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Modeling Nonlinear Free Vibrations of Wind Turbine Tower

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ABSTRACT: The goal of this paper is to study the large amplitude free vibration of wind turbine tower which modeled as a variable cross-section beam with eccentric mass. The effects of variable axial force due to gravity are also taken into account. The nonlinear governing equations of motion and the corresponding boundary conditions of the system are obtained using the Hamilton's principle as well as Euler-Bernoulli's assumptions. Then a numerical finite difference scheme is utilized to find the natural frequencies and the mode shapes of the system. Using Galerkin method, the partial differential equations governing dynamic of the system are reduced to ordinary differential equations in terms of the end displacements which are coupled due to the presence of the transverse eccentricity. These temporal coupled ordinary differential equations are then solved analytically using the multiple time scales perturbation technique. The obtained analytical results are compared with the numerical ones and excellent agreement is observed. The results of this research can be used to study the effect of the eccentric tip mass, variable cross-section and gravity on the large amplitude vibration of wind turbine tower for improved dynamic performance.

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1- Introduction

In recent years, the extraction of electrical energy from wind power has been attracted many attentions due to the growing use of renewable energies. In many countries, the development of wind farms are of great importance in the production of electrical power from wind energy and considerable efforts have been devoted to the design and development of wind turbines [1].

From an engineering viewpoint, wind turbines are complex structures with many design considerations. Due to the permanent interaction with wind power, the dynamic behavior of the structure is a fundamental issue in the design of wind turbines. For large turbines, the tower is one of the important structural parts of the turbine and its vibrational characteristics have direct effects on the whole dynamic response of the turbine [2]. The dynamic analysis of wind turbine tower has been investigated both theoretically and experimentally. Most theoretical studies have been performed by Finite Element (FE) analysis in commercial softwares, while analytic models are also reported. Mortagh et al. [3] proposed a simple lumped mass analytical model and obtained the natural frequencies and mode shapes of a tubular tower. Wang et al. [4] present a mathematical model for the tower and rotor of the wind turbine based on the thin beam theory and investigated the free and forced vibrations of the system. The transform matrix method is a simple, but efficient technique which has been used in vibration analysis of wind turbine tower. In reference [5], a transform matrix based method is reported for bending and torsional vibration of tubular towers. Furthermore, Feyzollahzadeh and Mahmoodi [6] proposed an analytical method for dynamic analysis of wind turbine tower based on the transform matrix. In the present study, a mathematical model is proposed for vibrational behavior of wind turbine tower. A varying cross section beam with an eccentric mass mounted at the top is considered as the tower and the nacelle of turbine. The Hamilton's principle is utilized to obtain the differential equations of the system. A matrix based method is developed for obtaining natural frequencies and mode shapes of the tower. By employing these natural modes, the governing equations of the system reduce to temporal equations in terms of end deflections of the beam and then solved using perturbation theory. The results are verified by numerical simulations and the effects of different parameters on the vibrational behavior of the system are investigated.

2- Mathematical Formulation

Fig. 1 illustrates the structural model of a tubular wind turbine. The tower is considered as a beam with variable cross section carrying an eccentric mass M which corresponds to the nacelle and turbine blades. In this figure, l is the length of the beam, J is the mass moment of inertia of the mass, a and b are transverse and axial eccentricities of the tip mass and g is the gravity constant.

Considering the Euler-Bernouli hypothesis along with nonlinear stretching of the midplane of the beam, the governing differential equations of the system can be obtained by using Hamilton's principle as:

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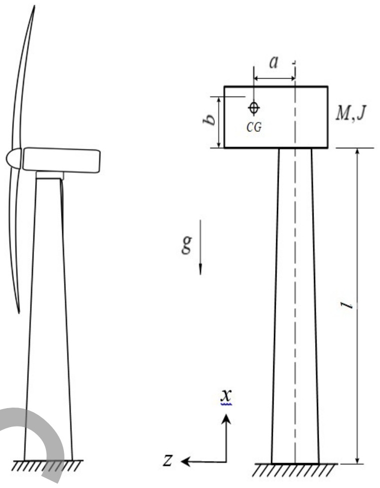

Figure 1. Structural model of wind turbine tower.

