



## Nonlocal Analysis of Chaotic Vibration, Primary and Super-Harmonic Resonance of Single Walled Carbon Nanotube Considering Thermal Effects

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**ABSTRACT:** In this article, a nonlinear elastic Bernoulli–Euler beam model is presented to investigate the chaotic behavior and primary and superharmonic resonance of single walled carbon nanotubes embedded in a visco-elastic medium at an elevated temperature. Using the Galerkin method and fourth-order Runge-Kutta method the governing equation is solved. The bifurcation diagram and largest Lyapunov exponent are employed to detect the critical amplitude of external force of periodic and chaotic response of single walled carbon. Having known the critical values, phase portrait and Poincare maps are presented to observe the periodic and chaotic behavior of the system. Moreover, the amplitude–frequency response for the primary superharmonic resonance of system is derived with the multiple scale method to investigate the feasibility of jump phenomenon. The sensitivity of jump phenomenon are studied for the selected viscoelastic foundation parameters, detuning parameter and external amplitude load. The results show that the amplitude of external force, viscoelastic foundation parameters, detuning parameter and temperature change in the cases of high and low temperature have a significant effect on the frequency response with jump phenomenon of system. In addition, the chaotic vibration of carbon nanotube can be controlled by changing of amplitude of external force.

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### 1- Introduction

Carbon nanotubes exhibit remarkable electronic, mechanical, physical and chemical properties such as exceptionally high strength-to-weight and stiffness-to-weight ratios as well as superior electrical and thermal conductivities over any other existing material known. Since many nonlinear dynamic phenomena e.g. spring hardening, jump, bifurcation and chaotic motions, which essentially modify sensitivity and performance of Carbon NanoTubes (CNTs), have been discovered experimentally, a comprehensive understanding of all characteristics of nonlinear mechanical response of carbon nanotubes at resonance frequencies under different boundary and environmental conditions is of high importance. Therefore, many research efforts have been focused on these intrinsic nonlinear behaviors of nano-structures. From the research status of vibrational behavior of CNTs, one can be found many researches were focused to investigate the effects axial thermal force, magnetic field, fluid velocity effect, amplitude of excitation, viscoelastic foundation parameters and nonlocal coefficient analysis on the linear and nonlinear dynamic behavior of CNTs have also been investigated [1-5]. Since many nonlinear dynamic phenomena such as chaotic vibrations and jumping at frequency responses of CNTs were analytically and numerically studied.

The main objective of this work is to examine the nonlinear nonlocal primary and superharmonic resonance and chaotic

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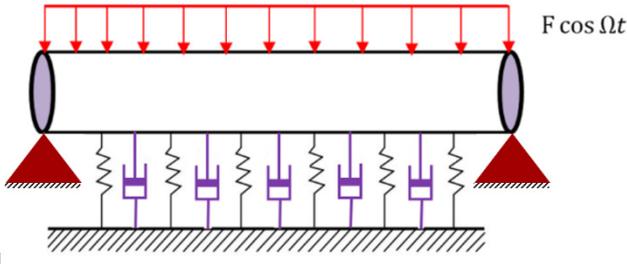
vibration of a single walled CNTs subjected to transverse harmonic forces surrounding to viscoelastic foundation. Using the Galerkin methods and multiple time scales perturbation technique, the nonlinear frequency response of a Single Walled Carbon NanoTube (SWCNT) in the case of primary and superharmonic resonances cases subjected to transverse harmonic force are analytically studied. The influences of amplitude of excitation, viscoelastic foundation, detuning parameter, temperature change and small scale on the instability condition and occurrence of jumping phenomenon at primary and superharmonic resonances response of a SWCNT are investigated. In addition, the chaotic domains of excitation amplitude according to bifurcation diagram and largest Lyapunov exponent are detected by applying the fourth-order Runge-Kutta method. The periodic and chaotic vibration of a SWCNT are demonstrated in phase plane and Poincare map corresponding to the selected external amplitude values.

