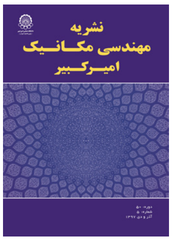

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Electro-mechanical Analysis of Rotating Cylinder Made of Functionally Graded Piezoelectric Materials: Sensor and Actuator

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ABSTRACT: In this paper, based higher shear deformation theory, electro-elastic equation of functionally graded material axisymmetric thick-walled cylinders in general form is presented. The displacements, stresses and electrical potential in clamped-clamped cylindrical shells analytically are calculated. The presented approach leads to the definition of new formulation to study thick shells based on shear deformation theory. The mechanical equilibrium equation obtained by energy method and for finding electrical equilibrium equation used Maxwell and Gauss equations. The governing equation solved in general form (independent of the order of shear deformation theory) by the coupled electro-mechanical using eigen vectors In this study, all mechanical and electrical piezoelectric material properties, were considered to follow an identical power law in the radial direction. The results obtained in the present paper have been compared with findings of plane elasticity theory. For investigating the effect of higher order approximations on displacements and stresses and electrical potential, a comparison between the results of first and third-order shear deformation theory have been studied. The numerical results show that the higher-order approximations must be applied in electro-elastic analysis of cylindrical shells made of functionally graded piezoelectric material. Finally, some numerical results are presented to study the effects of mechanical and electrical loading on the stresses, displacements and electrical potential of the cylinder.

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1- Introduction

The use of distributed sensors and actuators integrated with structures has become the subject of focus in recent years, and these structures have been termed intelligent or smart structures [1]. Functionally Graded Piezoelectric Materials (FGPM) are a new class that has been widely used as intelligent or smart structures in the engineering applications [2]. A number of papers considering various aspects of FGPM have been published in recent years [3–5].

In the present work, an attempt has been made to find the formulation for the FGPM thick cylindrical shell subjected to electro-mechanical loading, by using the higher-order shear deformation theory. The material properties are assumed to vary in the radial direction of the cylinder. The governing equations in the axisymmetric case and elasto-static state, which are a system of ordinary differential equations with constant coefficients, have been solved using the eigenvalues method. The results are compared with those derived by the Plane Elasticity Theory (PET) [6] for some load cases. Finally, the conclusions drawn from the present study are reported.

2- Mathematical Model

Geometry of a FGPM cylindrical shell under mechanical and electrical loading is shown in Fig.1.

It is based Shear Deformation Theory (SDT) on the displacement field [7]

$$U_z = \sum_{i=0}^{n_{SDT}} u_i z^i, U_x = \sum_{i=0}^{n_{SDT}} w_i z^i \tag{1}$$

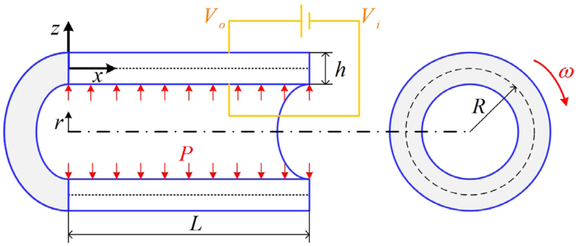

Fig.1. FGPM cylindrical shell.

Taking into account both the direct piezoelectric effect and the converse piezoelectric effect, distribution of the electric potential is given [8]:

$$\phi = \left(\frac{V_o - V_i}{h}\right) z - \left(\frac{V_o + V_i}{2}\right) + \left(z^2 - \left(\frac{h}{2}\right)^2\right)\psi(x) \tag{2}$$

The electric field strength is described by a function of the electric potential as follows:

$$E_x = -\frac{\partial \phi}{\partial x}, E_z = -\frac{\partial \phi}{\partial z} \tag{3}$$

Using the displacements of Eq. (1), the strains are given by

$$\begin{cases} \varepsilon_x = \frac{\partial U_x}{\partial x} = \sum_{i=0}^{n_{SDT}} \frac{dw_i}{dx} z^i \\ \varepsilon_\theta = \frac{U_z}{r} = \frac{1}{R+z} \sum_{i=0}^{n_{SDT}} u_i z^i \\ \varepsilon_z = \frac{\partial U_z}{\partial z} = \sum_{i=0}^{n_{SDT}} i u_i z^{i-1} \\ \gamma_{xz} = \frac{\partial U_x}{\partial z} + \frac{\partial U_z}{\partial x} = \sum_{i=0}^{n_{SDT}} i w_i z^{i-1} + \sum_{i=0}^{n_{SDT}} \frac{du_i}{dx} z^i \end{cases} \tag{4}$$

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For an orthogonal anisotropic piezoelectric material, considering that the geometry and loading of the shell are axisymmetric, the linear constitutive equations coupling the piezo-thermo-elastic field are expressed in the cylindrical coordinate system

$$\begin{Bmatrix}\sigma_z\\ \sigma_\theta\\ \sigma_x\\ \tau_{zx}\end{Bmatrix}=\begin{bmatrix}C_{11}&C_{12}&C_{13}&0\\ C_{12}&C_{22}&C_{23}&0\\ C_{13}&C_{23}&C_{33}&0\\ 0&0&0&C_{55}\end{bmatrix}\begin{Bmatrix}\varepsilon_z\\ \varepsilon_\theta\\ \varepsilon_x\\ \gamma_{zx}\end{Bmatrix}-\begin{bmatrix}e_{11}&0\\ e_{12}&0\\ e_{13}&0\\ 0&e_{35}\end{bmatrix}\begin{Bmatrix}E_z\\ E_x\end{Bmatrix} \tag{5}$$

$$\begin{Bmatrix}D_z\\ D_\theta\\ D_x\end{Bmatrix}=\begin{bmatrix}e_{11}&e_{12}&e_{13}&0\\ 0&0&0&0\\ 0&0&0&e_{35}\end{bmatrix}\begin{Bmatrix}\varepsilon_z\\ \varepsilon_\theta\\ \varepsilon_x\\ \gamma_{zx}\end{Bmatrix}+\begin{bmatrix}\xi_{11}&0\\ 0&0\\ 0&\xi_{33}\end{bmatrix}\begin{Bmatrix}E_z\\ E_x\end{Bmatrix} \tag{6}$$

