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Natural Frequency Analysis of Rotating Thin-Walled Beams with Embedded Shape Memory Alloy Wires Subjected to Uniform Temperature Field

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ABSTRACT: In this paper, ¬free vibration analysis of the rotating thin-walled composite beams with embedded shape memory alloy wires is represented. Pre-strained shape memory alloy wires are embedded in the middle of the cross section of thin-wall composite beam, symmetrically. The one-dimensional thermo-mechanical constitutive law suggested by Liang-Rogers is applied to model the thermomechanical behavior of shape memory alloy wires. The differential governing equations are extracted by using the extended Hamilton's principle based on first-order shear deformation theory. By heating the thin-walled beam, strain recovery operation will produce a tensile force along the longitudinal thin-walled beam. In order to solve the governing equations, the extended Galerkin method is used. The effect of rotational speed, recoverable strain limit, pre-twist angle, number of shape memory alloy wire and temperature difference on the natural frequency in temperature above the austenite finish are illustrated. It is found that the natural frequencies of rotating thin-walled beam increase as the number of shape memory alloy wires and compressive pre-strained shape memory alloy wires increases. In addition, results are in good agreement with those obtained in the literature.

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1- Introduction

Shape Memory Alloys (SMAs) are a class of metallic alloys that can return to initial configuration in response to stress and/or temperature changing. These SMA are comprised of two crystallography phases, namely, austenite (high-temperature phase) and martensite (low-temperature phase). In the composite structures, SMA wires can be embedded into the matrix such as carbon fibers. When an SMA wire is heated, the martensite phase transformed to austenite phase. Therefore, by heating up and changing phase to austenite a compression in the SMA wires are introduced and natural frequencies are changed. Lau [1] presented both theoretical and experimental vibration behavior of SMA composite beam with different boundary conditions. In another study, Lau et al. [2] are evaluated natural frequency of an SMA composite beam with clamped boundary condition as a theoretical and experimental. Barzegari et al. [3] investigated the natural frequency of an SMA composite beam with different boundary condition based on the three various models as Euler-Bernoulli, Timoshenko and third order beam theory.

In the last years, the theory of rotating blades modeled as thin-walled composite beams with arbitrary closed cross-sections frequently used by researchers [4-7]. For instance, Fazelzadeh and Hosseini [5] studied the vibration characteristics of rotating Functionally Graded Materials (FGM) thin-walled beam under high temperature gas flow. Librescu et al. [6] investigated the behavior of rotating thin-walled beam which made of FGM subjected to temperature environment.

To the best of author's knowledge, no investigation has been done on the vibration characteristics of rotating thin-walled composite beam with embedded SMA wires. The goal of this paper is to present the natural frequency analysis of rotating thin-walled SMA composite beam. The dynamic equations are obtained using extended Hamilton principle based on the first order shear deformation theory. It is assumed the SMA wires are embedded in the thin-walled beam with compressive pre-strained load. The result elucidated that by increasing numbers of SMA wires and angular velocity, the first three natural frequencies increase.

2- Thermo-Mechanical Behavior of SMA Material

The axial force introduced in the longitudinal direction of SMA wires due to temperature field is expressed as [3]:

$$P_z = \frac{\left[\alpha_c E(\xi) + \Theta\right](T - T_0) + \psi(\xi)(\xi - \xi_0)}{1 - \dfrac{E(\xi)A_{SMA}}{E_C A_C}} A_{SMA} \tag{1}$$

where α_c, E_c, A_c are coefficient of thermal expansion, Young's moduli and the cross-sectional area composite beam without SMA wires of matrix respectively. Θ and A_{SMA} are coefficient of thermal expansion and the cross-sectional areas of the embedded SMA wires, respectively. T, T_0 are the temperature field and reference temperature fixed at 20^oC. ξ, ψ indicate martensite fraction of SMA wires and phase transformation tensor, respectively.

