



## Stability of a Blade with Varying Rotation Speed by Considering the Lagging Motion

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**ABSTRACT:** Principal parametric resonance in rotating blades with varying rotating speed is investigated in this paper. In the presented model, the lagging-axial coupling motion due to Coriolis force is considered. The governing equations of motion are the available equations in the literature based on the exact geometrical formulation for unshearable blades. The rotating speed of the blades is considered as a mean value perturbed by a small harmonic variation. The variation frequency of the perturbed value is considered twice the one of the lagging frequencies and/or one of the axial frequencies which causes the principal parametric resonance. The direct method of multiple scales is implemented to study the dynamic instability produced by the principal parametric resonance. A closed form relation which defines the stability region boundary under the condition of the principal parametric resonance is derived using the method of multiple scales. The current results are validated by comparison with the available results in the literature. After validation of the results, a comprehensive study has been adjusted for illustration of the rotating speed effects and mode number influences on the parametric stability region.

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### 1- Introduction

The principal parametric resonance of a clamped-free beam subjected to the principal parametric resonance and the internal resonance between the two modes was examined by Chin and Nayfeh [1]. Ghayesh and Balar [2] analyzed the nonlinear vibration and the parametric stability of an axially moving viscoelastic Rayleigh beam employing the Method of Multiple Scales (MMS). Özhan and Pakdemirli [3] employed the MMS to investigate a general model of continuous systems having the third order nonlinearity subjected to the parametric excitation. Sahoo et al. [4] employed the direct MMS to examine the transverse vibration of a nonlinear moving simply-supported Euler-Bernoulli beam model subjected to the principal parametric resonance in the presence of internal resonance.

Among the researches on the rotating structures, some researchers have been studied the rotating beams and blades stability. Crespo da Silva and Hodges [5] investigated the dynamic response and the stability of rotating beam considering the aerodynamic forces, analytically. Turhan and Bulut [6] applied one of the perturbation methods on the one and two degrees of freedom discretized equations of motion obtained by the Galerkin approach to determine the frequency response and nonlinear natural frequencies. Arvin et al. [7] applied the direct MMS on the third order nonlinear equations of motion to investigate the nonlinear flapping frequencies behavior and the associated backbone curves. Arvin et al. [8] studied the rotating Euler-Bernoulli beams subjected to the principal parametric excitation. According to the literature review, the study on the parametric

resonance of blades with varying rotation speed is poor and due to the presence of these structures and their prominent roles in the various industries, developing this aspect of instability examination seems to be essential. Hence in this paper, the principal parametric resonance of these blades regarding the coupling of the axial-lagging motions has been investigated. The existed coupling is due to consideration of the Coriolis forces. The considered governing equations of motion are associated with the unshearable blades which are formulated based on the exact geometrical formulation in Ref. [9]. In order to investigate on the dynamics of the blades subjected to the principal parametric excitation, the MMS has been applied directly on the equations of motion to deliver the phase and amplitude modulation equations. The acquired modulation equations have been employed to examine the stability of the trivial and nontrivial solutions. The presented bifurcation diagrams, demonstrate the stability and instability regions as a function of excitation amplitude and the detuning parameter for the different modes of the blade which is subjected to the parametric excitation, for different rotating speeds.

### 2- Methodology

Aeroelastic forces besides accelerating and decelerating of the blades are some factors which produces the varying rotation speed. In this case, the blade rotation speed is considered as a constant mean velocity where is perturbed by a harmonic term i.e.  $\lambda_R(t) = \lambda_r (1 + \hat{\Omega} \cos \hat{\Omega}_f t)$ .  $\lambda_r$  is the mean rotation speed and  $\hat{\Omega}$  and  $\hat{\Omega}_f$  are, respectively, the amplitude and the frequency of the parametric excitation.

Applying the MMS on the equations of motion and performing a long mathematical procedure similar to Ref. [8], deliver the

modulation equations. Then the Routh-Hurwitz criterion (see Ref. [8]) has been employed to deliver the stability region.

### 3- Discussion and Results

The considered rotating blade for the numerical analysis is an Aluminum beam [8]. The rotation speed effects on the parametric excitation stability region have been studied here. The stability region for the first axial mode and also for the first three lagging modes besides the first axial mode has been depicted, respectively, in Figs. 1 and 2. As it is obvious the instability region for the lower modes is wider. Also, the instability region for the first three lagging modes is larger than the first axial mode. On the other hand, by increasing the lagging mode number, the possibility of simultaneous parametric resonance, i.e axial and lagging parametric instability occurrence, is clear. This phenomenon has been shown in Figs. 3 and 4. It can be seen that in the rotation speed about 1940rpm the 5th lagging and the 1st axial modes have been engaged simultaneously in the principal parametric resonance which is very destructive for the rotating blades. In this case an 1:1 internal resonance exists between the first axial and the fifth lagging modes.

