



## Free and Forced Vibration Analysis of Kelvin-Voigt Viscoelastic Nanoplate by Using Modified Couple Stress Theory

S. Salehi<sup>1</sup>, O. Rahmani<sup>1\*</sup>, A. Hoseini<sup>2</sup>

<sup>1</sup> Mechanical Engineering Department, University of Zanjan, Zanjan, Iran

<sup>2</sup> Department of Mechanical Engineering and Industrial Engineering, Buein Zahra Technical University, Qazvin, Iran

### Review History:

Received: 2 Feb. 2018

Revised: 30 Mar. 2018

Accepted: 29 Apr. 2018

Available Online: 28 May. 2018

### Keywords:

Forced vibration

Resonance phenomenon

Modified couple stress theory

Semi analytical Galerkin method

**ABSTRACT:** With the development of nanotechnology in the industrial applications and engineering sciences, analysis of the behavior of nanostructures has become important. In recent years, expansion and using of non-classical theories to predict the behavior of nanostructure materials has attracted the attention of researchers. In this paper, free and forced vibration of viscoelastic nanoplate on the Pasternak viscoelastic foundation will be studied. In this study, due to the inability of classical theories to describe the behavior of nano-dimensional structures, the non-classical modified couple stress theory has been used for express the size effect. By using the Galerkin semi-analytic method, free vibrations analysis for six different boundary conditions are discussed; also, forced vibration of rectangular viscoelastic nanoplate is studied by using the Navier method for simply supported boundary condition. Kelvin-Voigt model is used to simulate the behavior of viscoelastic nanoplate. In the results analysis section, the effect of small-scale factor, structural viscoelastic coefficient, linear elastic coefficient of foundation, external damping coefficient of foundation and shear coefficient of foundation on the natural frequency, maximum dynamic deflection and resonance phenomenon are presented.

### 1- Introduction

With the advancement of nanoscience, one of the main issues of researchers in recent years is the analysis and recognition of the behavior of small-scale structures. Due to the huge cost and the need for exact equipment for laboratory and experimental work on nano dimensions, using of non-classical theories has increased. Nanoplates are a subdivision of microstructures that improve their electrical properties, strength and flexibility by their atomic makeup. In this paper, by using modified stress coupling theory, free and forced vibration analysis of Kelvin-Voigt viscoelastic nanoplate on the Pasternak foundation has been studied and influence internal viscoelastic coefficient, elastic coefficient, shear foundation coefficient and small scale factor on natural frequency, maximum amplitude deflection and resonance phenomenon have been investigated.

Simsek [2] analytical and numerical solution procedures are proposed for vibration of an embedded microbeam under the action of a moving microparticle based on the Modified Couple Stress Theory (MCST) within the framework of Euler–Bernoulli beam theory. In this paper influences of the material length scale parameter, the Poisson's ratio, the velocity of the microparticle and the elastic medium constant as well as the solution procedures on the dynamic responses of the microbeam. In a study by Ma et al. [3] a non-classical Mindlin plate model is developed using a modified couple

stress theory. The equations of motion and boundary conditions are obtained simultaneously through a variational formulation based on Hamilton's principle. The new model contains a material length scale parameter and can capture the size effect, unlike the classical Mindlin plate theory.

### 2- Methodology

In Fig. 1, a schematic of the problem is shown.

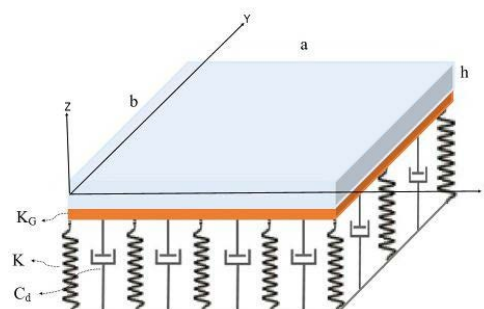


Fig. 1. General schematic of the problem

Using the energy method and Eqs. (1) to (4), equations of motion for different boundary conditions were derived. In this paper, free vibrations for the six different boundary condition and in the case of forced vibration simply support boundary condition have been studied.

\*Corresponding author's email: [omid.rahmani@znu.ac.ir](mailto:omid.rahmani@znu.ac.ir)

$$\sigma = (1 + \eta \frac{\partial}{\partial t}) E \varepsilon \quad (1)$$

$$U = \frac{1}{2} (\sigma_{ij} \varepsilon_{ij} + m_{ij} \chi_{ij}) \quad (2)$$

$$\chi_{ij} = \frac{1}{4} (e_{ipq} \varepsilon_{qp,j} + e_{j pq} \varepsilon_{qi,p}) \quad (3)$$

$$m_{ij} = 2\mu l^2 \chi_{ij} \quad (4)$$

For solving equations in free and forced vibration, semi-analytical method and Navier solution have been used, respectively.

### 3- Discussion and Results

In Fig. 2, the effect of small-scale factor on natural frequency is shown. The frequency ratio obtained from the modified couple stress theory to the Classical Theory (CT) frequency according to the small-scale factor is presented.

