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Optimal Trajectory Planning of a Quadrotor Based on Minimum Effort

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ABSTRACT: The optimal design of path planning for unmanned aerial vehicles with many potential applications ranging from mapping to supporting rescue operations will improve their performance. Hence, the aim of this paper is to determine the optimal trajectory of quadrotor robot based on minimizing engine torque in point-to-point motion. First, the dynamic equations of quadrotor motion are derived in state space form by using Newton's method. In this investigation, the computational method to solve the trajectory planning problem is based on the indirect solution of open-loop optimal control problem. The Pontryagin's minimum principle (PMP) is used to obtain the optimality conditions, which is lead to a standard form of a two-point boundary value problem. Finally, to evaluate the efficacy of the proposed method, numerical simulation is performed for a quadrotor and the optimal trajectory is designed based on minimize torque. The results illustrate the power and efficiency of the method to overcome the high nonlinearity nature of the problem such as path optimization of multi-rotor helicopters (tri, quad, hexa, octa, etc.).

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1- Introduction

Quadrotor Unmanned Aerial Vehicles (UAVs) have attracted more and more attention in recent years. In order to make unmanned vehicle technology more useful, it is critical to consider the path planning optimization of the system in point-to-point maneuvers, since it increases the efficiency and economic usage of quadrotor systems. In a typical quadcopter application. The vehicle which operates in three dimensional spaces, has six degrees of freedom, and has differential constraints, including limited speed and maximum acceleration [1]. The basic dynamical model of the quadcopter is the starting point for all of the studies but more complex aerodynamic properties has been introduced as well [2]. Hamel et al. [3] proposed a model for the dynamics of a four rotor vertical take-off and landing vehicle known as an X4-flyer. The model incorporates the airframe and motor dynamics as well as aerodynamic and gyroscopic effects due to the rotors for quasi-stationary flight conditions. On the other hand, finding optimal trajectory for a quadcopter is a difficult task because of complex dynamics. Korayem et al. [4] used the indirect solution of the optimal control problem to determine the dynamic load capacity of a flexible link manipulator. The Pontryagin's minimum principle has been used for path planning of the flexible link manipulator. Lai et al. [5] considered a time-optimal movement of the hovering quadrotor helicopter between two configurations. A Non Linear Programming (NLP) method is proposed to solve a set of highly nonlinear differential equations. However, since the quadrotor helicopter system is nonlinear, it will be a difficult task to find a feasible solution for the formulated NLP problem. Therefore, genetic algorithm is used to generate feasible solutions for the time-optimal problem. This paper proposes an indirect solution of the optimal control problem for path planning of quadrotor systems with different cost function. The indirect solution method is based on Pontryagin's minimum principle which is a suitable approach in the cases where the system has a large number of degree of freedom or optimization of the various objectives is targeted. The generalized Euler-Lagrange formulation is used to derive dynamic equations of a quadrotor system. Hamiltonian function for a proper objective function is formed, then using the PMP, optimality necessary conditions are obtained. The obtained equations lead to a standard form of a boundary value problem which is solved by numerical techniques. The obtained simulation results demonstrate the accuracy and merits of the proposed method.

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2- Quadrotor Dynamics and Kinematics

The quadcopter structure is presented in Fig. 1 including the corresponding angular velocities, torques and forces created by the four rotors. The different motions of a quadrotor are generated by varying the speed of four rotors as well as changing the thrust of each blade. The earth-frame denotes a frame that everything discussed can be referenced and the body-frame is a frame attached to the helicopter.

The absolute linear and angular position of the quadrotor is defined in the inertial frame *x, y, z* axes with P and η, respectively. Pitch angle θ determines the rotation of the quadcopter around the *y*-axis. Roll angle ϕ determines the rotation around the *x*-axis and yaw angle ψ around the *z*-axis. Vector q contains the linear and angular position vectors

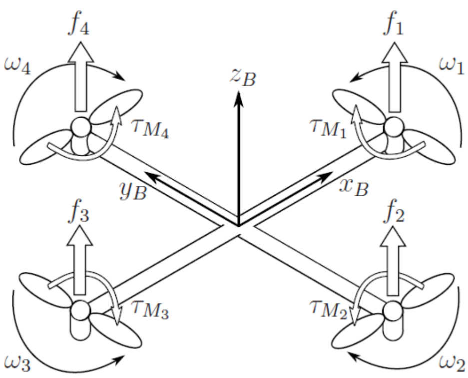

Fig. 1. Schematic view of a quadrotor system

$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad \eta = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}, \quad \mathrm{q} = \begin{bmatrix} P \\ \eta \end{bmatrix} \tag{1}$$

