, oil gas and petrochemical industries. In this regard, Amabily et al. studied the dynamic behavior and stability of fluid conveyed shell [1]. Reddy and Wang [2] analyzed the dynamics of beams containing fluid flow using finite element method.

In this paper, dynamical stability and vibration analyses of a thin-walled piezoelectric shell under internal fluid flow and the external harmonic load is investigated using energy methods. The results of this paper especially may be used for the measurement and stability control of fluid conveyed pipes (i.e. instability alert sensors).

2- Piezoelasticity Theory

The subsequent characterization of electromechanical coupling covers the various classes of piezoelectric materials. Details with respect to definition and determination of the constants describing these materials have been standardized by the Institute of Electrical and Electronics Engineers. In this regard, stresses, and strains on the mechanical side, as well as flux density and field strength on the electrostatic side, may be combined as follows

$$\begin{Bmatrix} \sigma \\ D \end{Bmatrix} = \overbrace{\begin{bmatrix} C^E & -e \\ e^T & \in^{\varepsilon} \end{bmatrix}}^{\mathbf{C}} \begin{Bmatrix} \varepsilon \\ E \end{Bmatrix}, \tag{1}$$

where $\{\sigma\}$, $\{\varepsilon\}$, $\{D\}$ and $\{E\}$ are stress, strain, electric displacement and electric field vectors, respectively, and $[C]$, $[e]$ and $\{\in\}$ are matrices of elastic stiffness, piezoelectric and dielectric constants, respectively. Furthermore, the coefficients of thermal expansion, pyroelectric and temperature change are shown by $\{\lambda\}$, $\{p\}$ and $\Delta\Theta$, respectively.

3- Cylindrical Shell Model

Based on the shell models, the displacement components of an arbitrary point along x , θ and z coordinates are denoted by $\tilde{U}$, $\tilde{V}$ and $\tilde{W}$, respectively, which are expressed in the following form

$$\tilde{U}(x,\theta,z,t) = u(x,\theta,t) - z\,\frac{\partial w(x,\theta,t)}{\partial x} \tag{2}$$

$$\tilde{V}(x,\theta,z,t) = v(x,\theta,t) - z\,\frac{1}{R}\frac{\partial w(x,\theta,t)}{\partial \theta} \tag{3}$$

$$\tilde{W}(x,\theta,z,t) = w(x,\theta,t), \tag{4}$$

where u , v and w are the components of the tube midplane

Corresponding author, E-mail: Shooshta@basu.ac.ir

displacement and t is the time.

According to Donnell's nonlinear shell theory, the strain-displacement relations can be written as

$$\varepsilon_{xx} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 - z\frac{\partial^2 w}{\partial x^2}, \tag{5}$$

$$\varepsilon_{\theta\theta} = \frac{1}{R}\frac{\partial v}{\partial \theta} + \frac{w}{R} + \frac{1}{2R^2}\left(\frac{\partial w}{\partial \theta}\right)^2 - \frac{z}{R^2}\frac{\partial^2 w}{\partial \theta^2}, \tag{6}$$

$$\gamma_{x\theta} = \frac{1}{R}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x} + \frac{1}{R}\frac{\partial w}{\partial \theta}\frac{\partial w}{\partial x} - \frac{2z}{R}\frac{\partial^2 w}{\partial x\,\partial \theta}. \tag{7}$$

4- Energy Functions

The total potential and kinetic energy of the piezoelectric shell can be expressed, respectively as

$$U_S = \frac{1}{2}\int_0^L \int_A \begin{bmatrix} \sigma & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ -E \end{Bmatrix} dA\,dx, \tag{8}$$

$$T_S = \frac{1}{2}\rho_S \iiint_\Lambda \vec{V}\,\vec{V} d\Lambda \tag{9}$$

where A, L, ρ_s and $\vec{V}$ are the cross-section area, total length of the shell, density and velocity vector of the shell, respectively, and '.' is the dot product operator. The work W due to applied radial excitation is

$$W_H = \int_0^{2\pi}\int_0^L (f_x u + f_\theta + f_r w)\,dx r d\theta. \tag{10}$$

The total energy associated with the fluid flow is defined as

$$E_F^T = \frac{1}{2}\rho_f \iiint_\Gamma \vec{V}_f . \vec{V}_f \, d\Gamma, \tag{11}$$

where Γ is the cylindrical fluid volume.

