

## Amirkabir Journal of Mechanical Engineering

Amirkabir J. Mech. Eng., 50(5) (2018) 333-336 DOI: 10.22060/mej.2017.11917.5216



# Static and Dynamic Pull-in Instabilities Analysis of Partially Affected Clamped Nano Actuators: The Substrate Effect

A. Noghrehabadi<sup>1</sup>, A. Haghparast<sup>1</sup>, M. Ghalambaz<sup>2\*</sup>

<sup>1</sup>Department of Mechanical Engineering, Shahid Chamran University, Ahvaz, Iran

<sup>2</sup> Department of Mechanical Engineering, Dezful Branch, Islamic Azad University, Dezful, Iran

**ABSTRACT:** Many researches have been carried out for modeling of micro/nano electromechanical systems instabilities, in which both movable and substrate electrodes are at the same size; however, there is no research considering the static and dynamic pull-in instabilities of micro/nano actuators with a smaller substrate electrode in the presence of small size effects. In the present study, the static and dynamic behaviors of partially affected clamped micro/nano actuators are investigated and the effects of position and length of the substrate electrode are analyzed. The non-linear Euler-Bernoulli governing equation of the beam motion and the corresponding boundary conditions are derived using the Modified couple stress theory. Finite element method is utilized to solve the governing equations. In order to investigate the accuracy of the utilized finite element method, the obtained results are compared with those available in the literature and a good agreement between them was found. The results demonstrate that a decrease of the substrate electrode length leads to an increase of the required pull-in voltage and the pull-in capillary force. Moreover, a small reduction in the pull-in deflection of the nano-beam is observed because of the decrease of the substrate electrode. Finally, a new parameter, named as balanced size effect-capillary force which changes the trend of the behavior of the nano-beam, is introduced.

#### **Review History:**

Received: 26 August 2016 Revised: 9 January 2017 Accepted: 22 January 2017 Available Online: 28 January 2017

Keywords:

Nano-beam Clamped-clamped beam Partially affected Modified couple stress theory

## **1-Introduction**

Because of simple structure, only a small number of mechanical components and low levels of voltage are needed to actuate micro/nano electromechanical systems [1]. In such devices, a flexible and moveable beam/plate is overhung on the top of a fixed beam/plate and a voltage difference is applied between them. Applying voltage difference between two electrodes results in deformation of the moveable electrode and its deviation toward the fixed (substrate) electrode.

In micro and nano-scale structures, the behavior of a material is highly dependent on the size of the structure [2]. Sizedependent behavior is an inherent feature of the material in a given beam if its size features, such as thickness or diameter, are close to internal material length scale parameter [3]. Since classical continuum theory cannot explain small scale effect or size-dependent behaviors of micro/nano structures, non-classical continuum theories, such as modified couple stress theory [4], are recently considered for nano-structures simulation.

For the purpose of encompassing all the possibilities, easier analysis, and optimizing already available conditions, improvement and development of nano-beam models have greatly interested the researchers. By developing a model for cantilever beam in static mode, considering surface stress effects and electrostatic force, Wang et al. [5] investigated the effect of displacement and change in length of the substrate electrode in presence of surface stress effects and electrostatic force.

As the literature suggests, so far, no research is carried out on partially affected clamped nano-beams with small-scale effect. The aim of this study is to develop a comprehensive model for clamped nano-beams in order to create an actual state by taking partial effects into consideration. Considering the small size effects, under the influence of electrostatic and capillary forces, static and dynamic influences of partially affected nanostructures are studied.

## 2- Methodology

In order to consider small-scale effects, Euler-Bernoulli nonlinear motion equation and its corresponding boundary conditions are obtained using non-classical theory of modified couple stress. Considering Von-Karman strains, nonlinear equations of mid-plane stretching are added to the system. The partially affected clamped micro/nano-beams are modelled as Fig.1 in order to analyze static and dynamic instability.

