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Analytical Solution of Heat Transfer in Laser-Irradiated Skin Tissue with Surface Heat Convection Using Dual Phase Lag Model

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ABSTRACT: The temperature distribution in the laser-irradiated biological tissue is investigated considering heat convection. The first- and the second-degree burn times are predicted using estimation of thermal damage. A non-Fourier equation of bio-heat transfer based on dual phase lag model is employed. The transport behavior of laser light in the tissue is regarded as highly absorbed and effects of the phase lag times on thermal response and thermal damage are explored considering different heat convection coefficients. The Laplace transform with discretization technique and also using boundary conditions, a set of algebraic equations in Laplace domain is generated which are solved by numerical Laplace inverse transform. The results show that the highly absorbed laser light in the tissue plays an important role in the burned skin time. Also, convective heat transfer boundary condition on the surface provides different results, even by considering the natural convection on the surface, and the first- and the second-degree burns are postponed at least 0.02 second.

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1- Introduction

One of the challenges in thermal therapy is to achieve the adequate temperature in the target area of a patient's body. If the temperature of the body region exposed to heat rises excessively or the heating duration is prolonged, the outcome might have irreversible side effects. There are several models of bio heat transfers to analyze the thermal behavior in tissue. In 1997, Tzou [1] presented a dual-phase-lag (DPL) model of the biological heat transfer equation. In the DPL model, in addition to heat flux, a temporal phase delay is also considered for temperature gradient which results in the addition of two-time constants in the classic Fourier law. The amounts of these temporal phase delays are determined empirically. Ahmadikia et al. [2] used the Laplace transform method to analytically solve the parabolic and hyperbolic heat transfer equations for skin in two finite and semi-infinite domains, under boundary conditions of constant, periodic and pulse train heat flux. Liu et al. [3] have used the second-order Taylor series expansion to solve the biological heat transfer equation of the DPL model by assuming a strongly scattering tissue and the effect of displacement over the skin surface. Liu and Wang [4] used the DPL model to numerically simulate and estimate the amount of thermal damage sustained by biological tissues due to the application of highly absorbing laser radiation and investigated the effects of blood perfusion rate and produced metabolic energy on thermal damage. In this paper, the effect of convectional heat transfer on skin surface by considering the application of strongly absorbing laser radiation is studied. The aims of this research are to analytically solve the biological heat transfer equation of the DPL model and obtain the distribution of temperature within skin tissue, to investigate the heat damage resulting from strongly absorbing laser radiation in the presence of convectional heat transfer over skin surface, and to explore the effect of phase delay time on the damage depth.

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2- Methodology

The formulation is based on following DPL model of biological heat transfer equation.

$$k\left(1+t_T\frac{\partial}{\partial t}\right)\frac{\partial^2 T}{\partial x^2}= \left(1+t_q\frac{\partial}{\partial t}\right)\left[rc\frac{\partial T}{\partial t}+w_b r_b c_b\left(T-T_b\right)-q_m-q_r\right] \quad (1)$$

The initial temperature has been considered as constant and equal to blood temperature (T_b). Tissue depth is always considered as an insulator and by assuming the laser radiation to be strongly-absorbent, the boundary condition for skin surface takes the form of Eq. (2) and a pulsing laser radiation (φ_{in}) has been considered.

$$q\left(0^+,t\right)=\phi_{in}\left(1-R_d\right)+h\left(T_f-T\right) \quad (2)$$

First, variable H is defined as $H=T-T_b$. After applying Laplace transform with respect to time (t), Eq. (1) will become:

(3)

If i and j indicate the node number and subinterval number, respectively, then, in the jth subinterval (which is situated between nodes i and $(i+1)$, Eq. (3) can be written as:

(4)

Applying temperature and heat flux continuity in the interfaces of two subintervals j-1 and j, an approximate value of $\tilde{H}$ is determined as Eq. (5).

$$\tilde{H}_j(x) = \frac{1}{\sinh(\lambda l)}\left\{\left(\tilde{H}_i - \frac{f_j}{\lambda^2}\right)\sinh\lambda(x_{i+1}-x) + \left(\tilde{H}_{i+1} - \frac{f_j}{\lambda^2}\right)\sinh\lambda(x - x_i)\right\} + \frac{f_j}{\lambda^2} \quad (5)$$

