Amirkabir Journal of Mechanical Engineering

Amirkabir J. Mech. Eng., 51(3) (2019) 1-3
DOI:

Computation Algorithms of Feasible Sets and Robust Feasible Sets for Constrained Linear Time-Invariant Systems Parametrized with Orthonormal Functions

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ABSTRACT: Feasible sets and robust feasible sets have an indispensable role in a priori stability guarantee of the constrained systems and in model predictive control. This article presents two algorithms for generating these sets for constrained linear time-invariant systems. Because the conventional algorithms for generating these sets must be applied iteratively over time, they are incapable of dealing with systems having the input vector constructed in any domain other than the time domain. The new algorithms, presented in this article, remove this limitation by treating the input vector monolithically over the time horizon. The presented algorithm for computation of the robust feasible set is capable of incorporating disturbances as well as parametric uncertainties that can be formulated as polytopes. For the verification, the results of the proposed algorithms were compared with results of the conventional methods in a similar circumstance. Finally, examples are presented to compare the computation times of the proposed algorithms with the conventional ones and to illustrate the effect of input vector parametrization, employing orthonormal functions, on the feasible region and the robust feasible region. Results showed that the parametrization improved the feasible set and robust feasible set.

Review History:

Received:
Revised:
Accepted:
Available Online:

Keywords:

Feasible set
Robust feasible set
Parametrization
Orthonormal functions
Model predictive control

1- Introduction

Feasible set is one of the major elements of control algorithms for constraints systems. The set is defined as a subset of the state space for which there exists a control law that satisfies all constraints. The volume of these sets relates closely to the control horizon and grows with the growth of the control horizon [1]. Similarly, robust feasible set is defined as a subset of the state space for which there exists a control law that satisfies all system's constraints for every possible disturbance scenario [2]. These sets are positively invariance [3] and they provide a priori knowledge of recursive feasibility and robust recursive feasibility for model predictive control method [4]. The problem of the computational complexity of model predictive control has evoked a number of solutions. Among the solution methods, parametrization of the input vector has proved to be a successful scheme to provide a reduction in computational burden as well as to guarantee stability [5, 6]. This paper provides algorithms to systematically determine feasible sets and robust feasible sets for systems in which the input vector is parametrized via orthonormal functions.

2- Background

The following uncertain dynamical system with state and input constraint is considered in the paper:

$$x(i+1)=A_{cl}\left(w^p(i)\right)x(i)+B\left(w^p(i)\right)v(i)+Ew^d(i),\quad i=0,1,\cdots,N_p-1 \tag{1-a}$$

$$x(0)=x_0 \tag{1-b}$$

$$Kx(i)+v(i)\in\mathbb{U},\qquad i=0,1,2,\cdots,N_p-1 \tag{1-c}$$

$$x(i)\in\mathbb{X},\qquad i=0,1,2,\cdots,N_p-1 \tag{1-d}$$

$$x(N_p)\in\mathbb{X}_T. \tag{1-e}$$

where the uncertain terms $A_{cl}(w^p(i))$, $B(w^p(i))$ and $w^d(i)$ are confined inside some polytopes which contain the origin as an interior point. For such system the feasible set $X_F(N_p)$ is defined as:

$$\mathbb{X}_F\left(N_p\right)=\left\{x_0\in\mathbb{R}^n\,|\,\exists v(i),Kx(i)+v(i)\in\mathbb{U},x(i)\in\mathbb{X},\ i=0,1,\cdots,N_p-1,x\left(N_p\right)\in\mathbb{X}_T\right\} \tag{2}$$

Similarly, the robust feasible set is defined as:

$$\mathbb{X}_{RF}\left(N_p\right)=\left\{x_0\in\mathbb{R}^n\,\middle|\,\exists v(i),Kx(i)+v(i)\in\mathbb{U},x(i)\in\mathbb{X},i=0,\cdots,N_p-1,x\left(N_p\right)\in\mathbb{X}_T,\ \forall\left(w^p(i),w^d(i)\right)\in\mathbb{W}^p\times\mathbb{W}^d\right\}. \tag{3}$$

3- Results and Discussion

3- 1- Parametrization of linear time-invariant systems via orthonormal functions

The general form of orthonormal functions is obtained from

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Takenaka-Malmquist equation [7]:

$$Ù_j(z)=\frac{\sqrt{1-|\xi_j|^2}}{z-\xi_j}\prod_{k=1}^{j-1}\left[\frac{1-\bar{\xi}_k z}{z-\xi_k}\right],\ j=1,2,\cdots \tag{4}$$

in which $\xi_j \in \mathbb{C}, |\xi_j|<1$ are stable poles of the functions and $\bar{\xi}$ is the complex conjugate of ξ. For $\xi_j=\bar{\xi}_j=a, 0\le a<1$ the network reduces to Laguerre network which is the simplest orthonormal basic functions.

