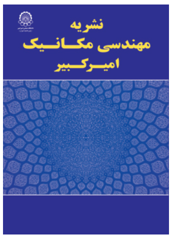

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Determination of Critical Speeds and Divergence Instability Boundary for a High-Speed Double- Helical Planetary Gear System

M. Karimi Khoozani[1], M. Poursina[2*], A. Pourkamali Anaraki[1]

[1] Department of Mechanical Engineering, Shahid Rajaee Teacher Training University, Tehran, Iran
[2] Department of Mechanical Engineering, University of Isfahan, Isfahan, Iran

ABSTRACT: High-speed planetary gears, or more generally, gyroscopic systems are not preserve energy and therefore subjected to instability. In this research, the dynamic equations for double- helical planetary gear system in 3-D space and considering 6-DOF for each member are extracted. Then, the system stability in the range of critical speed is investigated. In the extraction of equations, the constant mesh stiffness is assumed and the gyroscopic effects due to rotating carrier are considered. The critical speeds of gyroscopic systems occur at speeds in which one or more natural frequencies are zero. To calculate the critical speeds, the eigenvalue problem of the system is solved by numerical methods. In order to validate the equations and the process of extraction of critical speed, the obtained results for a high-speed spur planetary gear system are compared with the results of the existing research. Finally, by plotting the variations of the real and imaginary parts of the Eigenvalues of the double-helical planetary gear system versus a range of carrier speeds investigate the system stability near critical speeds. The results of the current study indicate that the double- helical planetary gear system is stable at some critical speeds and in others subjected to divergence instability.

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1- Introduction

Critical speeds of gyroscopic systems happen at particular speeds where one or more of the eigenvalues vanish. Calculation of the critical speeds and the speed regions with divergence instability is important in the design of rotating machinery, including high-speed planetary gears such as turbo-fan and turbo-jet.

By studying the available literatures in the field of stability of gears, it is observed that there is no references on the stability of double- helical planetary gear systems. Therefore, in the current study, the stability of a double- helical planetary gear system investigated in the range of critical carrier speeds by considering the gyroscopic effects due to rotating carrier and constant mesh stiffness.

2- Methodology

2- 1- Dynamic model

To extract the dynamic model in 3-D space for double- helical planetary gear set, each member has three translational and three rotational degrees of freedom. In this model, the following assumptions are considered:

(1) The bodies of the gears are rigid;

(2) Flexibilities of the mesh stiffness are modeled by linear springs acting on the plane of action normal to gear tooth surface;

(3) The mesh stiffness of tooth pair are constant;

(4) The gear teeth at the mesh interfaces are in constant contact;

(5) All planets gears are similar to one another;

(6) The frictional forces due to tooth sliding are neglected;

(7) The left and right sides of double- helical gears are of the same in geometry;

(8) The manufacturing error and damping of the system are neglected;

(9) The gyroscopic effects due to rotating carrier are considered;

Fig. 1 shows a double- helical gear planetary gear system with one planet.

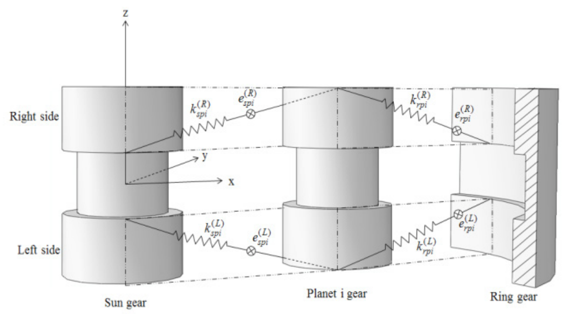

Fig. 1. Lumped mass model of double helical planetary gear system with one planet

2- 2- The overall equation of the system

The overall equations of the system with 18(N+3) degrees of freedom, N is the Number of planet gears, in the matrix form are proposed as follow:

$$\mathbf{M}\ddot{\mathbf{q}}(t)+\mathbf{G}\dot{\mathbf{q}}(t)+(\mathbf{K}_m+\mathbf{K}_b-\Omega_c^2\mathbf{K}_g)\mathbf{q}(t)=\mathbf{F}(t) \quad (1)$$

where, $\mathbf{M}$ is the mass matrix, $\mathbf{G}$ is the gyroscopic matrix,

Corresponding author, E-mail: mehrdad.poursina@gmail.com

$\mathbf{K}_m$ is the mesh stiffness matrix, $\mathbf{K}_b$ is the bearing stiffness matrix, Ω_c is the carrier speed, $\mathbf{K}_g$ is the stiffness matrix due to gyroscopic effect, $\mathbf{F}(t)$ is the excitation vector due to transmission error and external torque, $\mathbf{q}$, $\dot{\mathbf{q}}$ and $\ddot{\mathbf{q}}$ are the displacement, velocity and acceleration vectors, respectively.

2- 3- The process of critical speeds extraction

To extract the critical speeds, substitute the $\mathbf{q} = \Phi \exp(\lambda t)$ into the homogeneous form of the Eq. (1). At critical speeds, denoted Ω_{crit}, at least one eigenvalue vanishes. Substitution of $\lambda = 0$ gives equation (2):

$$(\mathbf{K}_b + \mathbf{K}_m)\Phi - \Omega_{crit}^2 \mathbf{K}_g \Phi = \mathbf{0} \tag{2}$$

where, Φ is the eigenvector, λ is the eigenvalue and $\mathbf{\Omega}_{crit}$ is the critical speed.
Eq. (2) represents an eigenvalue problem where Ω_{crit}^2 is the eigenvalue.

3- Results and Discussion

The basic parameters of a double- helical planetary gear system are presented in Table 1.