5- Solution Methodology

In this study, mode expansion analyses besides Lagrange equations of motion lead to a set of nonlinear discretized governing equations of motion as

$$[M]\{\ddot{q}\} + [D]\{\dot{q}\} + [K]\{q\} = \{F\}. \tag{12}$$

where $[M]$ and $[D]$ are the mass and damping matrices, respectively and $[K]$ is the stiffness matrix composed of linear and nonlinear terms as

$$[K] = [K_L + K_{NL}]. \tag{13}$$

Eq. (12) can be re-written as

$$\begin{bmatrix} M_{dd} & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} \ddot{q}_d \\ \ddot{q}_\varphi \end{Bmatrix} + \begin{bmatrix} D_{dd} & 0 \\ 0 & 0 \end{bmatrix}\begin{Bmatrix} \dot{q}_d \\ \dot{q}_\varphi \end{Bmatrix} + \begin{bmatrix} K_{dd} & K_{d\varphi} \\ K_{\varphi d} & K_{\varphi\varphi} \end{bmatrix}\begin{Bmatrix} q_d \\ q_\varphi \end{Bmatrix} = \begin{Bmatrix} F_d \\ F_\varphi \end{Bmatrix}. \tag{14}$$

Due to the existence of static coupling between mechanical and electric displacement, the parts of mass and damping matrices associated with $\dot{q}_\phi$ and $\ddot{q}_\phi$ are obtained zero. By means of second set of equations of Eq. (14), the amplitude of electric field vector can be calculated in terms of displacement vector:

$$\{q_\phi\} = -\left[K_{\phi\phi}{}^{-1}K_{\phi d}\right]\{q_d\}. \tag{15}$$

Eliminating q_ϕ in Eq. (14) and using Eq. (15), yields the modified equations of motion which are then solved and analyzed using the state space method, fourth order numerical integration, and energy spectrum analyses.

6- Results and Discussion

The variations of dimensionless natural frequency versus dimensionless flow velocity for the first four modes are shown in Fig. 1. It can be found that the imaginary part of frequency decreases with the increase of flow velocity for all modes until reaching the value zero and instability occurs in the system. The flow velocity at this point is defined as the critical flow velocity that is located at U_f^*=0.0044 for the first vibration mode. Within the zero-frequency area of the first mode ($0.0044 \le U_f^* \le 0.0089$), the real part of complex

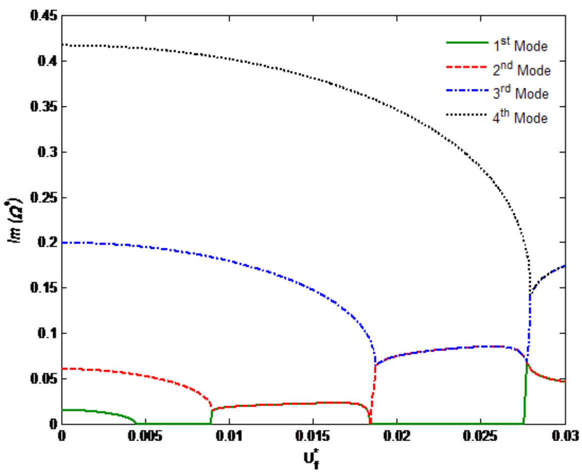

Figure 1. Dimensionless natural frequencies versus dimensionless flow velocity for 1st to 3rd mode.

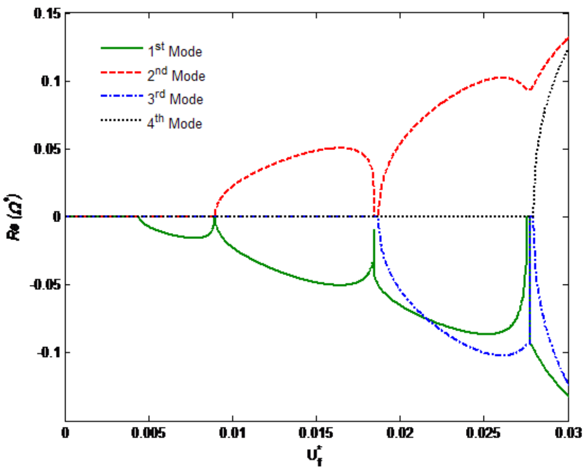

Figure 2. Dimensionless damping frequencies versus dimensionless fluid

frequency has increased as shown in Fig. 2. Furthermore, the magnitudes of natural frequencies of the first and second modes are merged within $0.0089 \leq U_f^* \leq 0.0185$ which is physically known as flutter instability phenomenon. The behavior of the system in the other modes is similar.

7- Conclusions

The most important results of this paper are listed below

1. By increasing the flow velocity, the natural frequencies for all modes decreased until reaching the divergence instability point (i.e. and $\mathrm{Im}(\Omega^*) = 0$ and $\mathrm{Re}(\Omega^*) \neq 0$)
2. For the range of $0.0089 \leq U_f^* \leq 0.0185$, the value of first and second modes are merged which is called the flutter instability.
3. The induction electric potential is increased by increasing the flow velocity which is due to increasing strain field of the shell. Hence, this fact may be considered to design new instability alert sensors.
4. Increasing the temperature increased smoothly the vibration amplitude of the system.

References

[1] M. Amabili, *Vibration and Stability of Shells and Plates*, Cambridge University press, University of parma, Italy, 2007.

[2] J. Reddy, C. Wang, *Dynamics of fluid-conveying beams*, 2004.

Please cite this article using:

A. Shooshtari and V. Atabakhshian, Stability Analyses and Dynamic Response of Fluid Conveyed Thin-Walled Piezoelectric Cylinder Under Harmonic Excitation, *Amirkabir J. Mech. Eng.*, 50(1) (2018) 151-162.

DOI: 10.22060/mej.2016.876