The orthonormal functions (4) have the ability to capture the control signal *v(i)* in Eq. (1) via a parametrization. For a single input system, the Laguerre parametrization is

$$v(i)=L(i)^T\eta,\ i=0,1,\cdots,N_p-1 \tag{5}$$

in which η is the vector of new decision variables. Using Eq. (5), the parametrized system is:

$$x(i+1)=A_{cl}\left(w^p(i)\right)x(i)+B\left(w^p(i)\right)L(i)^T\eta+Ew^d(i),\ i=0,1,\cdots,N_p-1 \tag{6-a}$$

$$x(0)=x_0 \tag{6-b}$$

$$Kx(i)+L(i)^T\eta\in\mathbb{U},\quad i=0,1,2,\cdots,N_p-1 \tag{6-c}$$

$$x(i)\in\mathbb{X},\qquad i=0,1,2,\cdots,N_p-1 \tag{6-d}$$

$$x(N_p)\in\mathbb{X}_T. \tag{6-e}$$

3- 2- Computation of feasible sets for the orthonormal parametrized systems

Because the decision variable vector η of the parametrized system Eq. (6) does not have a one-to-one correspondence with time, the traditional algorithm for generating feasible set, expressed in reference [1] for instance, fails to be applied. Therefore, a new algorithm should be devised to produce such set. The core idea is to use batch Eqs. (6) where all input and state elements are stack together in appropriate vectors. The batch equation for Eq. (6-a) of a nominal system, i.e. $A_{cl}(w^p(i))=A_{cp}$ $B(w^p(i))=B$, $w^d(i)=0$, is:

$$X(N_p)=S_X(N_p)x_0+S_U(N_p)Ã\eta \tag{7}$$

where S_X and S_U are appropriate convolution matrices. Using Eq. (7), the constraints which are shown in Eqs.(6-c) to (6-e) can be modelled as:

$$Ó_X X(N_p)\le\sigma_X \tag{8-a}$$

$$Ó_X S_X(N_p)x_0+Ó_X S_U(N_p)Ã\eta\le\sigma_X \tag{8-b}$$

$$Ó_U Ó_K S_X(N_p-1)x_0+Ó_U Ó_K S_U(N_p-1)Ã\eta+Ó_U Ã\eta\le\sigma_U \tag{8-c}$$

where $\Sigma_X,\Sigma_K,\Sigma_U$ are appropriate diagonal matrices of constraint sets. Similarly, σ_X,σ_U are appropriate vectors related to the constraints.

Using Eqs. (7) and (8), the algorithm of the feasible set X_F (N_p,N) for the parametrized system which is shown in Eq. (6) is:

$$\mathbb{X}_F(N_p,N)=\left\{x_0\left|\begin{bmatrix}Ó_X S_X(N_p) & Ó_X S_U(N_p)Ã\\ Ó_U Ó_K S_X(N_p-1) & \left(Ó_U Ó_K S_U(N_p-1)+Ó_U\right)Ã\end{bmatrix}\begin{bmatrix}x_0\\ \eta\end{bmatrix}\le\begin{bmatrix}\sigma_X\\ \sigma_U\end{bmatrix}\right.\right\} \tag{9}$$

3- 3- Computation of robust feasible sets for the orthonormal parametrized systems

Similar to Eq. (7) the batch equation for an uncertain system with a disturbance term w_d is:

$$X(N_p)=S_X\left(A(w^p);N_p\right)x_0+S_U\left(A(w^p),B(w^p);N_p\right)Ã\eta+S_U\left(A(w^p),E;N_p\right)W^d \tag{10}$$

Then, the robust feasible set for the parametrized system Eq. (6) is:

$$\mathbb{X}_{RF}(N_p,N)=\{x_0|\begin{bmatrix}Ó_X S_X\left(A(w^p);N_p\right) & Ó_X S_U\left(A(w^p),B(w^p);N_p\right)Ã\\ Ó_U Ó_K S_X\left(A(w^p);N_p-1\right) & \left(Ó_U Ó_K S_U\left(A(w^p),B(w^p);N_p\right)+Ó_U\right)Ã\end{bmatrix}\begin{bmatrix}x_0\\ \eta\end{bmatrix}\le\begin{bmatrix}\sigma_X-Ó_X S_U\left(A(w^p),E;N_p\right)W^d\\ \sigma_U-Ó_U Ó_K S_U\left(A(w^p),E;N_p\right)W^d\end{bmatrix},\forall w^p\in\mathbb{W}^p,w^d\in\mathbb{W}^d\} \tag{11}$$