### 2- Methodology

Fig. 1 shows a multi-walled CNT of length  $L$ , Young's modulus  $E$ , density  $\rho$ , cross-sectional area  $A$ , and cross-sectional moment of inertia  $I$ , resting on a linear viscoelastic medium.

Based on the nonlocal elasticity theory in conjunction with the von Kármán geometric nonlinearity and by considering the thermal effect, the dynamic governing equation of single





**Fig. 1. Schematic of carbon nanotube under viscoelastic foundation**

walled CNT subjected to transverse harmonic external force can be expressed as [7]:

$$\begin{aligned} & \rho A \left(1 - (e_0 a)^2 \nabla^2\right) \frac{\partial^2 w}{\partial t^2} + \left(1 - (e_0 a)^2 \nabla^2\right) C \frac{\partial w}{\partial t} + EI \frac{\partial^4 w}{\partial x^4} \\ & = \left(1 - (e_0 a)^2 \nabla^2\right) \left[ \frac{EA}{2L} \int_0^L \left(\frac{\partial w}{\partial x}\right)^2 dx \right] \frac{\partial^2 w}{\partial x^2} \\ & + \left(1 - (e_0 a)^2 \nabla^2\right) N_t \frac{\partial^2 w}{\partial x^2} + p(x,t) + F(t) \end{aligned} \quad (1)$$

In Eq. (1), the applied pressure to the outermost layer which is in direct contact with the surrounding viscoelastic medium based on the Winkler foundation model can be described by

$$p(x,t) = -K_E \left(1 - (e_0 a)^2 \nabla^2\right) w \quad (2)$$

In addition, the thermally induced force  $N_t$  is given by

$$N_t = -\frac{EA}{1-2\nu} \alpha_x \Delta T \theta_x \quad (3)$$

For a CNT with simply supported boundary conditions at both ends,  $w(x,t) = \sum_{k=1}^K W_k(t) \sin \frac{k\pi x}{L}$  are defined. Applying Eq. (1) to single walled CNT yields a set of the nonlinear equations. For the SWCNTs having simply supported condition at the two ends, the nonlinear vibration equations of an embedded SWCNTs can be written in term of the function

$W_k(t)$  as:

$$\begin{aligned} & \ddot{W}_k + \frac{C}{\rho A} \dot{W}_k + \left( \frac{k^4 \pi^4 EI}{\eta AL^4} + \frac{K_E}{\rho A} + \frac{k^2 \pi^2 N_t}{\eta AL^2} \right) W_k \\ & + \frac{k^4 \pi^4 E}{4\eta L^4} W_k^3 = \frac{F(t)}{\eta A} \end{aligned} \quad (4)$$

where

$$\eta = \rho \left( 1 + \left( e_0 a \frac{k\pi}{L} \right)^2 \right) \quad (5)$$

The non-dimensional form of Eq. (4) can be presented as:

$$\frac{d^2 u_k}{d\tau^2} + \mu \frac{du_k}{d\tau} + u_k + \alpha u_k^3 = \Lambda_k \cos \Omega \tau \quad (6)$$

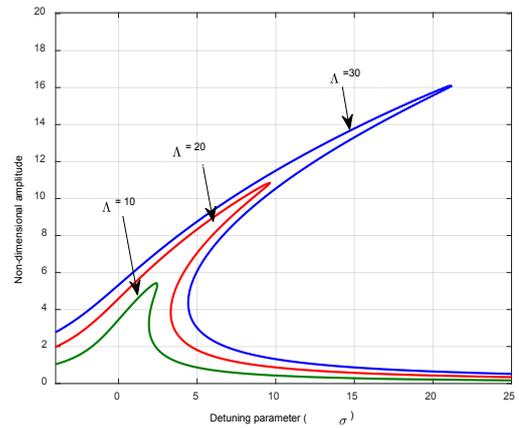
The analytical closed form of frequency response of SWCNT can be derived by applying the multiple time scale method in the cases of primary and super-harmonic resonances as following respectively:

