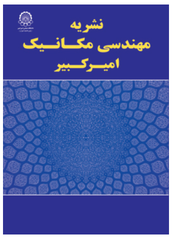

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Repair of Free Vibration Behavior of a Cracked Rotating Timoshenko Beam Using a Piezoelectric Patch and Applying Differential Transform Method

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ABSTRACT: This paper presents an analytical method for the application of piezoelectric patches to repair a rotating cracked beam. The beam equations of motion are obtained based on the Timoshenko beam theory including the effects of shear deformation and rotary inertia. The criterion applied for the repair is to modify the first natural frequency of the cracked beam towards that of the healthy beam applying a piezoelectric patch. Due to this, an external voltage is applied to actuate a piezoelectric patch bonded on the beam that decreases the effect of the crack on the vibration characteristics of the beam. First, the coupled equations of motion are discretized by applying the assumed modes method. Then, the cracked beam is modeled as numbers of healthy segments connected by two linear springs at the crack locations (one, extensional and the other, rotational). The compatibility requirements on the crack section and on the ends of the piezoelectric patch are considered to obtain the relationships between any two spans. Finally, applying the semi-analytical differential transform method, the natural frequencies and mode shapes of the system can be calculated. Numerical simulations are performed to assess the effects of different conditions on the repair moment coefficient. The presented model is validated by comparing the results with those available in the literature where, the natural frequencies are in a reasonably good agreement with the reported results.

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1- Introduction

The dynamics of the cracked structures have been the subject of many studies during the last decade. When a structural component is subjected to a crack, its stiffness is reduced which leads to a decrease in its natural frequencies. Masoud and Al-Said [1] exhibit a new crack localization algorithm based on a mathematical model. Loya et al. [2] modeled a beam as two segments connected by two massless springs (one extensional and another one rotational).

Piezoelectric materials have been widely used as sensors or actuators for their inherent coupling electromechanical characteristic. One interesting aspect is the interaction behaviors of the structures comprising piezoelectric and elastic materials. Wang et al. [3] presented a method to repair a cracked beam via piezoelectric patch. Ariaei et al. [4] studied an analytical method for the application of piezoelectric patches for the repair of cracked beams subjected to a moving mass. The criterion used for the repair is to alter the first natural frequency of the cracked beam towards that of the healthy beam using a piezoelectric patch. Chesne and Pezerat [5] presented a new class of distributed piezoelectric sensors, using laminate theory and the weak forms of the equation of motion for a beam.

The main objective of this study is to extend the piezoelectric patch for the repair of a cracked rotating Timoshenko beam. The criterion is modifying the first natural frequency of the cracked beam towards that of the healthy beam applying a piezoelectric patch in which the other natural frequencies are also very close to that of the healthy beam. The cracked beam is modeled as the number of segments connected by two massless springs at the crack location. A formulation based on the Differential Transform Method (DTM) will be developed. The natural frequencies are calculated and the effects of different conditions on the repair moment coefficient are assessed. Some of the results are validated against those reported in the literature where a reasonably good agreement is observed.

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2- Methodology

A simply supported beam of length L with an open crack at $x_2 = x_c$ is considered as in Fig. 1. Based on the repair concept presented by Ariaei et al. [4], a piezoelectric patch of length $x_3 - x_1$ is bonded at the bottom of the beam as shown in Fig. 1 where x_3 and x_1 are the distance from the left side of the beam to the right and the left ends of the patch, respectively.

The vibration amplitude of the transverse displacement and the rotation cross section angle of the beam is denoted by $w_i(x,t)$ and $\theta_i(x,t)$ on the interval $x_{i-1} \le x \le x_i$, respectively; where the sub-index i represents the i^{th} segment and i = 1, 2, 3, and 4 (see Fig. 1). The entire beam is now divided into

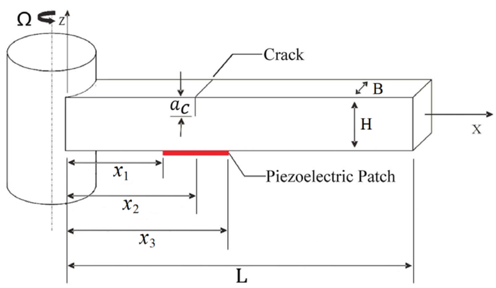


Fig. 1. A cracked rotating beam with a piezoelectric patch

four segments which are separated by the crack and the two ends of the piezoelectric patch. The criterion is to apply an external voltage in order to actuate a piezoelectric patch and to modify the first natural frequency of the cracked beam towards that of the healthy beam. According to Timoshenko theory, the governing differential equation for each segment of flap-wise bending motion i is given by:

$$-\rho A\frac{\partial^2 w_i}{\partial t^2}+\frac{\partial}{\partial x}\left(T\frac{\partial w_i}{\partial x}\right)+\frac{\partial}{\partial x}\left(\kappa AG\left(\frac{\partial w_i}{\partial x}-\theta_i\right)\right)=0 \quad (1)$$

$$-\rho I\frac{\partial^2 \theta_i}{\partial t^2}+\rho I\Omega^2\theta_i+\frac{\partial}{\partial x}\left(EI\frac{\partial \theta_i}{\partial x}\right)+\kappa AG\left(\frac{\partial w_i}{\partial x}-\theta_i\right)=0 \quad (2)$$

