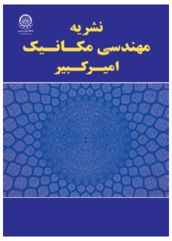

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Investigation of Torsional Static Behavior of Nano-rods Embedded in Elastic Medium Considering Surface Energy Effect

R. Nazemnezhad*, M. Aryanpour

Department of Engineering, Damghan University, Damghan, Iran

ABSTRACT: In this paper, the torsional static behavior of nano-rods under external torsional loads and embedded in elastic medium is investigated by considering the surface energy effect (the energy due to the surface shear modulus and the surface stress). For this purpose, surface stress components are obtained using the surface elasticity theory, and three types of external torsional loadings, uniform torque load, linear torque load, and sinusoidal torque load are considered. Then, the governing equation of motion of nano-rod is derived using the Hamilton's principle. The governing equation of motion is analytically solved for clamped-clamped and clamped-free boundary conditions; and the surface energy effect on torsional static behavior of nano-rod (rotational displacements) is investigated for various values of nano-rod radius and length, and torsional torque. In order to complete the investigations, effects of value and sign of the surface energy components on torsional static behavior of nano-rod are also considered. The obtained results show that the effect of the surface energy can be dependent on the geometrical parameters and the value and sign of the surface energy components. Results of the present study can be useful in design of nano-electro-mechanical systems like nano--bearings and rotary servomotors.

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1- Introduction

Torsional behavior of nano-structures is an important mechanical behavior that should be considered in design of nano-electro-mechanical systems like nano-bearings and rotary servomotors.

The aim of the present study is to investigate the surface energy effect on torsional static behavior of nano-rods embedded in elastic medium. To this end, governing equations of nano-rods is derived based on the surface elasticity theory [1]. Then, governing equations are solved analytically and for different boundary conditions the surface energy effect on torsional static behavior of nano-rods is investigated with considering various values for nano-rod radius, nano-rod length, and external torque.

2- Problem Formulation

Consider a nano-rod with length L $(0 \le x \le L)$ and radius R (see Fig.1) in x-y-z coordinates.

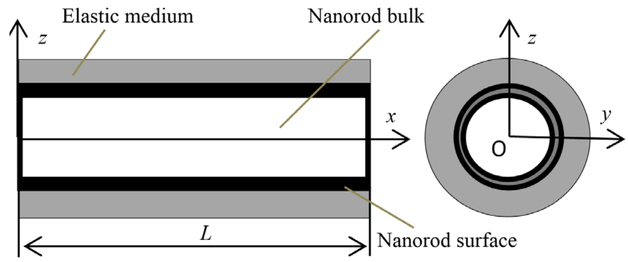

Fig.1. Schematic of nano-rod embedded in elastic medium with showing the nano-rod surface

The displacement field at any point of the nano-rod can be written as

$$u(x,t)=0;\ v(x,t)=-z\,\theta(x,t);\ w(x,t)=y\,\theta(x,t) \tag{1}$$

where $\theta\ (x,t)$ is the rotational displacement of the nano-rod about center O. By considering Eq. (1) as well as the surface elasticity theory, the nonzero bulk and surface stresses can be given, respectively

$$\sigma_{xy}=-Gz\frac{\partial\theta}{\partial x};\ \sigma_{xz}=Gy\frac{\partial\theta}{\partial x} \tag{2}$$

$$\tau_{xy}=2(\mu_0-\tau_0)\left(z\frac{\partial\theta}{\partial x}\right);\ \tau_{xz}=\tau_0 y\frac{\partial\theta}{\partial x} \tag{3}$$

where G is bulk shear modulus, τ_0 is residual surface tension under unconstrained conditions, and μ_0 is the surface Lamé constant.

Now, using the Hamilton's principle results in the governing equation and boundary condition as

$$(GI_P+C)\frac{\partial^2\theta}{\partial^2 x}-k_t\theta=T_e;\ \left.(GI_P+C)\frac{\partial\theta}{\partial x}\delta\theta\right|_0^L=0 \tag{4}$$

where

$$C=2\pi R^3(\mu_0-\tau_0);\ I_P=\iint(y^2+z^2)dA=\frac{\pi R^4}{2}$$

and k_t is the stiffness of the elastic medium, and T_e is the external torque.