$$\left[ \mu^2 + \left( \sigma - \frac{3}{8} \Lambda a^2 \right)^2 \right] a^2 = \frac{1}{4} \Lambda^2 \quad (7)$$

$$\left[ \frac{1}{4} \mu^2 + \left( \sigma - 3\alpha \Lambda^2 - \frac{3\alpha}{8} a^2 \right)^2 \right] a^2 = \alpha^2 \Lambda^6 \quad (8)$$

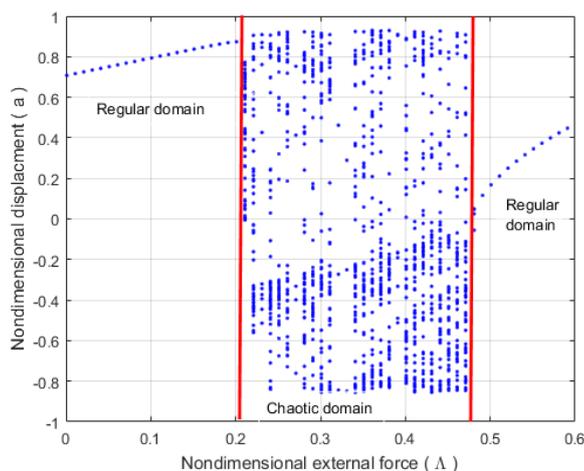
### 3- Discussion and Results

The material and geometric parameters of the CNT are taken to be, the Young's modulus  $E = 1.1 \text{ TPa}$ , the mass density  $\rho = 1300 \text{ kg/m}^3$ , the Poisson's ratio  $\nu = 0.3$ , the length  $L = 45 \text{ nm}$ , the outermost diameter  $d_0 = 3 \text{ nm}$ , the inner outermost diameter  $d_i = 2.32 \text{ nm}$ , the coefficient of thermal expansion in the cases of low temperature  $\alpha_x = -1.6 \times 10^{-6} \text{ K}^{-1}$ , the coefficient of thermal expansion in the cases of high temperature,  $\alpha_x = 1.1 \times 10^{-6} \text{ K}^{-1}$ , change of temperature  $\Delta T = 100 \text{ K}$ , the Winkler foundation modulus  $K_E = 10^7 \text{ N/m}^2$ , the damping coefficient  $C = 3 \times 10^{-15} \text{ Ns/m}^2$  and the scale coefficient  $e_0 a = 1 \text{ nm}$ . Fig. 2 shows the effect of non-dimensional amplitude of external force on the frequency response amplitude of single walled carbon nanotube for different values of  $\Lambda$  at primary resonance. It can be observed that increasing values of  $\Lambda$  intensifies the jump phenomena and makes the unstable region wider.



**Fig. 2. Primary frequency response of CNT**

A bifurcation diagram of excitation amplitude  $\Lambda$  for SWCNT is shown in Fig. 3. For regular response there is a one-by-one correspondence between excitation amplitude and nanotube displacement amplitude and in the chaotic response, any particular points on horizontal axis are mapped to multiple points of displacement amplitude.



**Fig. 3. Bifurcation diagram**

#### 4- Conclusions

In this paper, the nonlinear primary and superharmonic resonances and chaotic vibration of a single walled carbon nanotube embedded in viscoelastic foundation was investigated. The Galerkin method and multiple time scales perturbation method were implemented to drive the frequency response. The effects of amplitude of excitation, temperature change, high and low temperature conditions on the frequency resonance were investigated. Moreover, by employing the bifurcation diagrams and the largest Lyapunov exponent, the effects of amplitude of external force was investigated to predict the chaotic and periodic parameters.

The results show that these parameters have a significant effects on the dynamic behavior of SWCNT.

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