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Buckling Investigation of Cylindrical Shell Using Size-Dependent New Super Element

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ABSTRACT: In this paper, using modified couple stress theory, a new cylindrical shell element is introduced. Since classical continuum theory is unable to correctly compute stiffness and account for size effects in micro/nanostructures, higher-order continuum theories such as modified couple stress theory have taken on great appeal. In this paper, using modified couple stress theory and using shell model in place of beam model, buckling of nanotubes is investigated via the finite element method. The new cylindrical shell element based on the super element's shape function defined and the mass-stiffness matrix has been developed. In addition to modified couple stress cylindrical shell element, the classical cylindrical shell super element can also be defined by setting size effects parameter to zero in the equations. In special cases, in order to investigate the application of the equations developed, the cylindrical nanoshell buckling is studied using a modified couple stress cylindrical shell element and the results are validated using the analytical method. In addition, the effects of parameters such as size effects parameter, length, and thickness on cylindrical shell buckling are investigated.

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1- Introduction

Since classical continuum theories, due to their lack of intrinsic length scales, are unable to correctly predict the behavior of micro/nanostructures, use of higher-order theories which are able to account for size effects in computations has become popular [1-3].
Due to the topological structure of a nanotube in the form of a cylindrical shell, the use of a shell model is significantly more effective in correctly predicting nanotubes behavior than the use of a beam model [4].
Since the complexity of micro/nanostructures such as complicated loading or geometry, the use of the analytical method is not always possible, it is especially important to use other current methods such as Finite Element Method (FEM).
In the present paper, using the finite element method and modified couple stress theory which are able to take size effects into account and to model micro/nanostructures correctly, a new cylindrical shell element is introduced.

2- Element Definition and Relationships

A 16-node cylindrical super element at length L, radius R, and thickness h is considered according to Fig. 1.
Displacement of a point of the cylindrical shell element which can be represented by vector U with components u, v, and w along r, θ and x is expressed as follows:

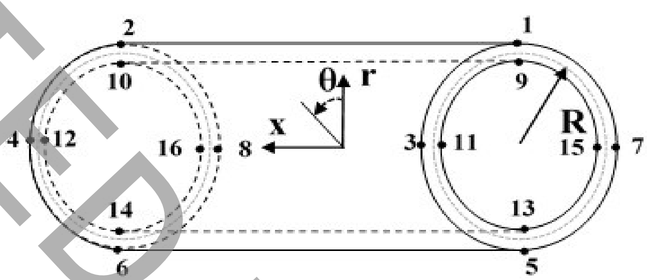

Figure 1. Cylindrical shell super element.

$$\mathbf{U} = \{u \quad v \quad w\}^T = \mathbf{N}_{3\times48}\,\mathbf{d}_{48\times1} \tag{1}$$

where N is the shape functions matrix and d is nodes displacements vector.
According to the modified couple stress theory, strain energy is defined as follows [5]:

$$U = \frac{1}{2}\int_{\Omega}\left(\boldsymbol{\sigma} : \boldsymbol{\varepsilon} + \mathbf{m} : \boldsymbol{\chi}\right) dV \tag{2}$$

Classical and non-classical components of the strain tensor for the cylindrical element are defined as:

$$\varepsilon_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \tag{3}$$

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$$\chi_{ij} = \frac{1}{4}\left(e_{ipq}\eta_{jpq} + e_{jpq}\eta_{ipq}\right) \tag{4}$$

where u_i, e_{ipq}, and η_{ipq} represent the components of the displacement vector, permutation symbol, and deviator stretch gradient tensor, respectively. The classical components of strain tensor are determined as follows:

$$\begin{aligned}
&\varepsilon_{xx} = \frac{\partial u}{\partial x},\ \varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{w}{r},\ \varepsilon_{rr} = \frac{\partial w}{\partial r},\\
&\gamma_{x\theta} = \gamma_{\theta x} = \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x},\\
&\gamma_{xr} = \gamma_{rx} = \frac{\partial u}{\partial r} + \frac{\partial w}{\partial x},\\
&\gamma_{\theta r} = \gamma_{r\theta} = \frac{\partial v}{\partial r} - \frac{v}{r} + \frac{1}{r}\frac{\partial w}{\partial \theta}
\end{aligned} \tag{5}$$

which can be expressed in the matrix form as:

$$\text{å} = \mathbf{LU} = \mathbf{LN\,d} = \mathbf{B\ d} \tag{6}$$

Higher order strain components are obtained as follows:

$$\begin{aligned}
&\chi_{rr} = \frac{1}{2}\left(\frac{1}{r^2}\frac{\partial u}{\partial \theta} - \frac{1}{r}\frac{\partial^2 u}{\partial \theta \partial r} + \frac{\partial^2 v}{\partial x\, \partial r}\right)\\
&\chi_{xx} = -\frac{1}{2}\left(\frac{1}{r}\frac{\partial v}{\partial x} + \frac{\partial^2 v}{\partial x\, \partial r} - \frac{1}{r}\frac{\partial^2 w}{\partial x\, \partial \theta}\right)\\
&\chi_{\theta\theta} = \frac{1}{2r}\left(\frac{\partial^2 u}{\partial r \partial \theta} - \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial x} - \frac{\partial^2 w}{\partial x\, \partial \theta}\right)\\
&\chi_{x\theta} = \chi_{\theta x} = \frac{1}{4}\begin{pmatrix} \frac{\partial^2 u}{\partial x\, \partial r} - \frac{1}{r^2}\frac{\partial v}{\partial \theta} - \\ \frac{1}{r}\frac{\partial^2 v}{\partial r \partial \theta} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} - \frac{\partial^2 w}{\partial x^2} \end{pmatrix}\\
&\chi_{rx} = \chi_{xr} = \frac{1}{4}\begin{pmatrix} -\frac{1}{r}\frac{\partial^2 u}{\partial x\, \partial \theta} - \frac{1}{r}\frac{\partial v}{\partial r} + \frac{v}{r^2} \\ -\frac{\partial^2 v}{\partial r^2} + \frac{\partial^2 v}{\partial x^2} + \frac{1}{r}\frac{\partial^2 w}{\partial r \partial \theta} - \frac{1}{r^2}\frac{\partial w}{\partial \theta} \end{pmatrix}\\
&\chi_{r\theta} = \chi_{\theta r} = \frac{1}{4}\begin{pmatrix} -\frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} - \frac{1}{r}\frac{\partial u}{\partial r} + \frac{\partial^2 u}{\partial r^2} \\ +\frac{1}{r}\frac{\partial^2 v}{\partial x\, \partial \theta} + \frac{1}{r}\frac{\partial w}{\partial x} - \frac{\partial^2 w}{\partial x\, \partial r} \end{pmatrix}
\end{aligned} \tag{7}$$

which can be expressed in the matrix form as:

$$\div = \bar{\mathbf{L}}\mathbf{U} = \bar{\mathbf{L}}\mathbf{N\,d} = \bar{\mathbf{B}}\ \mathbf{d} \tag{8}$$

The components of Cauchy stress tensor and the symmetric part of the higher-order stress tensor are determined as follows:

$$\text{ó} = \mathbf{C}\ \text{å} = \mathbf{C\ Bq} \tag{9}$$

(10)

According to the above equations the stiffness matrix of the new element defined as follows:

$$\mathbf{K} = \int_V \left(\mathbf{B}^T\mathbf{CB} + \bar{\mathbf{B}}^T\mathbf{D}\bar{\mathbf{B}}\right)dV \tag{11}$$

The element stress stiffness matrix $[K_\sigma^e]$ is defined for an isoparametric element as follows:

$$[\mathbf{K}_\sigma^e] = \int_V [\mathbf{G}]^T[S][\mathbf{G}]dV \tag{12}$$

3- Results and Discussion

In this section, attempts have been made to demonstrate the use of the new cylindrical shell element in solving the problem. Static linear analysis is the basis for a general buckling problem and the equilibrium equation can be stated as below:

$$[K]\{D\} = \{R\} \tag{13}$$

where {R} is an arbitrary load. Finally, the critical load of the buckling problem is defined as follows:

$$\det([K] + \lambda[K_\sigma]) = 0 \tag{14}$$

The dimensionless critical buckling load obtained for different radius/thickness scale are compared with analytical results in Table 1 that shows by increasing radius/thickness scale, critical axial buckling load will decrease, which is due to the decrease in rigidity. The critical axial buckling load obtained from couple stress theory are greater than that of analytical, which is due to the presence of one size parameter in couple stress theory.

Furthermore, the effect of length to radius ratio on dimensionless critical axial buckling load of nano cylindrical shell for different size effects, the effect of length scale parameter for different thickness and the effect of thickness of nano cylindrical shell were investigated in this paper. The results show that an increase in the length/radius scale leads to a decrease in the critical axial buckling load, which is due to the decrease in nano cylindrical shell rigidity. It is shown that with increasing size effect parameter, the effect of length increasing on dimensionless critical axial buckling load reduction is more and by increasing the size parameter, the critical axial buckling load will increase too, which is

Table 1. Dimensionless critical buckling load

R/h	Analytical [6]	Couple stress theory
30	0.0310	0.0354
35	0.0257	0.0304
40	0.0222	0.0246
45	0.0193	0.0211
50	0.0171	0.0183

due to the increase in rigidity. It is obvious that using this new cylindrical shell super element, *h/R* increment leads to increasing the critical axial buckling load.

4- Conclusions

Size-dependent cylindrical shell super element formulation is developed in this paper by using shell model in place of beam model and using the modified couple stress theory in place of classical continuum theory. The stiffness matrix and geometric stiffness matrix for cylindrical shell super element are developed in this paper, and by means of size-dependent finite element, formulation are extended to more precisely account for nanotube buckling. Based on the results, it is appropriate to use this cylindrical shell super element in micro/nano-scale problems. The results showed using this cylindrical shell element, the rigidity of the nano-shell are greater than that in the classical theory, and the critical axial buckling load obtained from couple stress theory are greater than that of analytical, which is due to the presence of one size parameter in couple stress theory. The findings indicate that the new cylindrical shell element is reliable for simulating micro/nanostructures and can be used for the analysis of size effect and has desirable convergence characteristic.

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