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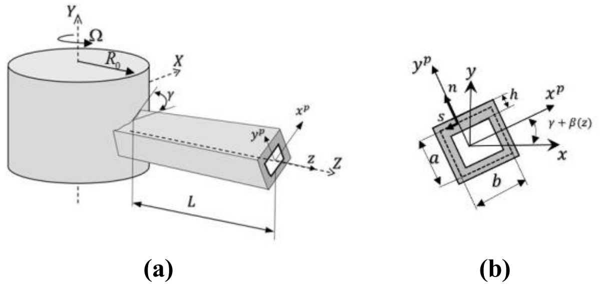

Fig. 1. (a) The schematic of rotating thin-walled beam, (b) Cross section of thin-walled beam

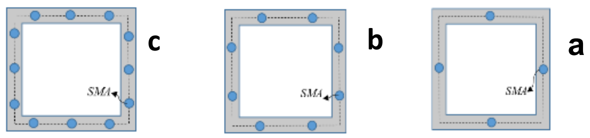

Fig. 2. Alignment of SMA wire (a) 4 numbers of SMA, (b) 8 numbers of SMA, (c) 12 numbers of SMA

3- Governing Equations of Motion

Fig. 1 indicated a straight and pre-twisted flexible thin-walled rotating beam of length L , mounted on a rigid hub of radius R_0 at the presetting angle γ . It is assumed that the thin-walled beam rotate at a constant angular velocity Ω about Y axis.

Three different alignments of SMA wire embedded in the thin-walled beam are shown in Fig. 2. The equations of motion and associated boundary conditions of rotating thin-walled beam can be obtained through the use of extended Hamilton's principle as follows [6].

Governing equations:

$$\delta u_0 : \left[a_{44}(z)(u_0'+\theta_y)+a_{45}(z)(v_0'+\theta_x)\right]' -b_1\ddot{u}_0+b_1\Omega^2 u_0+\Omega^2\left[R(z)u_0'\right]'-\left[P_z u_0'\right]'=0 \tag{2a}$$

$$\delta v_0 : \left[a_{55}(z)(v_0'+\theta_x)+a_{54}(z)(u_0'+\theta_y)\right]' -b_1\ddot{v}_0+\Omega^2\left[R(z)v_0'\right]'-\left[P_z v_0'\right]'=0 \tag{2b}$$

$$\delta\theta_x : \left[a_{33}(z)\theta_x'+a_{32}(z)\theta_y'\right]'-a_{55}(z)(v_0'+\theta_x) -a_{54}(z)(u_0'+\theta_y)-(b_4(z)+b_{14}(z))(\ddot{\theta}_x-\Omega^2\theta_x) -(b_6(z)-b_{13}(z))(\ddot{\theta}_y-\Omega^2\theta_y)=(M_x^T)' \tag{2c}$$

$$\delta\theta_y : \left[a_{22}(z)\theta_y'+a_{23}(z)\theta_x'\right]'-a_{44}(z)(u_0'+\theta_y) -a_{45}(z)(v_0'+\theta_x)-(b_5(z)+b_{15}(z))(\ddot{\theta}_y-\Omega^2\theta_y) -(b_6(z)-b_{13}(z))(\ddot{\theta}_x-\Omega^2\theta_x)=(M_y^T)' \tag{2d}$$

