Fatigue crack growth analysis via Wiener degradation model with random effects

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ABSTRACT

Aerospace structures reliability is analyzed in order to increase the availability and decrease the stochastic failures of the system. A degradation-based modeling method is an effective approach for reliability assessment. Degradation models are usually developed based on degradation data or understandings of physics behind the degradation processes of products or systems. Stochastic models such as the Wiener process are one of the powerful tools in this field, especially the analysis of damage expansion and fatigue crack growth. This study presents a survey of degradation modeling approaches with consideration of random effects frequently used in engineering programs. Firstly, Wiener processes are used to model the degradation process of the product, which considers measurement errors simultaneously with random effects. Moreover, the closed-form expressions of some reliability quantities such as the Probability Density Function (PDF) are derived. Then, the Maximum Likelihood Estimation (MLE) method based on the Expectation-Maximation (EM) algorithm is presented to estimate the unknown parameters in the degradation models. Finally, a practical case study of fatigue crack growth using proposed models is provided and compared with the basic Gamma process to demonstrate the superiority and effectiveness of the Wiener process. It is shown that the Wiener process model estimates fatigue crack growth path better than the Gamma model and by adding the measurement error parameter to the model, its accuracy is increased.

KEYWORDS

Wiener processes, Random effects, Measurement errors, Gamma process, Fatigue

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1. Introduction

Degradation is a process that occurs under the influence of internal and external factors such as environmental and operational conditions within a system or component. One of the modes of degradation is damage accumulation over time, which is usually an irreversible process, and when the accumulated damage exceeds a natural or predetermined threshold level, the ultimate failure can occur and can cause severe losses. Therefore, it is imperative to study and model the mechanisms of system degradation to predict and prevent potential failures in order to effectively prevent subsequent losses. In recent years, extensive research has been conducted in this area that can be classified into two broad categories: the data-driven and physics-of-failure based models [1, 2].

In recent years, degradation process-based reliability analysis has been extended to reduce product development time, and the Wiener process has always been the interest of researchers as one of the methods of stochastic process modeling. Using the data collected under fatigue loading conditions, Mishra and Vanli [3] proposed a new method for predicting the remaining useful life of a structure from Lamb wave sensor using regression and Wiener process modeling. Omar and his colleagues [4] studied the fatigue and contact wear for the bearing of the motor pump by Gamma and Wiener processes. They presented a comparison between Wiener and Gamma processes and identified their advantages, drawbacks well as their principle uses. Zhuang et al. [5] investigated wear of revolute joints of a lock mechanism in an aircraft using the Wiener process and showed the cumulative wear of each revolute joint over time.

The authors' investigation shows that the fatigue crack growth analysis using the Wiener process has not been studied simultaneously with random effects and measurement error. For this purpose, in this paper, at the first, the Wiener model with measurement error and random effects are presented and then the Expectation-Maximation (EM) algorithm will be used to estimate the parameters. In the next step, will be demonstrating the measurement error and random effects model using the fatigue crack growth data reported in Wu and Ni [6] and the results will be compared with the Gamma model.

2. Methodology

Wiener process is also called Gaussian process or Brownian motion with drift. In general, a Wiener process can be expressed as [7]

$$X(t) = \mu\Lambda(t) + \sigma B(\Lambda(t)) \tag{1}$$

where $X(t)$ represents system degradation, μ is the drift parameter showing the rate of degradation, σ is the volatility parameter, $B(\cdot)$ is the standard Brownian motion, and $\Lambda(t)$ is a monotone increasing function representing a general time scale.

A system often fails when the degradation process arrives at a certain critical degradation level (h). The lifetime T of the system is then determined as the first instant at which the degradation process $X(t) \geq 0$ exceeds the level h:

$$T = \inf\{t \geq t_0; X(t) \geq h\} \tag{2}$$

The PDF of the life time T of (1) to h is given by:

$$\begin{aligned} f_T(t) &= -\frac{dR_T(t)}{dt} = \frac{h}{\sqrt{2\pi\sigma^2(\Lambda(t))^3}} \exp\left(-\frac{(h-\mu\Lambda(t))^2}{2\sigma^2\Lambda(t)}\right)\frac{d\Lambda(t)}{dt} \\ &= \frac{h}{\sqrt{\sigma^2(\Lambda(t))^3}} \phi\left(\frac{h-\mu\Lambda(t)}{\sigma\sqrt{\Lambda(t)}}\right)\frac{d\Lambda(t)}{dt} \end{aligned} \tag{3}$$

