Amirkabir Journal of Mechanical Engineering

Amirkabir J. Mech. Eng., 51(3) (2019) 1-3
DOI:

Free Vibration Analysis of Nanotube-Reinforced Composite Conical Shell in High-Temperature Environment

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ABSTRACT: In this research, free vibration analysis of functionally graded carbon nanotube reinforced composite conical shells subjected to the high-temperature environment is investigated. The material properties of functionally graded carbon nanotube reinforced are assumed to be graded through the thickness direction. Two kinds of carbon nanotube reinforced composites including uniformly distributed in which carbon nanotubes are distributed uniformly through the shell thickness, and functionally graded in which carbon nanotubes are graded with three different distributions, are considered. The effect of thermal loading is considered as initial stress. Applying Hamilton's principle based on the classic theory and considering Von Karman strain-displacement relation, the governing equations are obtained. The analytical Galerkin method together with beam mode shapes as weighting functions is employed to solve the equations of motion. The results are compared with those presented in the literature. In addition, the effect of various parameters such as thermal loading, boundary conditions, and different geometrical conditions are studied. It is shown that the initial thermal stresses have significant effects on the natural frequencies and cannot be neglected. Moreover, the critical buckling temperature rise of the shells can be extracted from the presented diagrams of the fundamental frequency parameters versus the temperature rise.

Review History:

Received:
Revised:
Accepted:
Available Online:

Keywords:

Free vibration
Conical shells
Functionally-graded carbon nano-tube
Functions of the beam
Temperature.

1- Introduction

With attention to the need for high strength and light structures, it is necessary to increase the ratio of strength to weight for these structures. In recent years, by the development of nanoscience, Carbon Nanotube Reinforced Composite (CNTRC) shells have been used as a new approach in the construction of shells.

Mirzaei and Kiani [1] studied the thermal buckling of Functionally Graded Carbon Nanotube Reinforced composite (FGCNTRC) conical shells. A semi-analytical method for studying the buckling of moderately thick carbon nanotube reinforced composite conical shells under axial compression is presented by Hosseini and Talebitooti [2]. Shen [3] studied thermal buckling and post-buckling of CNTRC cylindrical shells with a higher order shear deformation shell theory. Shu [4] presents the first endeavor to apply the global method of Generalized Differential Quadrature (GDQ) to the free vibration analysis of composite laminated conical shells. Jam and Kiani [5] studied the buckling of FGCNTRC conical shells subjected to lateral pressure. Free vibration analysis of embedded FGCNTRC conical, cylindrical shells and annular plates using the Variational Differential Quadrature (VDQ) method is carried out by Ansari et al. [6]. Recently, Mehri et al. [7] investigated the bifurcation and vibration responses of a Carbon Nanotube (CNT) reinforced functionally graded conical shells.

To the best of the author's knowledge, the effect of boundary conditions on vibration and thermal buckling analysis of the CNTRC conical shell in the high-temperature environment is not available in open literature. The present research investigates the vibration and thermal buckling of FGCNTRC conical shells with different boundary conditions using the Galerkin method. Four types of distribution of CNTs are considered in this article.

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2- Problem Formulation

The coordinate system in a conical shell is shown in Fig. 1. The length is denoted as L, semi-cone angle φ, small radius

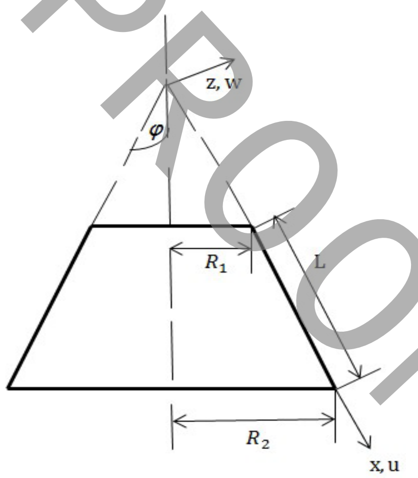

Figure 1. Conical shell coordinate system.