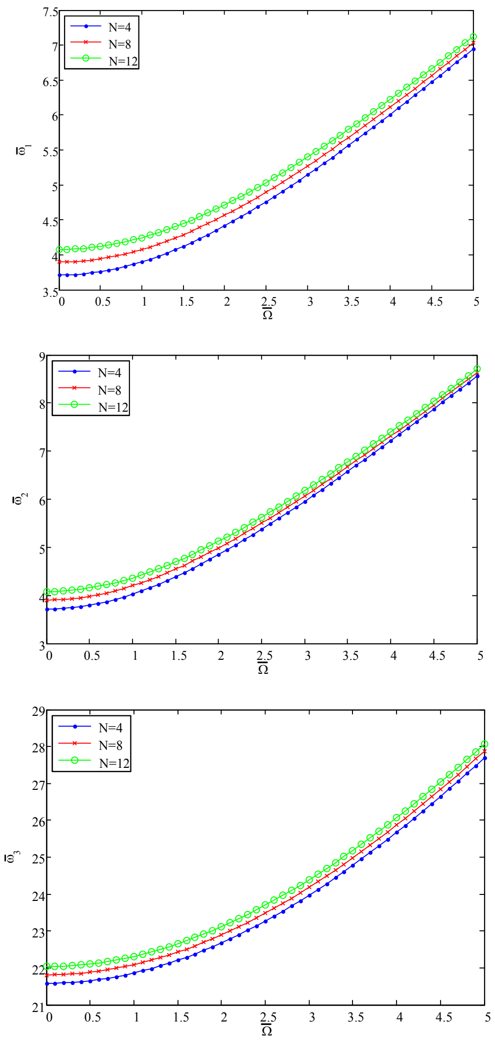

Fig. 3. variation of first three natural frequencies as a function of angular velocity for three different numbers of SMA wire embedded in the thin-walled blade

Boundary conditions:

At $z=0$,

$$u_0=v_0=\theta_x=\theta_y=0 \tag{3a}$$

At $z=L$,

$$\begin{aligned} &\delta u_0 : a_{44}(z)(u_0'+\theta_y)+a_{45}(z)(v_0'+\theta_x)=P_z u_0', \\ &\delta v_0 : a_{55}(z)(v_0'+\theta_x)+a_{54}(z)(u_0'+\theta_y)=P_z v_0', \\ &\delta\theta_y : a_{22}(z)\theta_y'+a_{23}(z)\theta_x'=M_y^T, \\ &\delta\theta_y : a_{33}(z)\theta_x'+a_{32}(z)\theta_y'=M_x^T \end{aligned} \tag{3b}$$

4- Method of Solution

In order to solve the coupled partial differential equations (Eq. (2)), extended Galerkin's method is applied. In this method, we express the displacement fields as a product of weight function and generalized coordinates as follows [5]:

$$u_0(z,t)=U^T q_u\ ,v_0(z,t)=V^T q_v$$
$$\theta_x(z,t)=\Theta_x^T q_x\ ,\ \theta_y(z,t)=\Theta_y^T q_y \tag{4}$$

where $\boldsymbol{U}$, $\boldsymbol{V}$, Θ_x and Θ_y are the weight functions that satisfied the boundary conditions, and $\boldsymbol{q_u}$, $\boldsymbol{q_v}$, $\boldsymbol{q_x}$ and $\boldsymbol{q_y}$ are time dependent vector of generalized coordinates. By substituting displacement fields (Eq. (4)) into governing equations, the matrix form of equations are achieved as

$$[M]\{\ddot{q}(\mathrm{t})\}+[K]\{q(\mathrm{t})\}=0 \tag{5}$$

where M and K are the mass and stiffness matrix, respectively. Also, $q(\mathrm{t})$ is the generalized coordinate system. The natural frequency of dynamic rotating thin-walled beam system are obtained by substituting the generalized coordinates as $q(t)=\eta e^{\omega t}$ in Eq. (5). So we have,

$$\omega^2=-[M]^{-1}[K] \tag{6}$$

5- Result and Discussion

In the present paper, the vibration characteristics of rotating thin-walled beam with embedded SMA wires are analyzed. For example, Fig. 3. Shows the influence of number of SMA wires on the first three natural frequencies of rotating thin-walled composite SMA beam as a function of rotating speed Ω . In this figure we take $\varepsilon_L=6.7\%$, $T=70^o\mathrm{C}$, $\gamma=0$ and $\beta_0=45^o$.

The results show that by increasing the number of SMA wires (N) the first three natural frequencies increase. This is to be expected, because the increase of number of SMA wires yields an increase tensile force due to strain recovery action of the pre-strained SMA wires in the longitudinal directions. Also, it can be observed that the first three natural frequencies increase with the increase of rotating speed.

6- Conclusions

The natural frequency analysis of rotating thin-walled beams with embedded shape memory alloy wires under uniform temperature field have been investigated based on the first order shear deformation theory and using extended Galerkin's technique. The dynamic equations are obtained using extended Hamilton principle. It is supposed the SMA wires embedded in the thin-walled beam with compressive pre-strained load. Here, the effect of number of SMA wires and rotating velocity are presented. The result reveal that by increasing numbers of SMA wires and angular velocity, the first three natural frequencies increase.

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