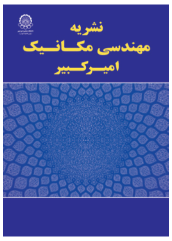

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Dynamic Analysis of Asynchronous Low-Velocity Impacts on Laminated Composite Plate

M. Kavousi Sisi, M. Shakeri*, M. Sadighi

Mechanical Engineering Department, Amirkabir University of Technology, Tehran, Iran

ABSTRACT: Studying asynchronous low-velocity impacts on the structure is one of the applicable problems in this field. In this research, the dynamic analysis of asynchronous low-velocity impacts with arbitrary times and locations on the orthotropic laminated composite plates has been investigated. The dynamic equations of motion are obtained using Hamilton's principle with the assumptions of small deformations and the Hertzian contact law is used for modeling the contact between target and impactors. Then, the closed form solution of the governing equations is obtained using double Fourier expansion of displacement and loading fields. The accuracy of the results has been checked by comparing them to those in the literature in conjunction with the example considering the convergence of the results. Several numerical examples showed that the times and locations of the impacts play an important role on the superposition of induced waves which affect maximum contact forces, minimum and maximum of the plate transverse displacements, as well as the plate, absorbed energy. Modeling asynchronous low-velocity impacts of drop test could be mentioned as one of the significant results of this study. This modeling can substitute for experimental tests and decrease the costs.

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1- Introduction

Foreign object impacts are one of the typical loadings which have different classifications such as single and multiple (simultaneous or asynchronous). A special form of asynchronous impacts is the repeated impacts.

Literature reports few theoretical studies on multiple impacts subjected to the laminated beams and plates [1, 2]. Impact-induced damages evaluation is another real case which is investigated for the stiffened composite laminated plates by Li et al. [3]. Recently, Kavousi Sisi et al. [4] have presented the theoretical solution of asynchronous/ repeated low-velocity impacts of multiple masses on laminated composite beams.

There are very few theoretical studies on the asynchronous/ repeated impacts despite many contributions to the single/ simultaneous multiple impact analysis of the composites. Whereas, the possibility of asynchronous/ repeated impacts is high in real. The main goal of the present work is to study asynchronous/ repeated impacts on the laminated plate assuming the CPT[1] and modified Hertzian contact law. The governing equations of the motion are obtained by Hamilton's principle and solved for SS[2] boundary conditions. All the impact parameters are arbitrary. Several examples are investigated with the emphasis on impact times and locations but are summarized here. Verification and the convergence of the results have been checked and ignored any kind of failure in the structure.

2- Dynamic Analysis of Multiple Mass Impacts

Fig. 1 is the schematic of laminated composite plate composed of K number of orthotropic layers under N low-velocity impacts. a , b and h are the width, height, and length, respectively. Mass, velocity, location and time of the impacts are arbitrary.

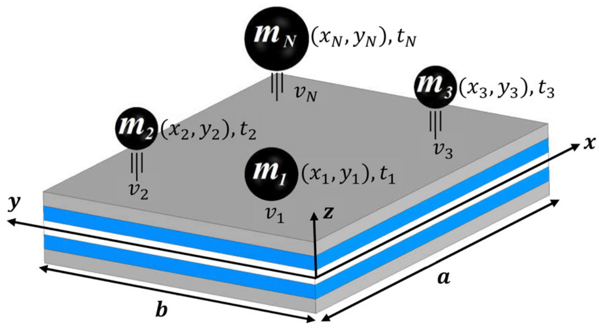

Figure 1. Geometry and the coordinate system of the problem

According to CPT and linear strain–displacement relations and neglecting the rotary inertia, dynamic governing equations of the transverse motion for a symmetrically laminated plate subjected to asynchronous impacts are obtained using Hamilton's principle as:

$$1)\ q = D_{11}\frac{\partial^4 w_0}{\partial x^4} + 2(D_{12} + 2D_{66})\frac{\partial^4 w_0}{\partial x^2 \partial y^2} + D_{22}\frac{\partial^4 w_0}{\partial y^4} + 4D_{16}\frac{\partial^4 w_0}{\partial x^3 \partial y} + 4D_{26}\frac{\partial^4 w_0}{\partial x \partial y^3} + I_1 \ddot{w}_0;$$

$$q = -\sum_{i=1}^{N} F_i(t)\delta(x - x_i)\delta(y - y_i)H(t - t_i)$$

$$2)\ \left(F_i(t) = m_i \ddot{W}_i(t)\right) H(t - t_i), i = 1, 2, ..., N;$$

1 Classical Plate Theory
2 Simply Supported

Corresponding author, E-mail: shakeri@aut.ac.ir

$$F_i(t) = K_i(\alpha_i(t))^{3/2};$$
$$K_i = \frac{4}{3}\frac{\sqrt{r_i}}{\frac{1-\nu_i^2}{E_i}+\frac{1-\nu_t^2}{E_t}}, \alpha_i(t) = w_0(x_i, y_i, t) - W_i(t) \qquad (1)$$