It is desired to solve Eq. (4) for two types of boundary conditions, i.e. clamped-clamped (CC) and clamped-free (CF), and for three types of external torques. The solution of Eq. (4) gives:

Corresponding author, E-mail: adehghan@yazd.ac.ir

a) Uniform torque

a.1) CC

$$k_t \neq 0 \qquad \bar{\theta} = \frac{GI_P}{K_t L^2}\left(\frac{e^{-\bar{x}\alpha}(-1+e^{\bar{x}\alpha})(-e^{-\alpha}+e^{\bar{x}\alpha})}{(1+e^{\alpha})}\right)$$

$$k_t = 0 \qquad \bar{\theta} = \frac{GI_P}{(GI_P+C)}\left(\frac{\bar{x}^2}{2}-\frac{\bar{x}}{2}\right)$$

a.2) CF

$$k_t \neq 0 \qquad \bar{\theta} = \frac{GI_P}{K_t L^2}\left(\frac{e^{-\bar{x}\alpha}\left(-1+e^{\bar{x}\alpha}\right)\left(-e^{-2\alpha}+e^{\bar{x}\alpha}\right)}{1+e^{2\alpha}}\right)$$

$$k_t = 0 \qquad \bar{\theta} = \frac{GI_P}{GI_P+C}\left(\frac{\bar{x}^2}{2}-\bar{x}\right)$$

b) Linear torque

b.1) CC

$$k_t \neq 0 \qquad \bar{\theta} = \frac{GI_P}{K_t L^2}\left(\frac{e^{-\bar{x}\alpha}\left(-e^{\alpha}+e^{\alpha+2\bar{x}\alpha}+e^{\bar{x}\alpha}\bar{x}-e^{2\alpha+\bar{x}\alpha}\bar{x}\right)}{-1+e^{2\alpha}}\right)$$

$$k_t = 0 \qquad \bar{\theta} = \frac{GI_P}{GI_P+C}\left(\frac{\bar{x}^3-\bar{x}}{6}\right)$$

b.2) CF

$$k_t \neq 0 \qquad \bar{\theta} = \frac{GI_P}{K_t L^2}\left(\frac{e^{-\bar{x}\alpha}\left(-e^{\alpha}+e^{\alpha+2\bar{x}\alpha}-e^{\bar{x}\alpha}\bar{x}\alpha-e^{2\alpha+\bar{x}\alpha}\bar{x}\right)}{\alpha\left(-1+e^{2\alpha}\right)}\right)$$

$$k_t = 0 \qquad \bar{\theta} = \frac{GI_P}{GI_P+C}\left(\frac{\bar{x}^3-3\bar{x}}{6}\right)$$

c) Sinusoidal torque

c.1) CC

$$k_t \neq 0 \qquad \bar{\theta} = \frac{-GI_P}{GI_P+C}\left(\frac{\sin(\pi\bar{x})}{\pi^2+\alpha^2}\right)$$

$$k_t = 0 \qquad \bar{\theta} = \frac{-GI_P}{GI_P+C}\left(\frac{\sin(\pi\bar{x})}{\pi^2}\right)$$

c.2) CF

$$k_t \neq 0 \qquad \bar{\theta} = \frac{-GI_P}{GI_P+C}\left(\frac{e^{-\bar{x}\alpha}\left(-\pi e^{\alpha}+\pi e^{\alpha+2\bar{x}\alpha}+\alpha e^{\bar{x}\alpha}\sin(\pi\bar{x})+\alpha e^{2\alpha+\bar{x}\alpha}\sin(\pi\bar{x})\right)}{\alpha\left(1+e^{2\alpha}\right)\left(\pi^2+\alpha^2\right)}\right)$$

$$k_t = 0 \qquad \bar{\theta} = \frac{GI_P}{(GI_P+C)}\left(\frac{\sin(\pi\bar{x})}{\pi^2}-\frac{\bar{x}}{\pi}\right)$$

where $\bar{x} = x/L$ and $\alpha = \left(k_t L^2/(GI_P+C)\right)^{0.5}$.

3- Results and Discussion

In Table 1 non-dimensional torsional displacement of nano-tube under constant external torque (T_e=1 n.N) for various values of nano-tube length and stiffness of elastic medium are compared with those given in Ref. [2].

The mechanical and geometrical properties of nano-tube are: E=1.095 TPa, G=0.4601 GPa, ν=0.19, D_{in}=0.5 nm, D_{out}=2 nm, L=5 nm. Table 1 shows that the derivation of the presented formulations is correct and the results are reliable.

For better representation of surface energy effect on torsional static behavior of nano-rods, Displacement Ratio parameter (DR) is defined as

$$\text{DR}=\frac{\text{Rotational displacement with considering surface energy effect}}{\text{Rotational displacement without considering surface energy effect}}$$

In Figs. 2 and 3 variations of DR versus the length of nano-rod are plotted for CC and CF boundary conditions, respectively.

