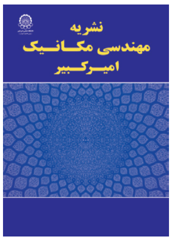

Amirkabir Journal of Mechanical Engineering

Amirkabir J. Mech. Eng., 50(5) (2018) 337-340
DOI: 10.22060/mej.2017.12173.5274

Bending and Free Vibration Analysis of Functionally Graded Nano-plate Using Trigonometric Higher-Order Plate Theory

A. Azizi, A. Setoodeh*

Department of Mechanical and Aerospace Engineering, Shiraz University of Technology, Shiraz, Iran

ABSTRACT: In this paper, bending and free vibration analyses of functionally graded nano-plates are investigated using a new trigonometric higher-order plate theory. The governing equations are developed by employing Hamilton's principle and then a Navier-type analytical solution for bending and free vibration of simply supported rectangular FG nano-plates is obtained. Furthermore, the nonlocal theory of elasticity is used to take into account the small scale effects. The mechanical properties of the functionally graded nano-plates are assumed to vary by a power law function through the thickness. In order to confirm the accuracy of the present theory, the obtained results from the present solution are compared with the existed results, and a very good agreement is achieved. Moreover, the effects of length-to-thickness ratio, aspect ratio and nonlocal parameter on the bending and free vibration solutions are investigated. The results demonstrate that the inclusion of nonlocal parameter in governing equations or increasing the power index, leads to reduction of the natural frequency and increasing of the deflection and in another word the nano-plate stiffness is reduced. Also, the impact of the nonlocal parameter and size effects is reduced by increasing the length of the nano-plate or aspect ratio. Furthermore, the present theory not only provides exact solution for the bending and free vibration of thick and moderately thick functionally graded nano-plates with minimum computational cost, but also exhibits the parabolic distribution of the shear stress through the thickness.

Review History:

Received: 17 November 2016
Revised: 23 January 2017
Accepted: 5 March 2017
Available Online: 13 March 2017

Keywords:

Functionally graded nano-plate
Trigonometric higher-order theory
Nonlocal theory
Bending
Free vibration

1- Introduction

Nano-scale devices are designed by using nano-beam, nano-membrane and nano-plate structures. Therefore, the modeling and analysis of nano-plates has been of interest to researchers. A survey in the literature shows that several researches have been done to study mechanical behavior of FG nano-plates. In most researches non-classical theories with one variable, including modified couple stress theory [1, 2] and the nonlocal theory of elasticity [3, 4] are used to analyze Functionally Graded (FG) nano-plates. The study of previous works demonstrates that higher-order plate theories are rarely used in researches involving nonlocal theory. Meanwhile, the trigonometric higher-order plate theory has not been used for this purpose yet. In this paper, for the first time a new trigonometric higher-order plate theory is employed to obtain accurate bending and free vibration responses of FG nano-plates. The results of the present theory are compared with those reported by previous researchers and good agreement is observed between the results.

The present article demonstrates Navier-type solutions for the bending and free vibration of moderately thick as well as thick FG nano-plates with minimum computational cost. Also, the present theory predicts the parabolic distribution of the shear stress through the thickness.

2- Methodology

In this paper, the bending and free vibration of rectangular FG nano-plates are investigated by employing a new trigonometric higher-order plate theory. The governing equations are derived based on Hamilton's principle and then the exact Navier-type solutions of bending and free vibration of such nano-plates with simply supported boundary conditions are obtained. The most popular non-classical continuum mechanics theories include the classical couple stress theory, strain gradient theory, nonlocal theory of elasticity and modified couple stress theory. The non-classical theories with one variable as nonlocal theory and modified couple stress theory, because of their simplicity and less computational cost, mostly are of interest to researchers. Therefore, the nonlocal theory is used for considering the size effects in this paper. The mechanical properties of FG nano-plates are changed by a power law function through the thickness.

Corresponding author, E-mail: setoodeh@sutech.ac.ir

3- Results and Discussion

Analytical solutions of bending and free vibration of rectangular FG nano-plates are obtained using Navier method. According to the results reported in Table 1, it can be observed that the present theory produces more accurate results in comparison with the solution based on third-order shear deformation theory (TSDT) of Ref. [5]. Also, it can be noted that the developed results are close to the three-dimensional (3-D) solution reported in reference [6]. Table 2 shows the capability and high accuracy of the present theory to estimate the natural frequency of thick ($a/h = 5$) and moderately thick ($a/h = 10$) FG nano-plates while the results are compared with the three-dimensional (3-D) solution achieved in reference [7].

Present theory shows that increasing the nonlocal parameter (μ) leads to predict lower natural frequency and higher deflection for FG nano-plates. It means that by increasing the nonlocal parameter (μ), the model predicts a softening behavior for FG nano-plates.

Based on the present theory and the developed results it can be seen that the increase of the power gradient index (n) yields to decrease of the natural frequency and increase of

Table 1. Dimensionless first natural frequency $\omega h\sqrt{\rho_t/G_t}$ of square nano-plate

μ	a/h	Present	TSDT [5]	3-D [6]
1	20	0.0218	0.0218	0.0211
	10	0.0850	0.0854	0.0827
2	20	0.0202	0.0202	0.0191
	10	0.0788	0.0791	0.0751
3	20	0.0189	0.0189	0.0176
	10	0.0737	0.0741	0.0692

Table 2. Dimensionless first natural frequency $\omega a^2/h\sqrt{\rho_t/E_t}$ of FG plate (μ=0)

a/h	n	Present	3-D [7]	Error %
10	0	5.770	5.779	0.16
	1	4.419	4.428	0.20
	5	3.767	3.774	0.19
5	0	5.282	5.304	0.42
	1	4.079	4.100	0.51
	5	3.391	3.405	0.41

the deflection of FG nano-plates. The power gradient index determines the fraction of metal and ceramic in the FG nano-plate. Fig.1 exhibits capability of the present theory in predicting the parabolic distribution of the transverse shear stress through the plate thickness. Four different values for the nonlocal parameter are selected and the plate is subjected to a sinusoidal loading. It is worth noting that μ=0 demonstrates the classical behavior of FG nano-plates by ignoring the size-effect.