Figs. 1 and 2 compare the result of the proposed algorithms with the result of references [1] and [2] for a special condition in which the orthonormal network generates same basic functions as the Finite Impulse Response (FIR) network, i.e. *a=0.N=*N_p. The system matrices are:

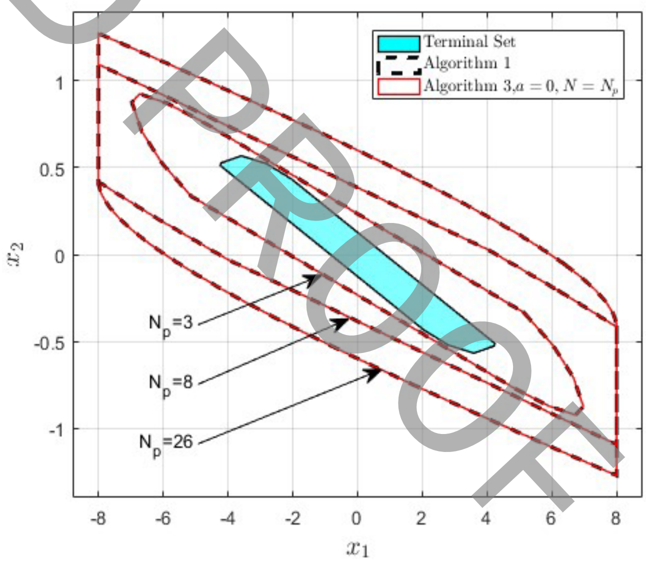

Figure 1. Comparison of feasible sets computed by the proposed algorithm and reference [1].

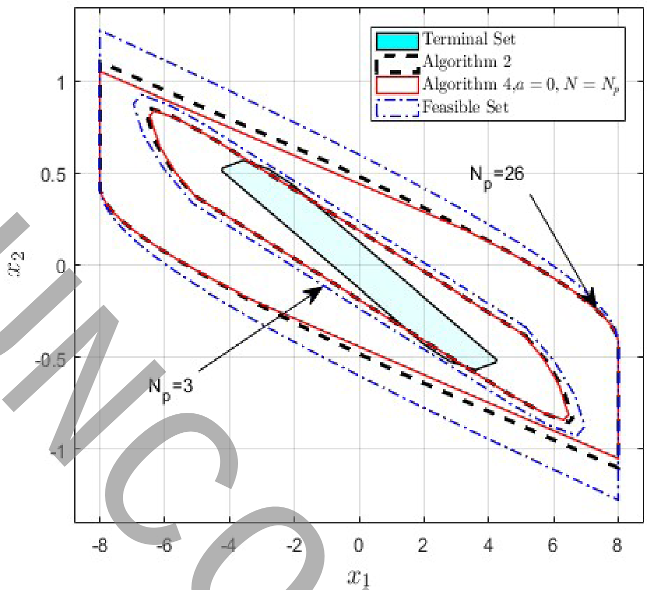

Figure 2. Comparison of robust feasible sets computed by the proposed algorithm and reference [2].

$$A=\begin{bmatrix}1.1 & 2\\ 0 & 0.95\end{bmatrix}, B=\begin{bmatrix}0\\ 0.08\end{bmatrix}, H_u=\begin{bmatrix}1\\ -1\end{bmatrix}, h_u=\begin{bmatrix}1\\ 1\end{bmatrix},$$
$$H_x=\begin{bmatrix}1 & 0\\ -1 & 0\\ 0 & 1\\ 0 & -1\end{bmatrix}, h_x=\begin{bmatrix}8\\ 8\\ 8\\ 8\end{bmatrix}, K=-\begin{bmatrix}1.19 & 7.88\end{bmatrix}. \tag{12}$$

4- Conclusions

This paper presents algorithms to determine feasible sets and robust feasible sets for systems in which the input vector is parametrized via orthonormal functions. Then, the algorithms are applied to a system where the Laguerre function is used to parametrize the input vector. The comparison of the result shows the validity of the proposed algorithms.

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