Table 1. Comparison of non-dimensional torsional displacement of nano-tube for various values of elastic medium stiffness and two boundary condition types

$\bar{x}$		0.5		1.0	
kt n.N	BC	Present study	Ref. [3]	Present study	Ref. [3]
0	CC	0.125	0.125	0.000	0.000
	CF	0.375	0.375	0.498	0.498
1	CC	0.124	0.124	0.000	0.000
	CF	0.362	0.362	-	-
10	CC	0.115	0.115	0.000	0.000
	CF	0.286	0.286	0.375	0.375

Figs. 2 and 3 show that in presence of the surface energy DR increases while it is reported that the surface energy decreases the transverse displacement of the nano-beams [3].

It is observed from Fig.2 that the surface energy effect is

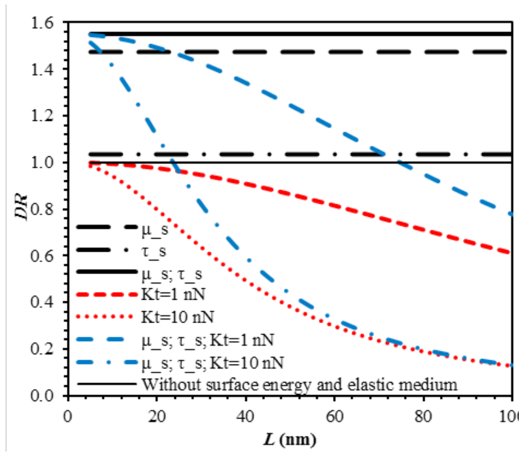

Fig.2. Variations of DR versus the length of nano-rod for clamped-clamped boundary condition

independent from the length of nano-rod. In addition, Fig.2 displays that components of the surface energy do not have the same effects on DR. Fig.2 also shows that the elastic medium decreases the DR and its effect depends on the length of nano-rod which is on the contrary to the surface energy effect. Furthermore, increasing the stiffness of the elastic medium increases its decreasing effect. Another point which can be obtained from Fig.2 is that when both the surface energy and the elastic medium effects are simultaneously considered for small length of nano-rod the effect of the surface energy is dominant while it is the other way round for large length of nano-rod. The last point from Fig.2 is that the curves considering both the surface energy and the elastic medium effects intersect the horizontal axis indicating that there is a specific length in which the increasing effect of the surface energy cancels the decreasing effect of the elastic medium. In Fig.3 variations of DR versus length of nano-rod is displayed for CF boundary condition. Fig.3 demonstrated that trends of curves are similar to those displayed in Fig.2. The only difference between these two figures is that the decreasing effect of the elastic medium on DR of nano-rod with CF boundary condition is more than that with CC one. In the other words, softer the boundary condition of the nano-rod is, more the decreasing effect of the elastic medium is.

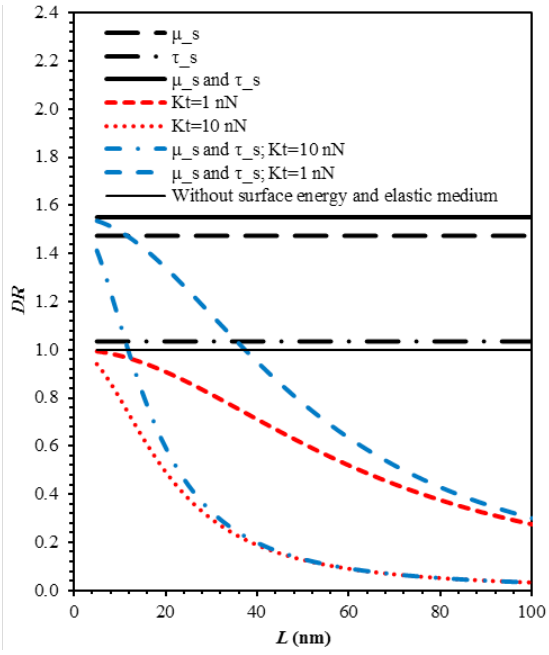

Fig.3. Variations of DR versus the length of nano-rod for clamped-free boundary condition

4- Conclusions

This study shows that the surface energy effect depends on its components sign and the radius of nano-rod, and its effect is not dependent on the length of nano-rod and boundary condition type. While it is observed that the elastic medium has a decreasing effect and its effect depends on the length and radius of nano-rod and the boundary condition type.

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