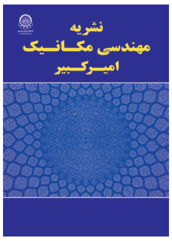

Amirkabir Journal of Mechanical Engineering

Amirkabir J. Mech. Eng., 50(1) (2018) 43-46
DOI: 10.22060/mej.2017.11764.5178

Nonlinear Free Vibration in Flexure Beams with an Intermediate Rigid Element and a Tip Mass

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ABSTRACT: A usual method in achieving a proper value for the ratio of constraint to degree of freedom stiffness in a compliant mechanism, is using an intermediate rigid element in its constitutive beams. This paper aims to study the nonlinear free vibration of a stiffened beam with a mass connected to its tip. Hamilton's principle is used to find nonlinear partial differential equations governing behavior of the beam. The mode-shapes of the normalized and linearized system are then found analytically and verified via Abaqus simulations. Using a single mode approximation, the first mode-shape of the system is used along with the Lagrange equations to find governing ordinary differential equations of degree of freedom and degree of constraint dynamic. These equations are then solved numerically using MATLAB. The Discrete Fourier Transform of dynamic responses show that the degree of freedom dynamic contains a single dominant frequency, while the constraint dynamic contains three main harmonics. It is observed that dominant frequencies are essentially natural frequencies of the linearized system which are available in a closed form. The suggested analytical formulations as well as the proposed frequency analysis, is expected to provide an effective approach for analytical dynamic modeling of more complex compliant mechanisms.

Review History:

Received: 1 July 2016
Revised: 24 March 2017
Accepted: 7 May 2017
Available Online: 17 May 2017

Keywords:

Flexure beam
Rigid intermediate element
Nonlinear oscillations
Mode shape
Frequency analysis

1- Introduction

The use of compliant devices in different dynamic systems such as single [1] and multi-axis resonators [2], high-speed scanners [3], energy harvesting devices [4] and nano/micropositioning systems [5] is well-established. These mechanisms provide a guided motion via elastic deformation, instead of employing sliding or rolling joints. This property has made them well-suited for the systems with high precision demand.

The main building blocks of these mechanisms are the flexible beams. Since flexure beams have a long slender geometry, Euler-Bernoulli's beam theory can be successfully utilized to model the static, dynamic and vibratory behavior of them. This theory may be used along with a linear or nonlinear beam formulation [6]. The theoretical and experimental modelings of large amplitude vibration of the Euler-Bernoulli beams have been extensively investigated in the prior art. For example, Nayfeh and Mook [7] have modeled and analytically solved nonlinear vibrations of beams. Moeenfard and Awtar [8] modeled geometric nonlinearities in the planar vibration of a beam flexure with a tip mass and provided analytical perturbation solutions for the endpoint axial and lateral displacements of the beam.

In compliant mechanisms, the stiffness of the system in the DOF direction has to be minimized while its stiffness in the constraint direction shall be maximized. To achieve so, the flexure beam is usually stiffened using an intermediate rigid element. As far as the authors know, the dynamic behavior of such beams has not been yet reported in the prior art.

This paper deals with the modeling dynamic behavior of a flexure beams with an intermediate rigid part and a tip mass. Hamilton's principle is utilized to find the governing equations of motion. Analytical mode-shapes of the system are derived in a closed form and verified using the commercially available finite element software Abaqus. A single-mode approximation is then used to find the ODEs governing the dynamics of the system. These ODEs are then solved numerically and analyzed using DFT.

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2- Methodology

The schematic view of a partially stiffed flexure beam with a tip mass is depicted in Fig. 1.