$$\left(EIw_{,xx}\right)_{,xx}+\rho A w_{,tt}+cw_{,t} \quad (1)$$

$$\rho A u_{,tt}+g\rho A-\left(EA\left(u_{,x}+0.5w_{,x}^{2}\right)\right)_{,x}=0 \quad (2)$$

where u and w are the axial and transverse displacements, respectively. Furthermore, EI denotes the flexural rigidity, ρ is the mass density, A indicates the area of the cross section of the beam and, c is the damping per unit length of the beam. The boundary conditions at the fixed end of the tower are $u=w=w_x=0$ while at the other end $(x=l)$, the following relations must be satisfied:

$$EA\left(u_{,x}+0.5w_{,x}^{2}\right)+Mu_{,tt}-Maw_{,xtt}+Mg=0 \quad (3)$$

$$EA\left(u_{,x}+0.5w_{,x}^{2}\right)w_{,x}-\left(EI\,w_{,xx}\right)_{,x} \quad (4)$$

$$EIw_{,xx}+Mbw_{,tt}-Mau_{,tt} \quad (5)$$

where κ denotes the radius of gyration of the tip mass. By using Galerkin method, the partial differential Eqs. (1) and (2) can be reduced to some ordinary differential . equations which may be solved numerically or by a perturbation method. In this paper, both methods are employed and the results have been provided below.

3- Results and Discussion

As a case study, a tubular tower with variable cross section is considered. In Fig. 2 the variation of first and second natural frequencies of the tower against the taper ratio is presented. As the taper ratio increases, the first frequency increases while the second one decreases. The transient vibrations of the tower in response to initial condition are depicted in Fig. 3. The static equilibrium deflection due to the weight of the nacelle is considered as the initial condition.

4- Conclusions

The development of wind turbines are of great importance due to their environmental benefits. Due to the direct interaction with wind power, there are many challenges in mathematical modeling of wind turbines. In this paper, a dynamic modeling of wind turbine tower with variable cross section developed. A method for determination of natural frequencies of the

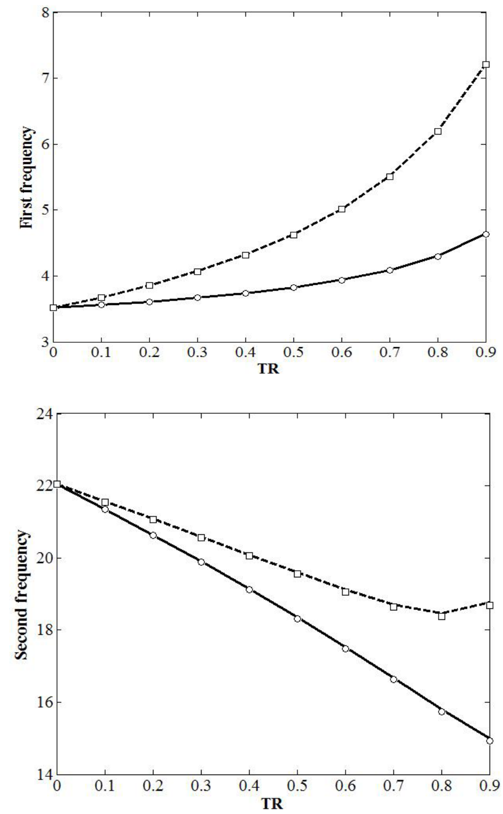

Figure 2. Variation of the natural frequencies of the tower against taper ratio.

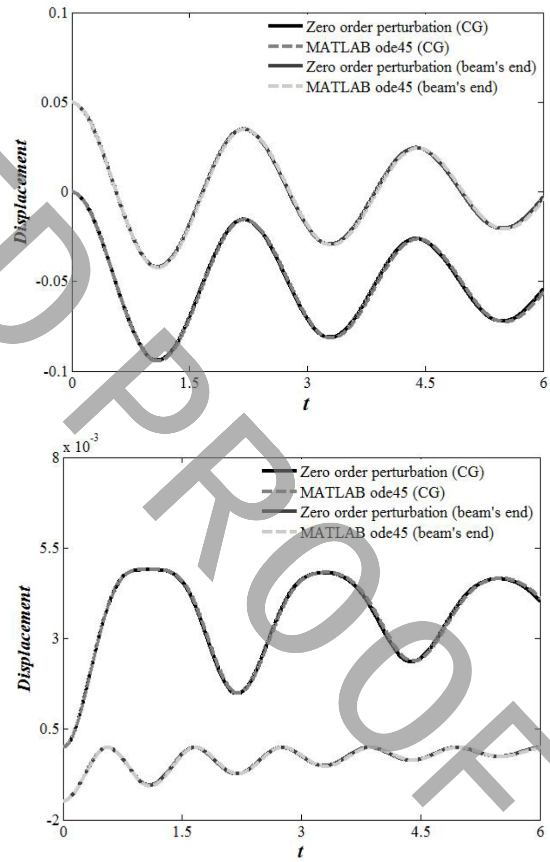

Figure 3. Transverse and axial vibrations of the tower.

tower proposed. Finally, the transient vibration of the structure in response to initial conditions investigated. This work provides an analytical framework for dynamic analysis of wind turbine towers that enables the designer to study the effect of various design parameters on the dynamic behavior of the tower.

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