The Basic Wiener process model is able to reflect the inherent randomness of the degradation itself, but it is unable to capture measurement errors introduced because of imperfect inspections. So, the degradation process is given by:

$$Y(t) = X(t) + \varepsilon \tag{4}$$

where $\varepsilon \sim N(0, \sigma_\varepsilon)$. In this case the lifetime T_e is defined as:

$$T_e = \inf\{t \geq t_0; Y(t) \geq h\} = \inf\{t \geq t_0; X(t) \geq h_e\} \tag{5}$$

where $h_e \sim N(h, \sigma_\varepsilon^2)$. As regards $\mu \sim N(\mu_\mu, \sigma_\mu^2)$ and $h_e \sim N(h, \sigma_\varepsilon^2)$, PDF of the life time T_e is [8]:

$$\begin{aligned} f(t) = &\left[\frac{h\sigma^2 + \mu_\mu\sigma_\varepsilon^2}{\left(\sigma_\varepsilon^2 + \sigma^2\Lambda(t)\right)\sqrt{\sigma_\varepsilon^2 + \sigma^2\Lambda(t) + \sigma_\mu^2\Lambda^2(t)}}\right. \\ &\left. -\frac{\sigma_\mu^2\sigma_\varepsilon^2\Lambda(t)\left(h - \mu_\mu\Lambda(t)\right)}{\left(\sigma_\varepsilon^2 + \sigma^2\Lambda(t)\right)\left(\sigma_\varepsilon^2 + \sigma^2\Lambda(t) + \sigma_\mu^2\Lambda^2(t)\right)^{3/2}}\right] \times \\ &\phi\left(\frac{h - \mu_\mu\Lambda(t)}{\left(\sigma_\varepsilon^2 + \sigma^2\Lambda(t)\right)\sqrt{\sigma_\varepsilon^2 + \sigma^2\Lambda(t) + \sigma_\mu^2\Lambda^2(t)}}\right)\frac{d\Lambda(t)}{dt} \end{aligned} \tag{6}$$

The observed degradation of a system may be very different because of unobservable endogenous factors and exogenous factors. Random effects models proved useful in dealing with these unobserved heterogeneities. One of the Wiener process models with random effects is [9]:

$$X(t) = \mu\Lambda(t) + \varsigma\mu B(\Lambda(t)) \tag{7}$$

In this case If be assumed $\kappa = 1/\mu \sim N(\mu_\kappa, \sigma_\kappa^2)$, then the PDF of lifetime T is defined as:

$$\begin{aligned} f_T(t) &= -\frac{dR_T(t)}{dt} \\ &= \frac{(h\sigma_\kappa^2 + \mu_\kappa\varsigma^2)\Lambda(t)}{\sqrt{2\pi[h^2\sigma_\kappa^2 + \varsigma^2\Lambda(t)]^3}} \exp\left(-\frac{(\mu h - \Lambda(t))^2}{2(\varsigma^2\Lambda(t) + \sigma_\kappa^2 h^2)}\right)\frac{d\Lambda(t)}{dt} \end{aligned} \tag{8}$$

One of the best stochastic process models, that it is appropriate when the gradual damage is monotonically increasing or decreasing over time, such as fatigue, corrosion, and crack growth is Gamma process. The PDF of this model for lifetime T is defined as[10]:

$$f_{\Delta L(t)}(y)=\frac{y^{\Delta\eta(t)-1}}{\beta^{\Delta\eta(t)}\Gamma(\Delta\eta(t))}\exp\left(-\frac{y}{\beta}\right) \quad y>0 \tag{9}$$

where $\Delta\eta(t)$ and β are the shape (function) and scale parameters of the gamma distribution, respectively.

Assume that the degradation processes of n independently tested units are inspected at ordered inspection times $t_1,\ldots,\ t_m$ with the degradation observations $\{\rho_i(t_j)=l_{ij}, i=1,\ldots,n,\ j=1,\ldots,m\}$, where ρ is depredation models mentioned above. By obtaining likelihood-function in each degradation model, the EM algorithm is used to estimate the unknown parameters.