The quadrotor is assumed to be rigid body and thus Newton-Euler equations can be used to describe its dynamics. In the inertial frame, the centrifugal force is nullified. Thus, only the gravitational force and the magnitude and direction of the thrust are contributing in the acceleration of the quadcopter

$$\begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = -g \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \frac{F_T}{m} \begin{bmatrix} C_\psi S_\theta C_\phi + S_\psi S_\phi \\ S_\psi S_\theta C_\phi - C_\psi S_\phi \\ C_\theta C_\phi \end{bmatrix} \tag{2}$$

In the body frame, the angular acceleration of the inertia $\dot{\omega}_B$, the centripetal forces $\omega_B \times (I\omega_B)$ and the gyroscopic forces Ã are equal to the external torque τ_B

$$I\dot{\omega}_B + \omega_B \times (I\omega_B) + \Gamma = \tau_B \tag{3}$$

The angular accelerations in the inertial frame are then attracted from the body frame accelerations with the transformation matrix W_η^{-1} and its time derivative

$$\ddot{\eta} = \frac{d}{dt}\left(W_\eta^{-1}\right)\omega_B + W_\eta^{-1}\ \dot{\omega}_B \tag{4}$$

For convenience of computing the optimal control problem, the system inputs are defined as

$$\begin{aligned} u_1 &= F_1 + F_2 + F_3 + F_4, \\ u_2 &= F_4 - F_2, \\ u_3 &= F_3 - F_1, \\ u_4 &= -F_1 + F_2 - F_3 + F_4 \end{aligned} \tag{5}$$

By using these equations, the optimal trajectory planning problem can be formulated.

3- Optimization Strategy

The purpose of trajectory planning is to move the quadcopter from the original location to the desired location by controlling the rotor angular velocities of the quadcopter. In this section, an indirect solution of optimal control problem is applied for the off-line global trajectory planning of a quadrotor. In comparison with other methods, the open-loop optimal control approach provides a powerful tool for designers to create various optimal paths via defining the proper performance measure. The optimal trajectory is then obtained by solving the $2n$ differential equations.

$$\begin{aligned} \dot{X}^*(t) &= \frac{\partial H}{\partial \lambda}\left(x^*(t), u^*(t), \lambda^*(t), t\right) \\ \dot{\lambda}^*(t) &= -\frac{\partial H}{\partial X}\left(x^*(t), u^*(t), \lambda^*(t), t\right) \\ H\left(x^*(t), u^*(t), \lambda^*(t), t\right) &\le H\left(x^*(t), \bar{u}(t), \lambda^*(t) t\right) \end{aligned} \tag{6}$$

In which the symbol (*) refers to the extremals of $x(t), u(t)$ and $\lambda(t)$; and $\bar{u}(t)$ denotes the admissible control value.

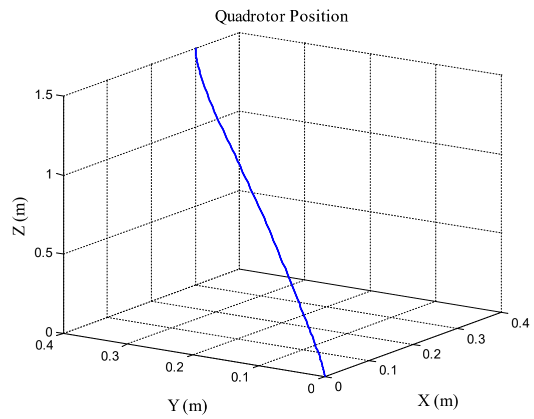

Fig. 2. The optimal paths between point A and B via minimum thrust

4- Results and Discussion

A simulation study has been performed to evaluate further the accuracy and efficiency of the quadrotor flight performance on finding the optimal path between two points with different objective functions. A quadcopter is considered by six degrees of freedom, which has four control inputs subject to saturation. The linear external force is the total thrust of the rotors. The path planning algorithm is developed to find the optimal trajectories that meet various objective functions, such as fuel efficient between two points. Hence, simulation studies are carried out for the quad-rotor helicopter which will be moved from the initial configuration X_0 at $t_0 = 0$ to the desired final configuration X_f at t_f. The initial and final velocities of the quad-rotor system are zero.

5- Conclusions

Quadrotors offer exceptional agility, with typically high thrust-to-weight ratios, and large potential for angular acceleration due to the outward mounting of the propellers. The trajectory generator is tasked with computing flight paths that achieve the task objective, while respecting the quadrotor dynamics. Hence, this paper presents a heuristic approach for optimal path planning that the optimization strategy is based on the indirect solution of the open-loop optimal control problem in point-to-point maneuver. The model predicts the effect of the thrust and torques induced by the four propellers on the quadrotor motion. Hamiltonian function for a proper objective function is formed, then using the PMP optimality necessary conditions are obtained. Finally, in order to verify the effectiveness of the proposed approach, several simulation studies on a quadcopter are performed for finding the optimal paths at point-to-point motion with different objective function like minimum effort. The results clearly indicate the effect of proposed approach on the performance improvement of quadrotor systems.

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