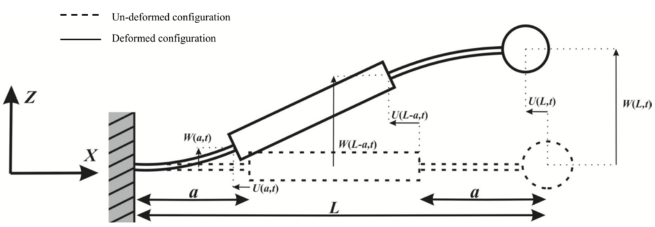

Figure 1. The deformed and un-deformed configuration of a flexure beam with an intermediate rigid element and a tip mass

The strain energy of this multi-body system can be expressed in terms of the displacement field of the system. Then using Hamilton's principle, the equations of motion and the corresponded boundary conditions of this system can be obtained. The normalized form of these equations can be linearized and homogenized for finding the mode shapes of the system as

$$\varphi_k(x) = c_1^{(k)}\sin(\beta_k x) + c_2^{(k)}\cos(\beta_k x) + c_3^{(k)}\sinh(\beta_k x) + c_4^{(k)}\cosh(\beta_k x) \qquad 0 < x < \alpha \tag{1}$$

$$\varphi_k(x)=c_5^{(k)}\sin(\beta_k x)+c_6^{(k)}\cos(\beta_k x)+c_7^{(k)}\sinh(\beta_k x)+c_8^{(k)}\cosh(\beta_k x) \quad 1-\alpha<x<1 \tag{2}$$

where $x=X/L$, φ_k is the k'th mode of the system, β_k is some coefficients of the k'th natural frequency and $c_i^{(k)}$ s are some constants which can be determined via satisfying boundary conditions.

In Fig. 2, the first mode shape obtained from the presented analytical technique is compared with that of FE simulations and excellent agreement is observed.

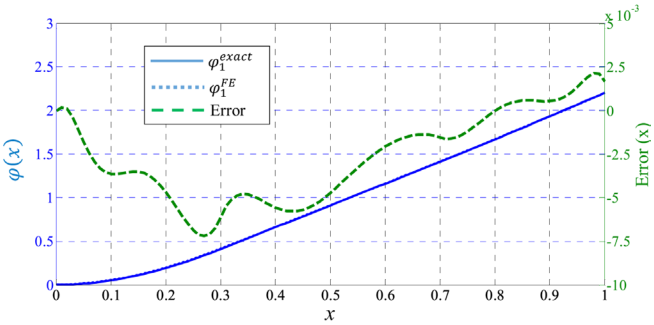

Figure 2. Comparison of the analytical and numerical prediction of the first mode shape of the system

Using a single-mode approximation, one can say

$$W(X,t)=\frac{\varphi(X)}{\varphi(L)}W_L(t) \tag{3}$$

By substituting this single mode approximation for the strain and kinetic energies along with using the Lagrange equations, the differential equations of motion can be obtained as

$$\frac{d^2u_\alpha(\tau)}{d\tau^2}+c_{11}w_t(\tau)\left(\frac{d^2w_t(\tau)}{d\tau^2}\right)+c_{11}\left(\frac{dw_t(\tau)}{d\tau}\right)^2+c_{12}u_\alpha(\tau)+c_{13}u_t(\tau)+c_{14}w_t^2(\tau)=0 \tag{4}$$

$$\frac{d^2u_t(\tau)}{d\tau^2}+c_{21}u_t(\tau)+c_{22}u_\alpha(\tau)+c_{23}w_t^2(\tau)=0 \tag{5}$$

$$\begin{aligned}&\frac{d^2w_t(\tau)}{d\tau^2}+w_t(\tau)+\frac{c_{32}}{c_{31}}\left(\frac{d^2w_t(\tau)}{d\tau^2}\right)w_t^2(\tau)\\&+\frac{c_{33}}{c_{31}}\left(\frac{d^2u_\alpha(\tau)}{d\tau^2}\right)w_t(\tau)+\frac{c_{34}}{c_{31}}\left(\frac{dw_t(\tau)}{d\tau}\right)^2w_t(\tau)\\&+\frac{c_{35}}{c_{31}}\left(\frac{dw_t(\tau)}{d\tau}\right)+\frac{c_{36}}{c_{31}}w_t(\tau)u_\alpha(\tau)\\&+\frac{c_{37}}{c_{31}}w_t(\tau)u_t(\tau)+\frac{c_{39}}{c_{31}}w_t^3(\tau)=f_z\end{aligned} \tag{6}$$

where u_α, u_t and w_t are the normalized axial displacements of the beam at $X=a$ and $X=L$,respectively and the normalized tip transverse displacement. Also f_z is the normalized tip force in the z direction.