By defining a distance between the two adjacent nodes as $l=x_i+1-x_i$, the Eq. (5) can be written as Eq. (6):

$$\tilde{H}_{i-1} - 2\cosh(\lambda l)\tilde{H}_i + \tilde{H}_{i+1} = \frac{f_{j-1}+f_j}{\lambda^2}\left[1-\cosh(\lambda l)\right] \quad (6)$$

By applying Eq. (6) to all the non-boundary nodes and also using boundary conditions, a system of linear algebraic equations in the matrix form of $[G]\{\tilde{H}\}=\{B\}$ will be presented. After solving the system of equations and computing vector $\{\tilde{H}\}$, the numerical inverse Laplace transform as represented in Ref. [5] is used for every $\tilde{H}_i$, and the temperature in the time domain is obtained.

3- Results and Discussion

Fig. 1 shows the skin surface temperature distribution in the case of $\tau_T < \tau_q$ for different values of convectional heat transfer coefficient, and for laser radiation with an intensity of q_0=20 kW/m^2. As can be observed, skin surface temperature diminishes with the increase of convectional heat transfer coefficient; indicating an increase of heat transfer from the skin surface.

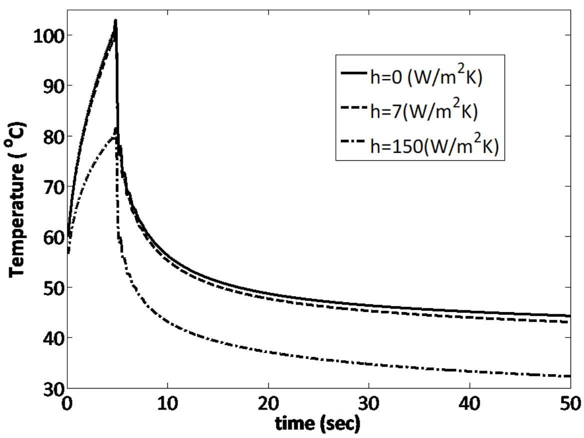

Figure 1. Skin surface temperature versus time for different values of convectional heat transfer coefficient

To evaluate the amount of heat damage, the activation energy of protein deformation reaction has been considered as E=4.6×105 (J/mol.K) and the frequency coefficient has been taken as A = 1.43 × 1072 (1/sec). To predict the first -and second- degree burn times, thermal damage parameter values have been shown in Fig. 2 in the case of $\tau_T < \tau_q$ and for various convectional heat transfer coefficients.

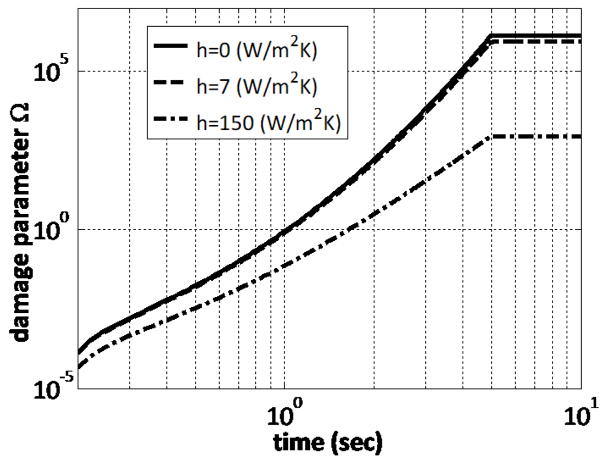

Figure 2. Logarithmic representation of thermal damage versus time for various convectional heat transfer coefficients

4- Conclusions

In this research, by applying strongly absorbing laser radiation on skin tissue and considering convectional heat transfer over the skin surface, the biological heat transfer equation based on the dual phase-delay model has been investigated analytically. The examined effects of phase delay times on temperature distribution indicate that the correct values of phase delay times have a significant influence on the obtained results and consequently affect the prediction of tissue burn time. It was found that the maximum skin surface temperature rises with the increase of phase-delay time resulting from thermal flux and diminishes with the increase of phase-delay time resulting from a temperature gradient. In general, it can be concluded that the consideration of even the slightest amount of convectional heat transfer over skin surface (e.g. natural convection) can affect the outcome and that ignoring the effect of this heat transfer can cause an error of at least 2.6% in the obtained results. Thus, it seems to be very important and necessary to consider the effect of convectional heat transfer over the skin surface.

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