The bending moment on the beam exerted by the piezoelectric in the non- dimensional form can be expressed as [4]:

$$M_p=\frac{1}{2}\sigma_x Dh(H+h)\frac{L_b}{EI}=-J_p\left(\theta_4\left(x_3^+\right)-\theta_2\left(x_1^+\right)\right) \quad (3)$$

In the equation above, J_p is the repair moment coefficient and is given by:

$$J_p=\frac{ge_{31}^2 L_b}{4C_p EI}D^2\left(H+h\right)^2 \quad (4)$$

The left segment of the crack has been treated as a beam connected to the right by two elastic springs (a rotational and an extensional) at the cracked section. The stiffness of the springs depends on the crack depth and on the geometry of the cracked section. Furthermore, the compatibility requirements for the left side and the right side of the piezoelectric can be expressed. Finally, applying the differential transform as the solution method leads to an eigenvalue problem to obtain the natural frequencies and the mode shapes of the system.

3- Results and Discussion

The fundamental natural frequencies obtained by applying DT method in this article are validated against those available in the reference [1] which used energy method for a healthy and cracked rotating beam (Fig. 2). That the differences are small and agreeable can be observed in Fig. 2; hence validating the results of this study.

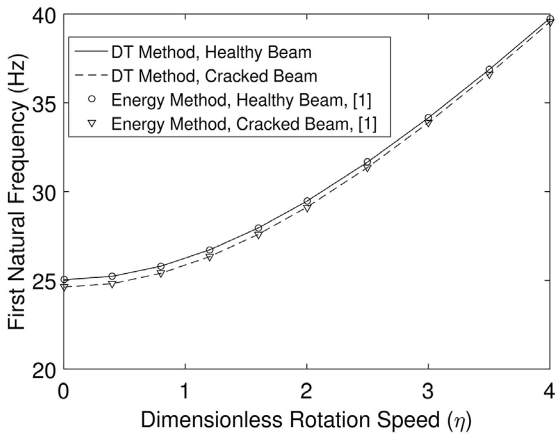


Fig. 2. Effect of the rotation speed on the fundamental natural frequency of a healthy and cracked beam

Now, the effects of the crack location and rotation speed on the repair moment coefficient are investigated in Figs. 3 and 4, respectively. It is observed that the repair moment coefficient is decreased by increasing the distance of the crack location to the clamped end and by increasing the rotation speed in Fig. 3 and Fig. 4, respectively. That is because of the fundamental natural frequency of the cracked beam to be closed to the healthy one by increasing the distance of the crack location to the clamped end and as it can be seen in Fig. 2, by increasing the rotation speed. This leads to a less essential value of applied voltage exerted to the piezoelectric.

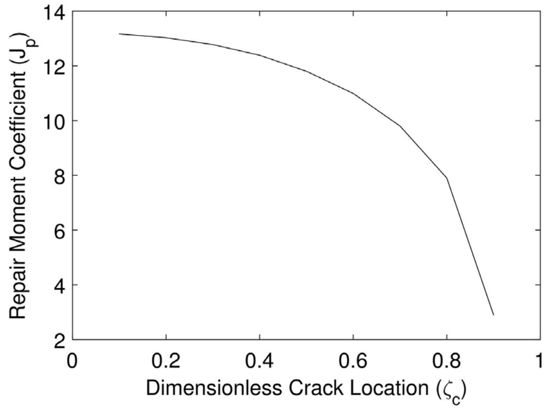
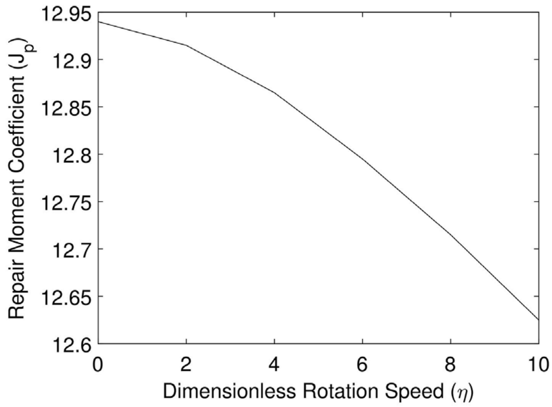


Fig. 3. Effect of the crack location on the repair moment coefficient



5- Fig. 4. Effect of the rotation speed on the repair moment coefficient

4- Conclusions

An analytical method on the use of a piezoelectric patch for the repair of a cracked Timoshenko beam was presented. The criterion used for the repair is to alter the first natural frequency of the cracked beam towards that of the healthy beam applying a piezoelectric patch. Numerical simulations were performed with respect to different conditions such as the rotation speed and crack location. It is observed that the repair moment coefficient is increased by an increase in the depth of the crack and a decrease in the angular velocity, hub radius, and distance of the crack location to the clamped end of the beam.

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