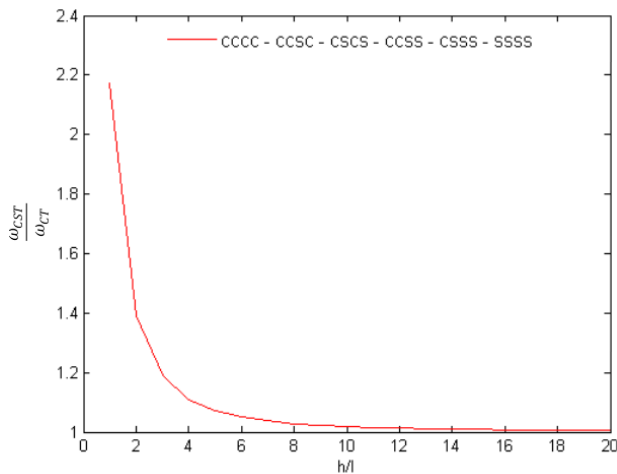


Fig. 2. Frequency ratio of MCST to CT frequency

According to Fig. 2 frequency derived from two classical and modified couple stress theory are near to each other for nanoplates that have a large thickness, while, for small thickness nanoplate, frequencies obtained from the two theories do not coincide and differ greatly. Considering external foundation damper and internal coefficient viscoelasticity for nanoplate leads to complex natural frequency, also, by increasing the values of these parameters, frequency ratio increases.

Considering length scale parameter leads to decrease the dynamic deflection and increase natural frequency, also creates resonant phenomenon at higher frequencies. Existence of linear elastic and shear coefficients of foundation increases the natural frequency and decrease maximum dynamic deflection.

### 4- Conclusions

According to the results obtained in this paper, considering the small scale factor, elastic coefficient and shear coefficient of the foundation increase the natural frequency of nanoplate. Furthermore external damper coefficient of foundation and

internal viscoelasticity coefficient of the nanoplate creates complex natural frequency.

### References

- [1] S. Pradhan, J. Phadikar, Nonlocal elasticity theory for vibration of nanoplates, *Journal of Sound and Vibration*, 325(1-2) (2009) 206-223.
- [2] M. Şimşek, Dynamic analysis of an embedded microbeam carrying a moving microparticle based on the modified couple stress theory, *International Journal of Engineering Science*, 48(12) (2010) 1721-1732.
- [3] H. Ma, X.-L. Gao, J. Reddy, A non-classical Mindlin plate model based on a modified couple stress theory, *Acta mechanica*, 220(1-4) (2011) 217-235.
- [4] S. Poursmaeeli, S. Fazelzadeh, E. Ghavanloo, Exact solution for nonlocal vibration of double-orthotropic nanoplates embedded in elastic medium, *Composites Part B: Engineering*, 43(8) (2012) 3384-3390.
- [5] L.-L. Ke, Y.-S. Wang, J. Yang, S. Kitipornchai, Free vibration of size-dependent Mindlin microplates based on the modified couple stress theory, *Journal of Sound and Vibration*, 331(1) (2012) 94-106.
- [6] A.G. Arani, R. Kolahchi, H. Vossough, Buckling analysis and smart control of SLGS using elastically coupled PVDF nanoplate based on the nonlocal Mindlin plate theory, *Physica B: Condensed Matter*, 407(22) (2012) 4458-4465.
- [7] A. Bakhsheshy, K.Khorshidi, Free vibration of functionally graded rectangular nanoplates in thermal environment based on the modified couple stress theory, *Modares Mechanical Engineering*, 14(15)(2015),323-330. (In Persian)
- [8] S.OmidDezyani, R. A. Jafari-Talookolaei, M. Abedi, H. Afrasiab, Vibration analysis of a microplate in contact with a fluid based on the modified couple stress theory, *Modares Mechanical Engineering*, 17(2),(2017),47-57. (In Persian)
- [9] G. A.Varzandian, S. Ziaei. Analytical Solution of Non-Linear Free Vibration of Thin Rectangular Plates with Various Boundary Conditions Based on Non-Local Theory. *Amirkabir Journal of Mechanical Engineering*, 48(4),(2017), 331-346.(In Persian)
- [10] M. Ghadiri, H. Safarpour, Free Vibration Analysis of a Functionally Graded Cylindrical Nanoshell Surrounded by Elastic Foundation Based on the Modified Couple Stress Theory, *Amirkabir Journal of Mechanical Engineering*, 49(4) (2018) 721-730.(In Persian)
- [11] R. Ansari Khalkhali, A. Norouzzadeh, R. Gholami, Forced vibration analysis of conveying fluid carbon nanotube resting on elastic foundation based on modified couple stress theory, *Modares Mechanical Engineering*, 15(3), (2015),27-34. (In Persian)
- [12] B. Akgöz, Ö. Civalek, Free vibration analysis for single-layered graphene sheets in an elastic matrix via modified couple stress theory, in: *Materials & Design*, 2012, pp. 164-171.
- [13] T. Aksencer, M. Aydogdu, Forced transverse vibration of nanoplates using nonlocal elasticity, *Physica E: Low-dimensional Systems and Nanostructures*, 44(7-8) (2012) 1752-1759.