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Analytical Calculation of Stress Intensity Factors for Unequal Cracks Emanating from a Circular Hole in an Infinite Plane

M. Hajimohamadi, R. Ghajar*

Department of Mechanical Engineering, K. N. Toosi University of Technology, Tehran, Iran

ABSTRACT: Thin plates are used in marine and aerospace structures. Geometric defects in components and industrial structures usually occur intentionally or unintentionally which hole is one of them. It is common that a crack initiate in the areas of stress concentration. Even if the crack is relatively small, it propagates and can lead to a dangerous situation. Stress intensity factor is one of the important parameters of the crack behavior. In this paper, by using the analytical solution of Muskhelishvili and finding suitable conformal mapping, behavior of two unequal and aligned cracks emanating from a circular hole is investigated. The effect of parameters such as load orientation, crack length and etc. is studied. The hole and the cracks are assumed to be traction free. The infinite isotropic plane is subjected to a uniform tensile loading at infinity in an arbitrary direction. To ensure the accuracy of the method, results are compared with some specific problems. In this paper, an explicit formula based on the geometric parameters of the problem is presented for stress intensity factor. Also and are obtained for various loading and cracks length.

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1- Introduction

Thin plates are used in marine and aerospace structures. Geometric defects in components and industrial structures usually occur intentionally or unintentionally which hole is one of them. It is common that a crack initiate in the areas of stress concentration.

Muskhelishvili [1] and Savin [2] combined complex variable functions method with conformal mapping, to solve the 2-D elastic analysis around the hole in the plane. By using complex mapping, Bowie [3] provided simple solution for a circular hole with one edge crack and a pair of symmetrical edge cracks in an infinite plate under uniform tension. Newman [4] developed an improved method of boundary collocation and combined with complex method of Muskhelishvili. By means of a boundary element method, Yan [5] calculated stress intensity factors for cracks emanating from circular and square hole in an infinite plate. Liu and Duan [6] investigated two unequal edge cracks originating from elliptical hole in an infinite plate under uniform tension. They obtained analytical solutions of mode II stress intensity factor (K_{II}) for the same problem under shear [7]. From the literatures, there are numerical and analytical solutions for mode I stress intensity factor (K_I) in symmetrical cracks emanating from the hole in an infinite plate under tension. In the present paper, K_I and K_{II} were calculated for unequal cracks emanating from circular hole in an infinite plane under remote tensile load at the arbitrary angle with respect to the cracks. Effects of crack length and angle of loading on K_I and K_{II} were considered.

Corresponding author, E-mail: Ghajar@kntu.ac.ir

2- Analytical Solution

Consider two collinear and unequal edge cracks emanating from a circular hole in an infinite plate. Plate is subjected to uniform remote tensile stress in arbitrary angle β with respect to the cracks as shown in Fig. 1.

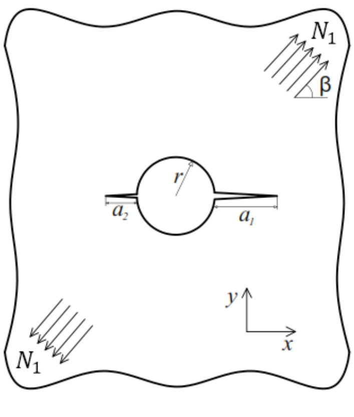

Fig. 1. An infinite plate with cracked hole under remote tensile load

Stress intensity factors can be defined by [7]:

$$\left[K_I + iK_{II}\right]_{Right\,Tip} = 2\sqrt{\pi}\,\frac{\phi'(1)}{\sqrt{\omega''(1)}}$$
$$\left[K_I + iK_{II}\right]_{Left\,Tip} = 2\sqrt{\pi}\,\frac{\phi'(-1)}{\sqrt{-\omega''(-1)}} \qquad (1)$$

A conformal mapping function $\omega(\zeta)$ which maps outside region of unit circle in the ζ plane to outside region of a circle with two unequal cracks in z plane, was obtained by combination of some simple mapping, as:

$$\omega(\zeta)=\lambda\left[M\left(\zeta+\frac{1}{\zeta}\right)+N+\frac{\zeta}{\sqrt{\zeta^2}}\sqrt{Q}\right]$$
$$Q=M^2\left(\zeta+\frac{1}{\zeta}\right)^2+N^2+2MN\left(\zeta+\frac{1}{\zeta}\right)-1$$
$$M=\frac{\delta_1+\delta_2}{4};\qquad N=\frac{\delta_1-\delta_2}{2} \tag{2}$$
$$\delta_i=\frac{1}{2}\left(1+l_i+\frac{1}{1+l_i}\right);\quad i=1,2;\quad l_i=\frac{a_i}{\lambda}$$

Using mapping function $\omega(\zeta)$ and assuming that $\varphi(\zeta)=\varphi_1(\omega(\zeta))$, stress function of $\varphi(\zeta)$ can be written as:

$$\varphi(\zeta)=B_1\omega(\zeta)+\varphi_0(\zeta) \tag{3}$$

The unknown $\varphi_0(\zeta)$ function in Eq.(3) was obtained from integral equation given by Savin [2]:

$$\varphi_0(\zeta)+\frac{1}{2\pi i}\int_\gamma \frac{\omega(\rho)}{\omega'(\rho)}\frac{\overline{\varphi'_0(\rho)}}{\rho-\zeta}d\rho+\overline{b_0}=\frac{1}{2\pi i}\int_\gamma \frac{f_1^0+f_2^0}{\rho-\zeta}d\rho \tag{4}$$