3- Results and Discussion

The solution of these equations for a sample initial condition is presented in Fig. 3.

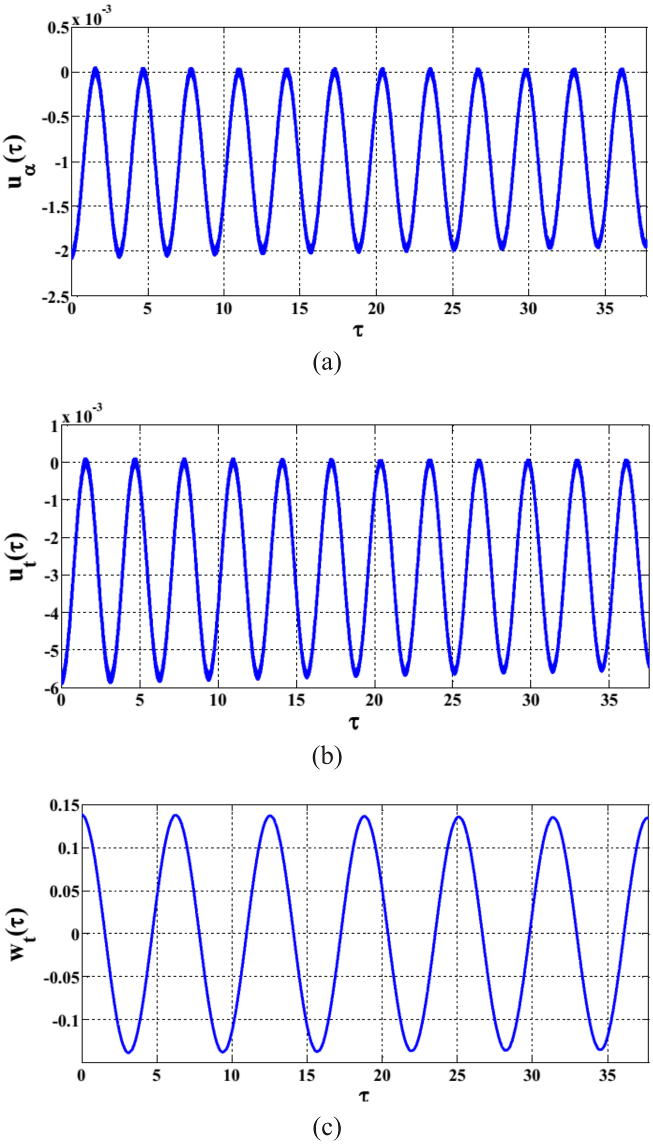

Figure 3. Normalized axial displacements of the beam at (a) *X=a* and (b) *X=L* and (c) normalized tip transverse displacement at *X=L*

The time response presented in these figures can be further analyzed using discrete Fourier transform to reveal the frequency content of them. It may be easily verified that the frequency content of the response of the nonlinear system are essentially those of the linear homogeneous system and can be analytically obtained by solving the following equation.

$$\begin{vmatrix}-\omega^2+c_{21} & c_{22}\\ c_{13} & -\omega^2+c_{12}\end{vmatrix}=0 \tag{7}$$

4- Conclusions

Understanding the nonlinear vibrations in flexure mechanisms is an essential first step to address better design insights. The existence of elastic stretching, geometric nonlinearities, and complicated boundary conditions make this investigation very difficult. In this paper, a nonlinear model is presented for the dynamic behavior of compliant beams with a tip mass and an intermediate rigid element. The exact mode-shapes of this system were derived and numerical solutions were provided for the axial and transverse dynamic of the system. The presented approach in this paper can effectively be used to model and simulate multi-body compliant mechanisms and provide a clear understanding of how different design parameters may affect the nonlinear dynamic performance of the system.

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Please cite this article using:

A. Davaq, H. Moeenfard, M. Moavenian, Nonlinear Free Vibration in Flexure Beams with an Intermediate Rigid Element and a Tip Mass, *Amirkabir J. Mech. Eng.*, 50(1) (2018) 125-136.

DOI: 10.22060/mej.2017.11764.5178