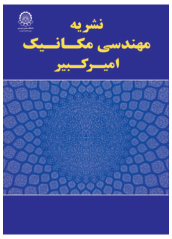

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Analysis of Static Pull-in Instability and Nonlinear Vibrations of an Functionally Graded Micro-Resonator Beam

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ABSTRACT: In This paper, the behavior of a functionally graded micro-resonator that is excited by the combination of DC electrostatic force and AC harmonic force, Casimir force, the uniform temperature change is investigated based on the Euler-Bernoulli beam theory and the nonlinear von-Karman strain. It is assumed that material properties follow exponential law distributions through the thickness direction. The principle of minimum total potential energy and the modified couple stress theory are used to derive the nonlinear governing differential equation of micro-beam. Static differential equations are solved by using the differential quadrature method. The effects of temperature change, material length scale parameter and power distributions model on pull-in voltage are investigated. Applying perturbation method with multiple scales technique and numerical integration of the second order nonlinear ordinary differential equation, an approximation for the response of the micro-beam to the primary-resonance excitation is obtained.

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1- Introduction

In the recent decades, developments in micro and nano technologies have facilitated the fabrication of large number of devices having a wide range of applications especially in the field of sensing and actuation, which are used in high-temperature environments, such as aerospace field and petroleum exploration [1, 2].

The composite materials have been used successfully in the aircraft industry, petroleum exploration and other engineering applications in macro scale. The traditional composite materials are weak in the environments with high-temperature and corrosion effects.

Micro-resonators are one of the most interesting elements which usually have high nonlinearities [3].

In most Micro-Electro Mechanical Systems (MEMS) micro-resonators, the core element in the system is a (C–C) micro-beam. In electrostatically actuated Functionally Graded Materials (FGM) micro-beam, when the applied direct current (DC) voltage between the two electrodes reaches a certain value, the functionally graded beam becomes unstable and collapses toward the fixed electrode which creates the short circuit. This leads to pull-in instability phenomenon. It can be used in micro-switch application but in micro-resonator, in order to achieve stable motions two electrodes are separated, and upper electrode vibrates in the equilibrium position.

Many papers have been published to study modeling MEMS structures. [3, 4] investigate controlling and optimization of the pull-in voltage.

2- Methodology

In many previous studies, static pull-in instability is modeled. Since the micro-beam thickness used in the micro resonator structure is at the micron scale, the size-dependent behavior should be considered in the static and vibrational analysis. This dependency has been reported in various experiments and is known as modified coupling stress theory.

According to this theory, the strain energy U_s in an isotropic elastic material occupying a region Λ can be written as follow

$$U_s = \frac{1}{2}\int_{\Lambda}(\sigma:\varepsilon + m:\chi)d\Lambda \tag{1}$$

In which, the von Karman strain tensor ε, stress tensor σ, the symmetric curvature tensor χ, the deviatoric part of the couple stress tensor and stress tensor m are respectively defined as

$$\varepsilon = \frac{1}{2}\left(\nabla u + (\nabla u)^T + (\nabla u)^T \nabla u\right) \tag{2-1}$$

$$\sigma = \lambda tr(\varepsilon)I + 2\mu\varepsilon \tag{2-2}$$

$$\chi = \frac{1}{2}\left(\nabla\theta + (\nabla\theta)^T\right) \tag{2-3}$$

$$m = 2l^2\mu\chi \tag{2-4}$$

where u is the displacement vector, μ and λ are lame's constants. *l* is a material length scale parameter, which is regards as a material property characterizing the effect of the couple stress. θ is the rotation vector expressed as

$$\theta = \frac{1}{2}curl(u) \tag{3}$$

Fig. 1 shows the (C–C) FGM micro-beam which is used in micro-resonator. Its components include a fixed electrode

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Corresponding author, E-mail: h_mohammadi@shirazu.ac.ir as a ground plane and a movable upper electrode made of a functionally graded materials with length *L*, width *b*, and thickness *h*, separated by a dielectric spacer with an initial gap $g_0(x)$. The (C–C) FGM micro-beam is excited by combination of DC and AC harmonic voltage.