4- Conclusions

In this paper, the nonlocal theory and a new trigonometric higher-order shear deformation plate theory are used for evaluation of bending and free vibration of rectangular FG nano-plates. A Navier-type analytical solution for bending and free vibration of FG nano-plates with simply supported boundary conditions is obtained. The results of present theory are compared with existing results. Good agreement is observed and the capability of the present theory to estimate the behavior of moderately thick and thick FG nano-plates is confirmed. The effects of the nonlocal parameter, side-to-thickness and aspect ratios on the bending and vibration responses are evaluated. Capturing the size-effect in the governing equations leads to predict lower stiffness for FG nano-plates. With increasing the length and aspect ratio,

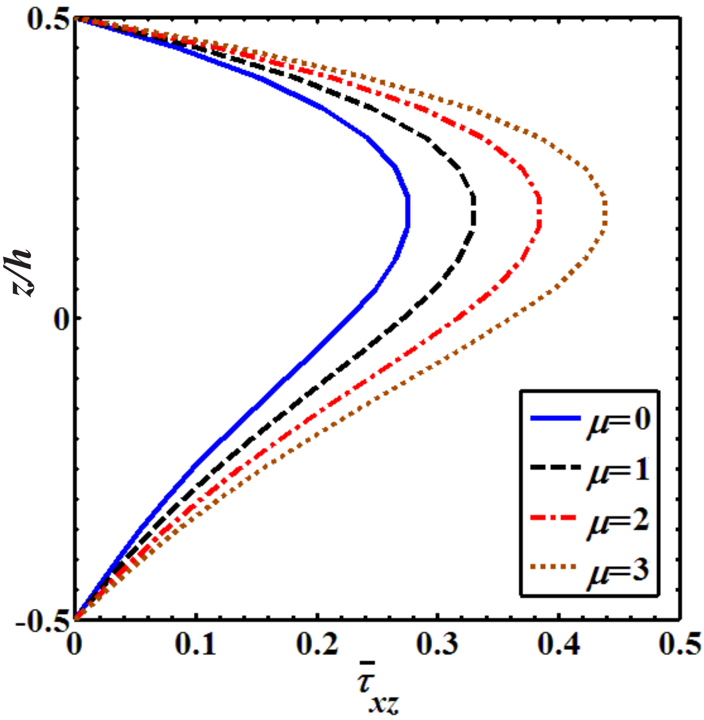

Fig. 1. Distribution of $\bar{\tau}_{xz}$ through the thickness for FG nano-plate under sinusoidal loading (a/h=10, n=2)

the effect of nonlocal parameter is reduced on the natural frequency of FG nano-plates. Also, the new trigonometric higher-order theory is able to predict the parabolic distribution of transverse shear stress through the thickness of FG nano-plates. Moreover, it is seen that the increase in power gradient index reduces the natural frequency. In other words, the increment of power index leads to a reduction in stiffness of FG nano-plates. The excellent accuracy in predicting the behavior of moderately thick as well as thick FG nano-plates with minimum computational cost is important advantage of the present theory compared to the similar ones.

References

[1] H.T. Thai, D.H. Choi, Size dependent functionally graded Kirchhoff and Mindlin plate models based on a modified couple stress theory, *Composite Structures*, 95 (2012) 142-153.

[2] L. He, J. Lou, E. Zhang, Y. Wang, Y. Bai, A size-dependent four variable refined plate model for functionally graded microplates based on modified couple stress theory, *Composite Structures*, 130(M1) (2015) 107-115.

[3] A. Daneshmehr, A. Rajabpoor, A. Hadi, Size dependent free vibration analysis of nanoplates made of functionally graded materials based on nonlocal elasticity theory with high order theories, *International Journal of Engineering Science*, 95 (2015) 23-35.

[4] M. Zare, R. Nazemnezhad, Sh. Hosseini Hashemi, Natural frequency analysis of functionally graded rectangular nanoplates with different boundary conditions via an analytical method, *Meccanica*, 50 (2015) 2391-2408.

[5] R. ghababaei, J.N. Reddy, Nonlocal third-order shear deformation plate theory with application to bending and vibration of plates, *Journal of Sound and Vibration*, 326

(2009) 277-289.

[6] E. Jomehzadeh, A.R. Saidi, Decoupling the nonlocal elasticity equations for three dimensional vibration analysis of nano-plates, *Composite Structures*, 93 (2011) 1015-1020.

[7] G. Jin, Z. Su, S. Shi, T. Ye, S. Gao, Three-dimensional exact solution for the free vibration of arbitrarily thick functionally graded rectangular plates with general boundary conditions, *Composite Structures*, 108 (2014) 567-577.

Please cite this article using:

A. Azizi and A. Setoodeh, Bending and Free Vibration Analysis of Functionally Graded Nano-plate Using Trigonometric Higher-Order Plate Theory, *Amirkabir J. Mech. Eng.*, 50(5) (2018) 337-340.

DOI: 10.22060/mej.2017.12173.5274