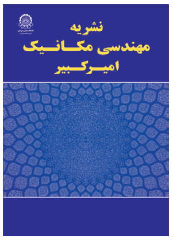

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Crack Identification in Postbuckled Plates Using Differential Quadrature Element Method and Sequential Quadratic Programming

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ABSTRACT: In this study, a method for crack identification in buckled plates using the differential quadrature element method and sequential quadratic programming method is proposed. The study consists of two parts. First, a numerical method is applied to compute the frequencies of buckled cracked plates. Cracks are assumed to be open and modeled as linear rotational spring. The governing equations are extracted considering the effects of shear deformations and initial geometric imperfection. The solution is assumed to be consist of static (postbuckling) and dynamic parts. The governing equations are converted into two different equation sets; postbuckling and vibration equations. The natural frequencies can be obtained solving these equation sets. In the second part, the sequential quadratic programming optimization method and the method introduced in the first part have been combined to identify crack parameters using natural frequencies. A weighted sum of squared errors between the measured and computed frequencies has been considered as the cost function and minimized to identify crack parameters. Finally, the accuracy and precision of the proposed method have been verified using some experimental and numerical case studies. The identification results show that the crack position, depth, and length can be predicted well by the presented method.

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1- Introduction

Plates are all-purpose elements in engineering structures. Their resistance to loads over critical buckling load makes them suitable for space structures, micro-switches, and many other engineering applications. But, these loads and their out-coming deformations make them susceptible for crack initiation. A lot of research has been performed on crack detection in beam and plate structures using modal properties. Hadjileonatiadis and Douka [1] used the kurtosis analysis introduced by Hadjileonatiadis [2] for crack detection in plates. Chang and Chen [3] and Fan and Qiau [4] employed 1-D wavelet analysis and 2-D continuous wavelet analysis of the modal shape of the cracked plate to find the shape and location of the crack, respectively. Moradi and Alimouri [5] used the differential quadrature method and bees algorithm to predict the location, depth and length of cracks using natural frequencies. Jingpin [6] used the nonlinear lamb wave-mixing method to identify micro-cracks in plates.

The following study has been performed to present a method for identification of crack parameters in the cracked postbuckled plate. After modeling the crack, the differential equation of motion of the plate vibrating around its postbuckled equilibrium state will be derived and solved by the differential quadrature element method to get the natural frequencies of the cracked plate. These frequencies are compared with those obtained by experimental modal analysis, and with the help of an optimization method, namely sequential quadratic programming method, will be minimized to find the location, length and depth the crack. Some case studies have been presented to verify the accuracy and precision of the proposed method.

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2- 2. Methodology

Fig. 1 shows a cracked rectangular plate of dimensions a and b, and thickness h. The crack is located at lc from the left loaded side and having length and depth of L_c and h_c, respectively. It is under compressive axial load which is perpendicular to the crack length.

The crack could locally decrease the plate stiffness. Reduction of stiffness causes a slope difference between the two sides of the plate at the crack location. This slope difference can be calculated using fracture mechanic principles and the definition of first mode stress intensity of the crack by the following relation [7]:

(1)

α_{bb} is the non-dimensional bending compliance coefficient

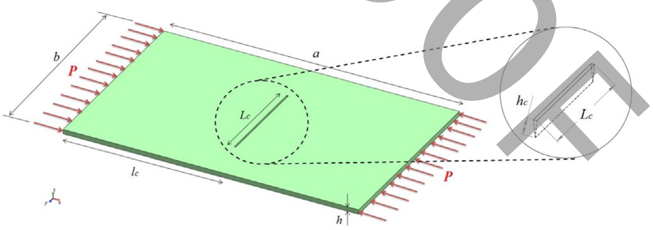

Figure 1. The cracked plate under uniaxial compressive load.

and σ_b is the nominal bending stress perpendicular to the crack. α_{bb} could be calculated using Eq. (2):

(2)

where g_b is a non-dimensional function of relative depth of the crack ($\zeta = hc/h$) which can be obtained by:

(3)

Considering the effects of shear deformations using Mindlin theory, the effects of initial geometric imperfection and the Von-Karman strain-displacement relations, the governing differential equations can be obtained. The plate is divided into several elements using Differential Quadrature Element Method (DQEM), the crack is modeled as a massless linear rotational spring between two or more elements, and the boundary and continuity conditions for elements are considered.