in which q is the expansion of the contact force (F_i) of the impactors. D_{ij};i,j=1,2,6 are bending stiffness matrix elements of the laminate. I_1, w_0 and $\ddot{w}_0$ are mass per unit area, transverse displacement of the plate and its second time derivative, respectively. δ and $H(t\text{-}t_i)$ stand for Dirac delta function and the Heaviside unit function used for considering the effect of i th impactor at time t_i. The definition of the other parameters could be found in [4].

For the plate initially at rest and SS boundary conditions, the conditions and analytical solution (Navier) for cross-ply laminated plate are written as:

$$\begin{aligned}
&w_0(x,0,t) = w_0(x,b,t) = w_0(0,y,t) = w_0(a,y,t) = 0\\
&M_{yy}(x,0,t) = M_{yy}(x,b,t) = M_{xx}(0,y,t) = M_{xx}(a,y,t) = 0\\
&\dot{w}_0(x,y,0) = 0\\
&W_i(t=t_i) = w_0(x_i,y_i,t_i), \dot{W}_i(t=t_i) = \upsilon_i,\ i=1,2,...,N
\end{aligned} \qquad (2)$$

$$\begin{aligned}
&w_0(x,y,t) = \sum_{m,n=1}^{\infty} W_{mn}(t)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\\
&q(x,y,t) = \sum_{m,n=1}^{\infty} Q_{mn}(t)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)
\end{aligned} \qquad (3)$$

in which $W_{mn}(t)$ and $Q_{mn}(t)$ are coefficients of Fourier series where m and n are a number of series' terms. The $m\times n+N$ non-linear second-order differential equations are solved by the Runge–Kutta method.

3- Results and discussion

All the material properties of the impactors and plates are reported in Table 1. Fig. 2 shows the verification of asynchronous impacts at the center of T300/934 laminated plate with lay- up $[0/90/0/90/0]_s$ and dimension 200×200×2.69mm (Ex1). Mass, tip radius and velocity of the impactor are 8.537gr, 6.35mm and -3000mm/s. The present study estimates the maximum contact force and transverse displacement of the plate center 8.3% more and 10% less than the results of Layerwise FEM method with 3D elasticity theory [6] which is adequate due to the calculation costs.

Table 1. Material properties of the plates and impactors

Material	E_{11} [GPa]	E_{22}* [GPa]	G** [GPa]	ν***	ρ [Kg/m³]	Ex	Ref
St	210	210	80.8	0.3	7960	1	[6]
Al	71	71	27	0.3	2700	2	[2]
T300/934	120	7.9	5.5	0.3	1580	1	[6]
T300/934	145.4	9.99	5.68845	0.3	1535.68	2	[2]

$$^{*}:\ E_{22} = E_{33};\ ^{**}: G_{12} = G_{13} = G_{23};\ ^{***}: \nu_{12} = \nu_{13} = \nu_{23} \qquad (4)$$

As a second example (Ex2), the effect of time interval between the two asynchronous impacts is studied on T300/934 laminated plate with $[0/90/90/0]_{2s}$ lay- up and dimension 150×150×2.54mm. The first and second impact locations and times are (30,75)mm , t_2µs and (120,75)mm , 0µs , respectively. Mass, tip radius and velocity of the impactors are 5gr, 6.35mm and -2000mm/s. All the results are reported after checking the convergence (n=m=20) and compared to single impact in Table 2. Transverse wave propagation speed in the plate (C_f) is obtained 2.9592×10⁶ mm/s, so the second wave reaches the location of the first impact after 30.4µs. Since the maximum contact force of single impact occurs at 34µs (Fig. 3), the second impact affects the first one for some t_2 values such as simultaneous impact (column 2 at t_2= 0µs in Table 2). The further increment of t_2 leads to constant maximum contact force and residual velocity of the 1st impactor and (as a result) absorbed energy by the plate at first impact (column 7 for $t_2 \geq$ 232µs in Table 2 and Fig. 4). These values are equal to the single impact.