R_1, and large radius R_2. The displacements in the longitudinal (x), circumferential (θ) and radial (z) directions in the shell are denoted as u, v and w, respectively.
The forces and moment resultant are calculated as follows:

$$\begin{bmatrix} N_x \\ N_\theta \\ N_{x\theta} \\ M_x \\ M_\theta \\ M_{x\theta} \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} \end{bmatrix} \begin{bmatrix} \varepsilon_{0x} \\ \varepsilon_{0\theta} \\ \varepsilon_{0x\theta} \\ k_x \\ k_\theta \\ \tau \end{bmatrix} - \begin{bmatrix} N_x^T \\ N_\theta^T \\ 0 \\ M_x^T \\ M_\theta^T \\ 0 \end{bmatrix} \qquad (1)$$

The governing differential equations of motion can be derived by using Hamilton's principle as:

$$\begin{aligned} &\frac{\partial N_x}{\partial x} + \frac{\sin(\varphi)}{R(x)}\left(N_x - N_\theta\right) + \frac{1}{R(x)}\frac{\partial N_{x\theta}}{\partial \theta} = \rho_t \frac{\partial^2 u_0}{\partial t^2} \\ &\frac{1}{R(x)}\frac{\partial N_\theta}{\partial \theta} + \frac{\partial N_{x\theta}}{\partial x} + \frac{2\sin(\varphi) N_{x\theta}}{R(x)} + \frac{\cos(\varphi)}{R(x)} Q_\theta = \rho_t \frac{\partial^2 v_0}{\partial t^2} \\ &-\left(\frac{\cos(\varphi)}{R(x)} N_\theta\right) + \frac{\partial Q_x}{\partial \alpha} + \frac{\sin(\varphi)}{R(x)} Q_x + \frac{1}{R(x)}\frac{\partial Q_\theta}{\partial \theta} \\ &+N_x^0\left(\frac{\sin(\varphi)}{R(x)}\frac{\partial w_0}{\partial x} + \frac{\partial^2 w_0}{\partial x^2}\right) = \rho_t \frac{\partial^2 w_0}{\partial t^2} \end{aligned} \qquad (2)$$

The displacement fields for a circular truncated conical shell are assumed to be in the following form:

$$\begin{aligned} u_0(x,\theta,t) &= A\left(\partial \varnothing(x)/\partial x\right)\cos(n\theta)\cos(\omega t) \\ v_0(x,\theta,t) &= B\varnothing(x)\sin(n\theta)\cos(\omega t) \\ w_0(x,\theta,t) &= C\varnothing(x)\cos(n\theta)\cos(\omega t) \end{aligned} \qquad (3)$$

With substituting Eq. (3) into Eq. (2) and using the Galerkin method, the whole system of differential equation has been discretized and the set of linear algebraic equations will be produced as:

$$\begin{bmatrix} I_1\omega^2 + k_{11} & k_{12} & k_{13} \\ k_{21} & I_1\omega^2 + k_{22} & k_{23} \\ k_{31} & k_{32} & I_1\omega^2 + k_{33} \end{bmatrix} \begin{Bmatrix} A \\ B \\ C \end{Bmatrix} = 0 \qquad (4)$$

3- Results and Discussion

To validate the results, they are compared with the previous works. At first, critical buckling temperature is compared with those of CNTRC conical shell by Mirzaei and Kiani [1], as shown in Table 1. The secondary comparisons are listed in Table 2, are demonstrated with the work done by Ansari et al. [6] which considers different types of CNTs distribution.
The variations of the frequency of CNTRC conical shells with ΔT for three different semi-cone angles are shown in Fig. 2. Both sides of the conical shell have simply supported boundary conditions and uniformly distributed (UD) -type is selected. It can be observed from this figure that ΔT with the occurrence of the critical buckling temperature decreases when the semi-cone angle is enhanced.
The variations of the fundamental frequency of CNTRC conical shells with ΔT for four different lengths to small radius ratios and two type CNT distributions are shown in Fig. 3. It can be seen from this figure that the frequency decreases as the temperature difference between the inner and outer surface increases. It is noteworthy that the sensitivity rate of

Table 1. The effect of CNTs distribution on critical buckling pressure.