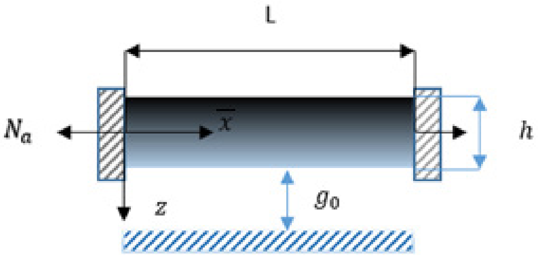

Fig. 1. Schematics of the (C–C) FGM micro-beams

From Eqs. (1) and (2) and boundary conditions, the motion equation for an FGM micro-beam subjected to electric field force, temperature change and geometric nonlinearity due to the mid-plane stretching can be derived as [4].

$$\frac{\partial^2 w}{\partial t^2}+\left(\frac{\partial^4 w}{\partial x^4}\right)+m_1\left(\frac{\partial^2 w}{\partial x^2}\right)+c\left(\frac{\partial w}{\partial t}\right)+\left(\frac{\partial^2 w}{\partial x^2}\right)\int_0^L\left(m_2\left(\frac{\partial w}{\partial x}\right)^2+m_3\left(\frac{\partial^2 w}{\partial x^2}\right)\right)dx=\frac{m_4 V_0^2}{(1-w)^2}+\frac{m_5 V_0^2}{(1-w)}+\frac{m_6}{(1-w)^4} \tag{4}$$

Using Galerkin method and considering the first vibrational mode and by applying multiple time scales technique the frequency response is

$$(F_1 V_{AC})^2=4\left(\sigma a\omega+a^3\left[\frac{5\xi_2^2}{12\omega^2}-\frac{3\xi_3}{8}\right]\right)^2+ca\omega^2 \tag{5}$$

Eq. (5) is an implicit function of the amplitude a of the periodic solution, as a function of the detuning parameter σ, quadratic stiffness ξ_2 and cubic stiffness ξ_3 terms, the damping coefficient C, and the amplitude of the harmonic excitation V_{AC} .

The geometric parameters of micro-beam are *L* = 410 μm, *b* = 100μm, g_0 = 1.18μm, *h*=1.5μm, the dimensionless material length scale parameter is set to be *h/l* = 3 unless stated otherwise, and the axial residual force *Na* = 0.

The influence of material property gradient index *n* on static pull-in of the FGM micro-beam with and without the effect of Casimir force is shown in Fig. 2.

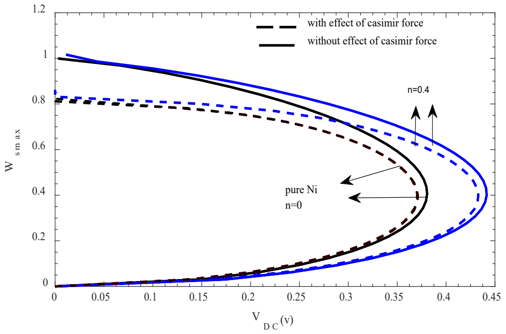

Fig. 2. The influence of the power law distributions

Fig. 3 depicts the effect of material property gradient index n on frequency response, which is obtained by the perturbation technique.

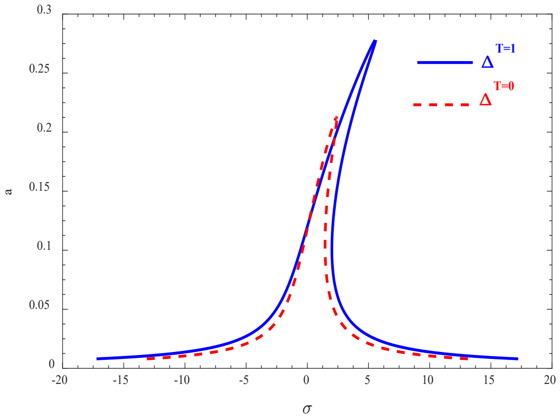

Fig. 3. The influence of temperature change in primary resonance.

The influence of temperature change on Frequency response curve is shown in Fig. 4.

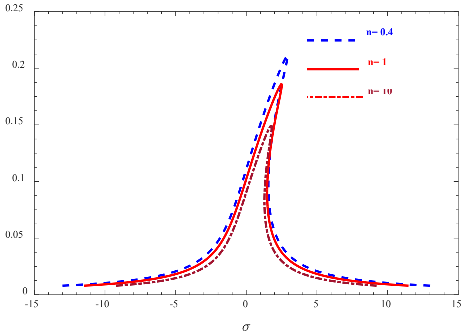

Fig. 4. The influence of the power law distributions in primary resonance.

3- Conclusions

This paper investigates static pull-in and nonlinear vibration of FGM micro-beams under combined electrostatic and intermolecular forces and temperature changes. The size dependent relations and temperature-dependency of the effective material properties are specially considered. The governing equation and boundary conditions are derived based on Euler–Bernoulli beam theory and von Kármán nonlinearity, and then the equations are solved numerically through DQ approximation to obtain the nonlinear natural frequency for clamped–clamped micro-beams. The parametric effects of the volume fraction profiles, temperature change, and size dependency of the material properties and intermolecular Casimir force on the static pull-in and non-linear vibration characteristics of the FGM micro-beam are discussed.

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