After determination of $\varphi_0(\zeta)$, stress function of $\varphi(\zeta)$ was obtained. Determining the derivative of stress function and second derivative of the mapping function, substituting in Eq.(1), the stress intensity factors at the right and left crack tips were obtained:

$$[K_I+iK_{II}]_{Right\,Tip}=-\sqrt{2\lambda\pi M}\,\frac{4B_1+2(B_2-iC_2)}{\sqrt{1+\dfrac{2M+N}{\sqrt{4M^2+N^2+4MN-1}}}} \tag{5}$$

$$[K_I+iK_{II}]_{Left\,Tip}=-\sqrt{2\lambda\pi M}\,\frac{4B_1+2(B_2-iC_2)}{\sqrt{1+\dfrac{2M-N}{\sqrt{4M^2+N^2-4MN-1}}}} \tag{6}$$

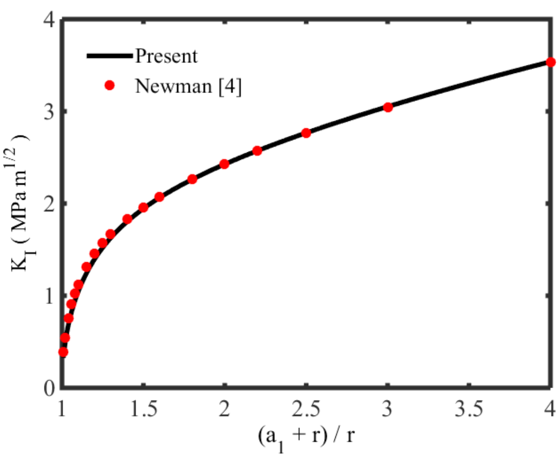

Fig. 2. Comparison of K_I for two equal cracks emanating from circular hole

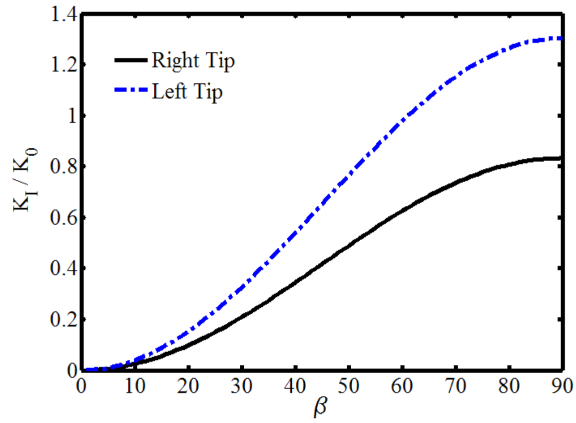

Fig. 3. K_I for two unequal cracks under different loading angle

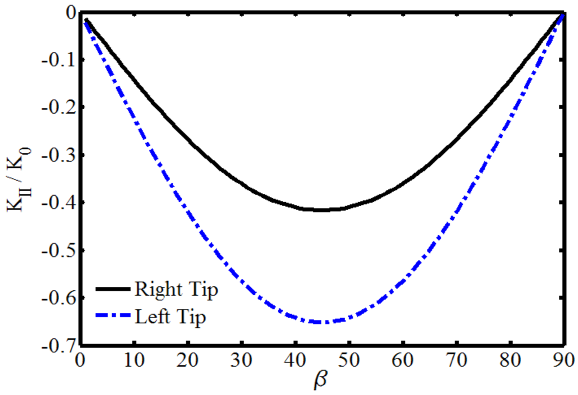

Fig. 4. K_{II} for two unequal cracks under different loading angle

3- Results and Discussion

K_I for two symmetrical cracks emanating from circular hole were compared with the results of Newman [4]. As shown in Fig. 2, the results of two methods were in good agreement, however for small cracks, it was observed up to 10% deviation.

Figs. 3 and 4 show the values of K_I and K_{II} for two unequal cracks of $a_1=0.5$, $a_2=1$ for different loading angles β, respectively.

It is observed that in the circular hole with two unequal cracks, in the case of $\beta=0^\circ$ which load is parallel to the cracks $K_I=0$, but for $\beta=90^\circ$ which load is perpendicular to the cracks, K_I is maximum. In these two cases, plane are not under shear load and $K_{II}=0$. Maximum K_{II} is occurred for $\beta=45^\circ$ due to maximum shear loading.

4- Conclusions

The problem of two unequal cracks emanating from circular hole in an infinite isotropic plane were investigated with the Muskhelishvili method. In order to use this method, a new conformal mapping function was presented. K_I and K_{II} of two cracks for different loading angles were obtained. The results show that by increasing the length of each cracks, K_I and K_{II} of two cracks increase. According to the results of other references, the presented formula has higher accuracy for medium and long cracks.

References

[1] N. Muskhelishvili, Some basic problems of the mathematical theory of elasticity, in, Nauka, Moscow, 1966.

[2] G.N. Savin, Stress concentration around holes, (1961).

[3] O. Bowie, Analysis of an infinite plate containing radial cracks originating at the boundary of an internal circular hole, Journal of Mathematics and Physics, 35(1-4) (1956) 60-71.

[4] J.C. Newman Jr, An improved method of collocation for the stress analysis of cracked plates with various shaped boundaries, (1971).

[5] X. Yan, Stress intensity factors for cracks emanating from a triangular or square hole in an infinite plate by boundary elements, Engineering Failure Analysis, 12(3) (2005) 362-375.

[6] S. Liu, S. Duan, Analytical solutions of cracks emanating from an elliptic hole in an infinite plate under tension, Chinese Journal of Mechanical Engineering, 27(5) (2014) 1057-1063.

[7] S. Liu, S. Duan, Analytical solutions of cracks emanating from an elliptical hole under shear, Chinese Journal of Aeronautics, 27(4) (2014) 829-834.

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