In partially affected nano-structures, the substrate electrode is shorter than movable electrode and shorter length of the substrate electrode prevents the movable electrode (nanobeam) from being fully influenced. In order to control location and length of the substrate electrode, the Heaviside function, H(x) is used in this model.

$$H(x) = H(x - D_1) - H(x - L + D_2)$$
(1)

Nonlinear differential equation governing the transverse displacement of nano-beam is as follows:

$$\left(EI + \mu Al^{2}\right)\frac{\partial^{4}w}{\partial x^{4}} + \rho A\frac{\partial^{2}w}{\partial t^{2}} - \left[\sigma_{r}bh + \frac{Ebh}{2L}\int_{0}^{L}\left(\frac{dw}{dx}\right)^{2}dx\right]\frac{\partial^{2}w}{\partial x^{2}} = q(x,t)$$
<sup>(2)</sup>

Corresponding author, E-mail: m.ghalambaz@iaud.ac.ir



Fig 1. Schematic view of a partially affected clamped nanoswitch

In order to simplify calculation of parameters, the governing equation and boundary conditions can be converted to non-dimensional mode. Thus, the obtained governing dimensionless equation for dynamic mode is:

$$(1+\delta)\frac{\partial^{4}W}{\partial X^{4}} + \frac{\partial^{2}W}{\partial T^{2}} - N_{S}\frac{\partial^{2}W}{\partial X^{2}} = H'\left(\frac{\beta}{(1-W(x))^{2}} + \frac{\gamma_{fr}\beta}{(1-W(x))} + \frac{\gamma_{ca}}{(1-W(x))}\right)$$
(3)

where

$$N_{s} = \left[ N + \eta \int_{0}^{1} \left( \frac{\partial W}{\partial X} \right)^{2} dX \right]$$

and dimensionless boundary conditions are:

$$W(0) = W(1) = 0, \quad \frac{\partial W}{\partial X}\Big|_{X=0,1} = 0 \tag{4}$$

In order to obtain the static governing equation, timedependent expressions are neglected.

For solving differential equations, numerical finite element method is used. The accuracy of finite element method is validated by comparing the results of previous researches. Using the proposed mode, the effects of dimensionless parameters (fringing field effect, size effect, location and length of the substrate electrode, electrostatic and capillary forces) on nano-beam instability are examined.

#### **3- Numerical Results**

Fig.2 indicates variation of dimensionless dynamic pull-in displacement parameter against pull-in capillary parameter for various values of the size effect for partially affected nano-beams. From Fig.2, it can be concluded that for partially affected nano-beams, at the intersection point of the graphs (point A). At point A, the change in the dimensionless size effect parameter has no effect on the pull-in displacement of nano-beams. At point A, a change in the trend of the displacement behavior of nano-beam as a function of size effect parameter can be seen. In the present study, Point A is defined as the balanced size effect-capillary force.

Fig.3 compares changes in dynamic pull-in voltage of nanobeams and dimensionless capillary force for various values of substrate electrode location and position (length) parameter. As shown in Fig.3, pull-in voltage and capillary force of clamped nano-beams with a shorter substrate electrode are more than nano-beams with a full-length substrate electrode.



Fig.2. Variation of pull-in displacement parameter against pullin capillary force parameter for different values of size effect

parameter and  $(\eta = 0, N = 0, \gamma_{fr} = 0, d_1 = 0, d_2 = 0.2)$ 



Fig.3. Variation of pull-in voltage parameter against pull-in capillary force parameter for different values of length and position of substrate electrode parameter  $(\gamma_{fr} = 0, \eta = 0, N = 0, \delta = 0.5, \gamma_{fr} = 0)$ 

#### **4-** Conclusions

Some of the most important results of this study can be summarize as:

• Shortening the length of substrate electrode increases the values of dimensionless pull-in voltage and capillary force parameters;

• Shortening the length of substrate electrode results in a slight reduction in pull-in displacement parameter for both static and dynamic modes;

• The increase of the size effect parameter increases pull-in voltage and capillary force parameters. In partially affected clamped nano-beams, the effect of size parameter on pull-in voltage and capillary force parameters is more than nano-beams with full-length electrode;

• Physically, a balanced size effect-capillary force point is found in the graphs. This point shows an advantage design condition in which by selecting a specified amount of capillary force, a nano-beam can be designed regardless of the size effect parameter.

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Please cite this article using:

A. Noghrehabadi, A. Haghparast, M. Ghalambaz, Static and Dynamic Pull-in Instabilities Analysis of Partially Affected

Clamped Nano Actuators: The Substrate Effect, Amirkabir J. Mech. Eng., 50(5) (2018) 333-336.

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