Table 1: Basic parameters of a double- helical planetary gear system

	Sun	Planet	Ring	Carrier
Number of teeth	47	39	125	-
Normal module (mm)		1.81		-
Helix angle (deg)		21.5		-
Normal pressure angle (deg)		22.5		-
Base radius (mm)	41.8	34.7	111.2	-
Mass (kg)	1.22	1.69	2.4	0.651
Mesh stiffness (N/μm)		564	531	-
$k_{y\varsigma}, k_{x\varsigma} (N/\mu m)$	100	100	1000	-
$k_{\theta y\varsigma}, k_{\theta x\varsigma}$ (1e6Nm/rad)	5	5	10	-

In order to analysis the stability of the system in the range of the critical carrier speeds, the diagram of variations of the real and imaginary parts of the system's eigenvalues versus the carrier speeds is plotted. Figs. 2a to 2e show the diagram of variation of the real and imaginary parts of the eigenvalue in the range of $0 < \Omega_c < 0.5$, $0.5 < \Omega_c < 1$, $1 < \Omega_c < 1.5$, $1.5 < \Omega_c < 2$ and $2 < \Omega_c < 2.5$ respectively.

The following results are obtained by investigation of the Figs. 5a to 5e:

- In the range of carrier speeds $0 < \Omega_c < 0.5$, the system has a divergence instability at the critical carrier speed of 0.36593 and stable at the critical carrier speed of 0.423762. Because at the critical carrier speed of 0.36593 the imaginary part of the eigenvalue becomes zero and only has a real part. While, at the critical carrier speed of 0.423762 the imaginary part of the eigenvalue becomes zero, but after this speed, the amount of the eigenvalue is still purely imaginary.

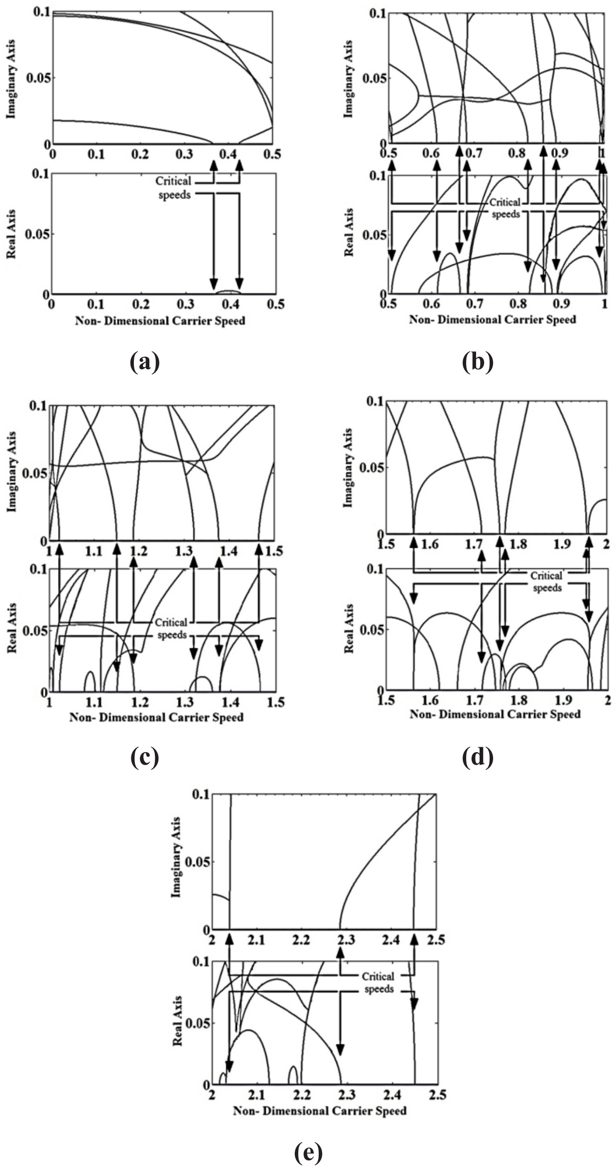

Fig. 2. Diagram of variation of the real and imaginary parts of the eigenvalue: (a) $0 < \Omega_c < 0.5$. (b) $0.5 < \Omega_c < 1$. (c) $1 < \Omega_c < 1.5$. (d) $1.5 < \Omega_c < 2$. (e) $2 < \Omega_c < 2.5$.

- In the range of carrier speeds $0.5 < \Omega_c < 2$, the system has a divergence instability at the critical carrier speeds of 0.5079, 0.613155, 0.682561, 0.825623, 0.861494, 0.891546, 1, 1.022982, 1.150256, 1.322876, 1.377269, 1.717047, 1.77044 and 2.039967 and stable at the critical carrier speeds of 0.66565, 0.993094, 1.187226, 1.46643, 1.562354, 1.758118, 1.954988, 2.28581 and 2.45034.

4- Conclusion

In this research, investigate the stability of a double- helical planetary gear system by extracting a linear time- invariant model in 3D space and considering the gyroscopic effects due to rotating carrier. For this purpose, after extracting the governing equations, the critical carrier speeds, in which one or more of the eigenvalues of the system vanish, are calculated. Then, the boundaries of the divergence instability are determined by plotting the diagram of the variety of the imaginary and real parts of the system's eigenvalues versus the range of carrier speeds. Divergence instability occurs when one of the imaginary eigenvalues of the system becomes real and positive. However, at critical speeds which the amount of the eigenvalue of the system is equal to zero, but after the critical speed, the amount of the eigenvalue is still purely imaginary, the system will be stable. The results of this study indicate that in a double- helical planetary gear system, at some critical speed, divergence instability occurs and stable in others.

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