$R_1/h = 50, \varphi = 30^o, h = 1$ mm, CC, $L = \sqrt{400R_1h}$

V_{CN}^*	CNTs distribution	[1]	Present
	uniformly distributed (UD)	394.06	397.41
0.12	FGX	407.61	411.48
	FGV	388.16	393.29
	FGA	382.25	386.35
		402.48	407.96
0.17	UD	417.33	421.85
	FGX	396.82	400.35
	FGV	390.44	394.96
	FGA	390.44	394.96

Table 2. Comparison of $\Omega = \omega R_1^2/h\sqrt{\rho^m/E^m}$ for CNTRC conical shell ($L/R_1 = 2, R_1/h = 20, \varphi = 30^o, SS$)

V_{CN}^*	CNTs distribution	[6]	Present
	UD	6.00	5.98
0.12	FGA	5.89	5.86
	FGX	6.61	6.60
	UD	7.51	7.49
0.17	FGA	7.43	7.41
	FGX	8.29	8.28

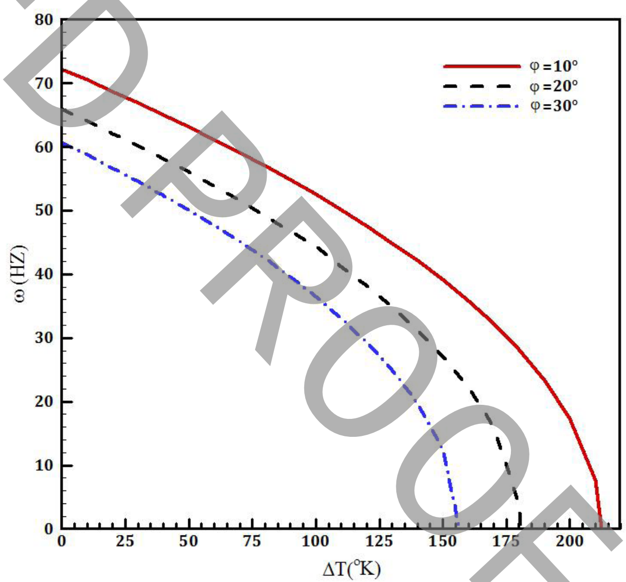

Figure 2. Variation of the frequency versus ΔT for the conical shell with different cone angles.

$L/R_1 = 2, h/R_1 = \frac{1}{40}, V_{CN}^* = 17\%, UD$

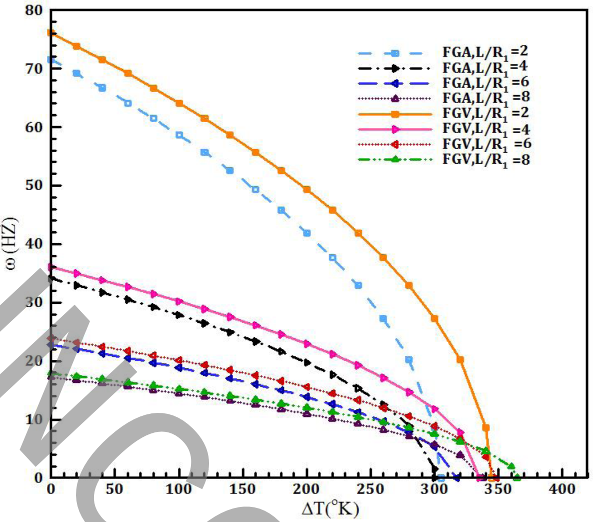

Figure 3. Variation of the fundamental frequency versus ΔT for the conical shell with different *L/R1*.

$$\frac{h}{R_1}=\frac{1}{20}, V_{CN}^{*}=12\%, \varphi=15°, SS$$

the fundamental frequency to ΔT increases with decreasing length to radius ratio.

4- Conclusions

The following main conclusions of the paper have been obtained:

- Conical shells with FGX type of CNTs distribution have the highest fundamental frequency because of the existence of more nanotubes in the outer surface that causes increasing stiffness of the shell.
- Fundamental Frequency increases when the volume fraction of CNTs rises. The highest fundamental frequency is occurred in $V_{CN}^{*}=0.28$ for ΔT=0 K. However, the maximum critical temperature buckling is occurred in $V_{CN}^{*}=0.17$.
- The effect of ΔT on reduction in frequency becomes more noticeable as the length to the radius of the shell is increased.

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