The stress resultants can be written as

$$\begin{Bmatrix}N_x^{(i)}\\ N_\theta^{(i)}\\ N_z^{(i)}\\ N_{xz}^{(i)}\end{Bmatrix}=\int_{-h/2}^{h/2}\begin{Bmatrix}\sigma_x(R+z)\\ R\sigma_\theta\\ \sigma_z(R+z)\\ \tau_{xz}(R+z)\end{Bmatrix}\frac{z^i}{R}dz,\left(i=0..\mathrm{n}_{SDT}\right) \tag{7}$$

Derivation of the equations of motion of the piezoelectric functionally graded cylindrical shell is accomplished by employing virtual work's principle. According to this principle, the motion equations are derived when the following equation holds considering the effect of in-surface and rotary inertias, the motion equations for a functionally graded piezoelectric cylindrical shell under mechanical loading and piezoelectric loading are written as:

$$\begin{aligned}&\frac{d}{dx}\left(RN_x^{(\mathrm{i})}\right)-iRN_{xz}^{(i-1)}=0\\ &\frac{d}{dx}\left(RN_{xz}^{(\mathrm{i})}\right)-iRN_z^{(i-1)}-N_\theta^{(i)}=\end{aligned} \tag{8}$$

$$-\int_{-h/2}^{h/2}\rho\omega^2(R+z)^2z^idz+P\left(R-\frac{h}{2}\right)\left(-\frac{h}{2}\right)^i \tag{9}$$

According to the Maxwell equation [9]:

$$\int_0^L\int_0^{2\pi}\int_{-\frac{h}{2}}^{\frac{h}{2}}\left(\frac{\partial D_x}{\partial x}+\frac{1}{R+z}\frac{\partial}{\partial z}\left((R+z)D_z\right)\right)(R+z)dzd\theta dx=0 \tag{10}$$

Therefore, the total solution for system of ordinary differential equations with constant coefficients is [10]

$$\{y\}_h=\sum_{\beta=1}^{2(2n_{SDT}+3)}C_\beta\{\eta\}_\beta e^{\lambda_\beta x}+\{K\} \tag{11}$$

where

$$\{y\}=\{u_0\quad u_1\quad\dots\quad u_{n_{SDT}}\quad w_0\quad w_1\quad\dots\quad w_{n_{SDT}}\quad\psi\}^T \tag{12}$$

The boundary conditions of cylinder at two end of cylinder assumed as follow

$$\begin{cases}\begin{Bmatrix}u_i\\ w_i\end{Bmatrix}_{x=0}=\begin{Bmatrix}u_i\\ w_i\end{Bmatrix}_{x=L}=\begin{Bmatrix}0\\ 0\end{Bmatrix}\quad ,i=0..n_{SDT}\\ \phi|_{x=0}=\phi|_{x=L}=0\end{cases} \tag{13}$$

3- Results and Discussion

In this section, numerical example is presented and discussed for verifying the accuracy of the present theory in predicting the electro-mechanical stress responses of cylinder. The displacements and circumferential stress of the cylinder for orders one (FSDT) and three (TSDT) of shear deformation theory caused by mechanical and thermal loading are reported in Figs 2 and 3.

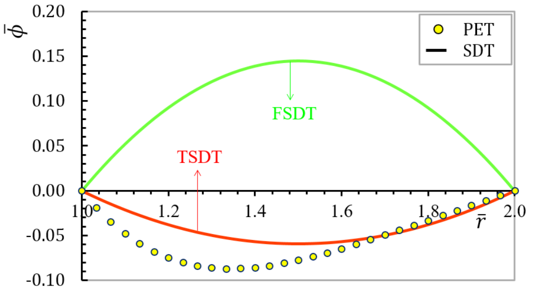

Fig.2. The electrical potential along the radial direction.

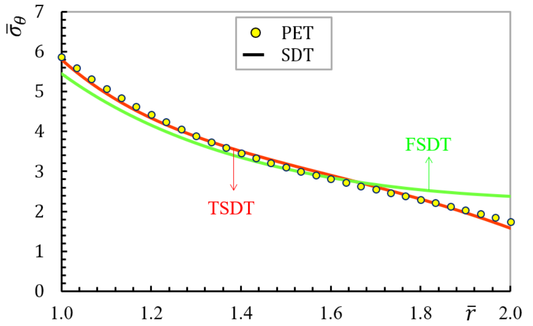

Fig.3. The hoop stress along the radial direction.

It is seen from Fig.2 that FSDT is not an acceptable solution. Thus, it is necessary to use a higher-order approximation. Fig.3 displays that HSDT is a very good approximation for hoop stress with respect to the PET results.

4- Conclusions

In the present study, making use of the HSDT, a general formulation for thick FGPM cylindrical shells under electro-mechanical loading has been presented. The properties graded along the radial direction. Based on the energy principle and the HSDT, the equilibrium equations have been derived. The system of ordinary differential equations with constant coefficients has been solved by using the eigenvalues method. Good agreement was found between the present results and PET. It concluded that the higher-order approximations must be applied in order to improve the accuracy of the theory.

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