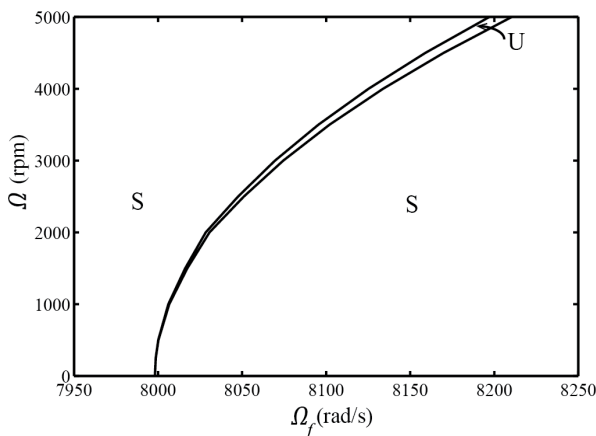


Fig. 1. The stability region for the first axial mode; S: the stable region, U: the unstable region

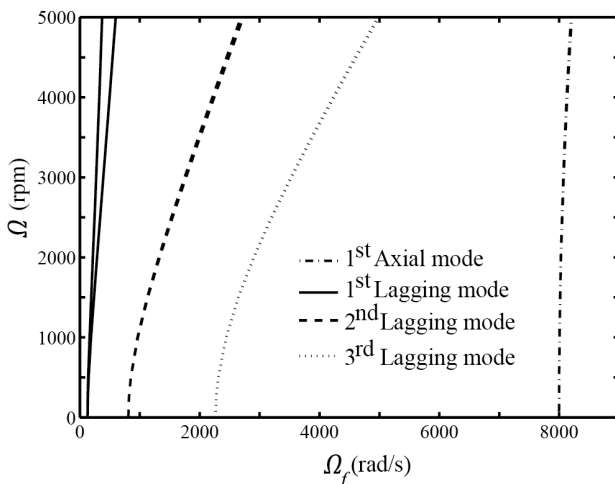


Fig. 2. The stability region for the first three lagging modes beside the first axial mode

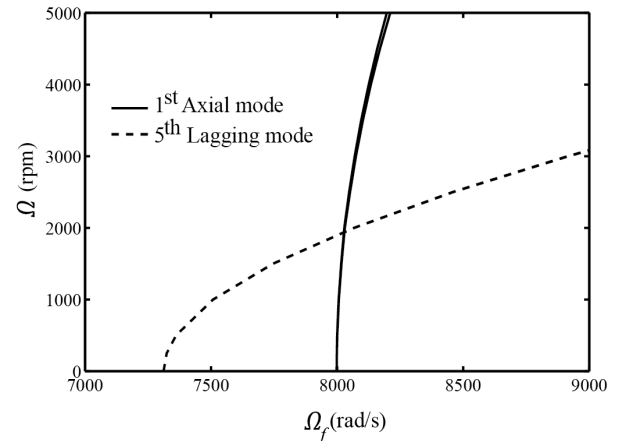


Fig. 3. The stability region for the fifth lagging mode beside the first axial mode

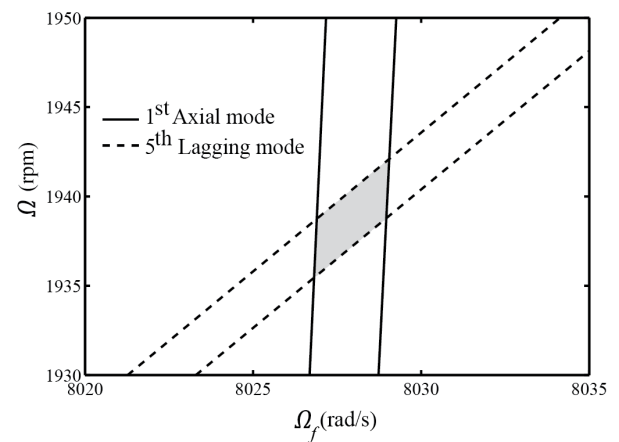


Fig. 4. The zoomed in of Fig. 3

### 4- Conclusions

The dynamic analysis of rotating blades subjected to the principal parametric resonance was investigated in this paper. The main findings are as follows; By increasing the rotation speed the instability range grows monotonically. The lower modes which possess the lower natural frequencies have wider range of instability. The instability regions predicted by the lagging modes are wider than the axial modes. The possibility of simultaneous parametric resonance between the excitation frequency and the lagging and the axial frequencies exists in a specific rotating speed. This phenomenon occurs when an 1:1 internal resonance exists between the first axial mode and one of the lagging modes.

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