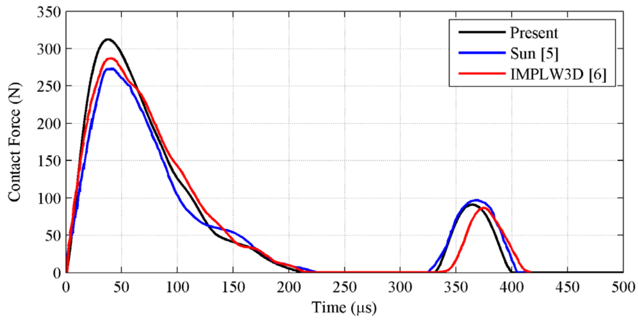

Figure 2. Result's verification for asynchronous impacts

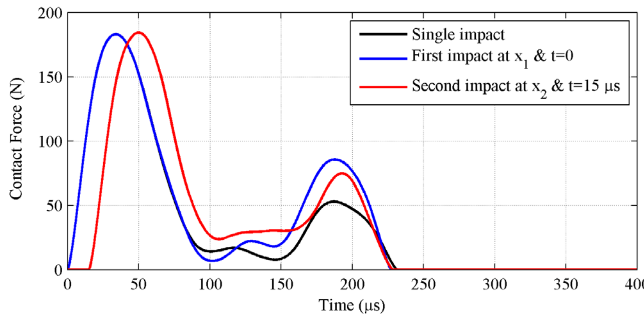

Figure 3. Comparison of contact force time history for single and two asynchronous impacts

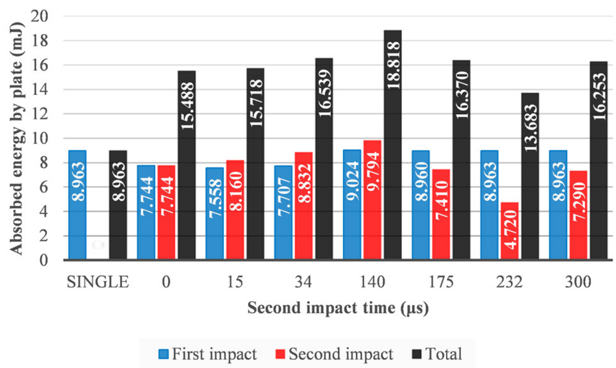

Figure 4. The effect of time interval between two asynchronous impacts on the absorbed energy by the plate

Table 2. Effect of time interval between two asynchronous impacts on maximum contact forces, transverse displacement of impact locations, transverse displacement of the plate and residual velocity of impactors

t_2 , μs	*Max* F_1 , N	*Max* F_2 , N	*Max* w_1 , mm	*Max* w_2 , mm	*Max* w , mm	b_{b1} , mm/s	b_{b2} , mm/s
Single impact	183.007	-	0.1283	-	0.1404	644.178	-
0	183.017	183.017	-0.1614	-0.1614	-0.2567	949.961	949.961
15	183.006	184.309	0.1656	-0.1659	-0.2537	988.262	857.927
34	183.007	182.924	0.1723	-0.1722	-0.2438	957.654	683.646
140	183.007	224.273	0.1548	0.1888	-0.2216	624.836	286.873
175	183.007	202.941	-0.1504	0.1728	-0.2051	645.091	1017.817
232	183.007	153.863	-0.1535	0.1723	0.1940	644.178	1453.307
300	183.007	158.276	-0.1616	-0.1742	-0.2064	644.178	1041.095

4- Conclusions

The main results for asynchronous impacts are:

- Time of the impact plays a key role in the positive and negative superposition of induced waves, thus the dynamic response is completely affected by it.
- By increasing the time interval between the impacts beyond a critical value, for every fixed distance interval, the next impacts will not affect the first contact force and the absorbed energy by the plate.

References

[1] K.Y. Lam, T.S. Sathiyamoorthy, Response of composite beam under low-velocity impact of multiple masses, *Composite Structures*, 44(2) (1999) 205-220.

[2] D. Chakraborty, M. Kumar, Response of Laminated FRP Composites under Multiple Impact Loading, *Journal of Reinforced Plastics and Composites*, 24(14) (2005) 1457-1477.

[3] D. Li, Y. Liu, X. Zhang, Low-velocity impact responses of the stiffened composite laminated plates based on the progressive failure model and the layerwise/solid-elements method, *Composite Structures*, 110 (2014) 249-275.

[4] M. Kavousi Sisi, M. Shakeri, M. Sadighi, Dynamic response of composite laminated beams under asynchronous/repeated low-velocity impacts of multiple masses, *Composite Structures*, 132 (2015) 960-973.

[5] Sun, C.T., Chen, J.K., 1985. "On the impact of initially stressed composite laminates". J Compos Mater, 19(6), November, pp. 490-504.

[6] C.T. Sun, S. Chattopadhyay, Dynamic Response of Anisotropic Laminated Plates Under Initial Stress to Impact of a Mass, *Journal of Applied Mechanics*, 42(3) (1975) 693-698.

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