3. Results and Discussion

In this study, the performance of the basic Wiener model, the Wiener with measurement error model, Wiener with random effects model, and the basic Gamma model based on data of fatigue crack growth of a batch of 2024-T351 aluminum alloy specimens [6] are compared. In the experiment, 30 units are subject to a constant amplitude fatigue test and the crack growth paths of 28 units are depicted in Figure 1.

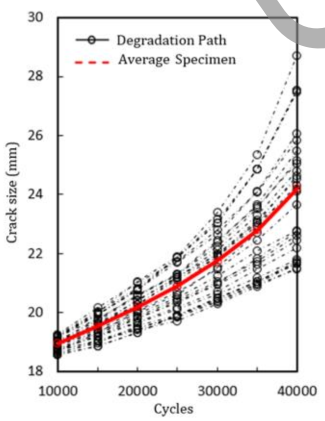

Figure 1. Fatigue crack growth paths of 28 testing units

As regards that the crack growth follows a power law, it can be considered that $\Lambda(t)=t^b$ which $\eta=b$. MLEs of the parameters in these models and the corresponding values of the Akaike Information Criterion (AIC) are listed in Table 1. The AIC is defined to be $AIC=-2L(S)+2P$, where P, S, L is the number of parameters, the observation and the maximum of the likelihood function of the model, respectively. From Table 1, it can be seen that the Wiener process with measurement error fits the data best. The Wiener process model estimates fatigue crack growth path better than the Gamma model and by adding the measurement error parameter to the model, its accuracy is increased.

Table 1. MLEs of 4 different Wiener process models for the crack growth data

Model	MLE	AIC
Basic Wiener	$\mu=0.0219,\ \sigma=0.0578,\ \eta=1.50$	184.11
Basic Gamma	$\beta=38.45,\ \eta=1.57$	230.27
Wiener with measurement error	$\mu_\mu=38.79,\ \sigma_\mu=11.33,$ $\sigma=0.0484,\ \sigma_\varepsilon=2.11\times10^{-12},$ $\eta=1.41$	-15.63
Wiener with random effects	$\mu_\kappa=35.11,\ \sigma_\kappa=10.96,$ $\varsigma=1.27,\ \eta=1.44$	-3.99

Estimated mean paths based on fatigue crack growth by the degradation models are given in Figure 2.

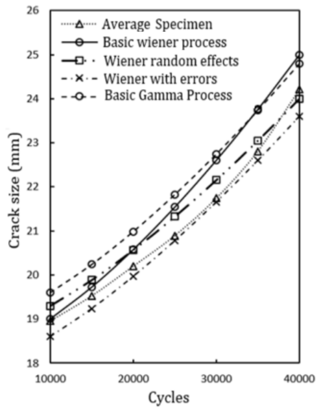

Figure 2. Estimated mean paths based on fatigue crack growth

To validate the estimated model, two samples 7 and 27 of 30 testing units are used to predict and estimate the degradation path by the Wiener process with random effects.

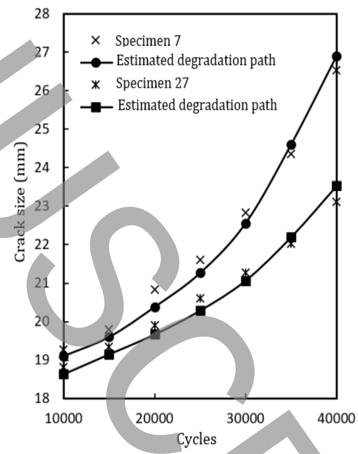

Figure 3. Comparison between the estimated path and experimental data of units 7 and 27

4. Conclusion

In this paper, the reliability of an aluminum alloy specimens under fatigue load is investigated using degradation processes. Stochastic process models are extensive, however, in this paper Wiener process with measurement errors, random effects and the basic Gamma process are used. In each process, the closed-form expressions of the Probability Density Function (PDF) of a lifetime are defined and the unknown parameters are estimated using the Expectation-Maximation (EM) algorithm. Then, each of these models is applied to analyze the fatigue crack growth data where the units were tested under the same conditions and the results of each degradation process are compared. The results show that the Wiener with measurement error model fits the data best. Degradation process models are extensive, therefore, for further investigation, similar study can be performed using models such as general path, inverse Gaussian process, and etc. It is also recommended to use machine learning techniques such as Support Vector Machines (SVM) and neural network to model degradation of a system and evaluate its results.

5. References

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