In order to solve the governing equations of the plate, the solution considered as the summation of the static solution and dynamic solution. The obtained equations will be converted to a set of nonlinear algebraic equations using DQEM to be solved by an arc-length strategy [8]. Then, inserting the static solution in remaining vibration equations and the corresponding boundary and continuity conditions, and then discretizing them using DQEM, an eigenvalue problem is obtained as:

$$\begin{bmatrix} A_{BB} & A_{BI} \\ A_{IB} & A_{II} \end{bmatrix} \begin{Bmatrix} X_B \\ X_I \end{Bmatrix} = -\omega^2 \begin{bmatrix} 0 & 0 \\ B_{IB} & B_{II} \end{bmatrix} \begin{Bmatrix} X_B \\ X_I \end{Bmatrix} \qquad (4)$$

The solution of this eigenvalue problem provides the natural frequencies and mode shapes of the plate. Natural frequencies of cracked structures are functions of crack parameters (length, depth, and location). Therefore, they could be used in an inverse problem to find the crack parameters using the experimental modal data and optimization algorithms. Here, the crack location, depth, and length are defined as the design variables and a weighted sum of squared errors between computed and measured natural frequencies is considered as the cost function. Due to the high computational cost of evaluating the postbuckling equilibrium state and consequently computing the natural frequencies of the cracked plate, it was decided to utilize a gradient-based optimization algorithm instead of using the common evolutionary algorithms, in which the number of function evaluations is high. Therefore, a gradient-based method namely, the sequential quadratic programming method is used as the optimizer.

Figure 2. Experimental test setup.

Table 1. Experimental crack cases and natural frequencies.

Case No.	P/P_{cr}	*Crack*			*Natural Frequencies*				
		l_c/a	L_c/b	h_c/h	ω_1	ω_2	ω_3	ω_4	ω_5
1	1.45	0.25	0.40	0.70	69.75	97.75	109.0	149.30	170.8
2	1.36	0.25	0.60	0.70	65.50	92.50	106.0	151.80	169.5
3	1.36	0.25	0.80	0.70	65.00	91.30	109.0	151.50	170.8
4	1.49	0.375	0.40	0.70	74.50	103.50	118.3	156.30	178.3
5	1.61	0.50	0.80	0.70	79.25	104.75	114.8	154.30	180.3

Table 2. predicted results of the experimental study.

Case No.	*Predicted crack*			*Error %*		
	l_c/a	L_c/b	h_c/h			
1	0.25	0.434	0.69	0.00	3.40	1.00
2	0.261	0.61	0.64	1.10	1.00	6.00
3	0.25	0.8	0.7	0.00	0.00	0.00
4	0.366	0.417	0.7	0.90	1.70	0.00
5	0.478	0.65	0.7	2.20	15.00	0.00

3- Results and Discussion

In order to verify the effectiveness of the proposed method, several case studies on simply supported cracked square plates made of Polyvinyl Chloride (PVC) is carried out. A square plate with length, thickness, Young's modulus, poison ratio and density equal to 0.5 m, 6 mm, 3.7 GPa, 0.3 and 1400 kg/m3 has been considered. A special fixture was designed and made to provide the simply supported edge conditions. An open edge crack was introduced by making a fine saw cut at the plate mid-span. Then, the plate was compressed axially to produce the state of postbuckling. Next, it was impacted by the hammer and the first five natural frequencies were measured by the data acquisition system. The test structure is shown in Fig. 2.

Some of the obtained frequencies are presented in Table 1 for a different amount of compressive load. Table 2 illustrates the results obtained by the proposed method using measured frequencies of Table 1. It can be seen that the proposed method has been identified the crack parameters with great accuracy.

4- Conclusions

In this study, a method for the identification of cracks in postbuckled plates using natural frequencies is proposed. Crack assumed to be open were modeled by massless rotational springs. Governing equations were extracted and converted into two sets of differential equations. In the first step, postbuckling equations were solved using DQEM and arc-length method. After that, using DQEM and inserting the postbuckling results into the vibration equations, the natural frequencies of cracked buckled plates were obtained. Finally, using SQP algorithm and the natural frequencies of the cracked plate, the crack parameters (location, depth, length) were obtained. The accuracy and integrity of the proposed method were examined using experimental and numerical data and showed that the crack location, size, and depth can be predicted well by the proposed method.

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