Free vibration analysis of functionally graded carbon nanotubes reinforced composite skew folded plates using isogeometric approach

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ABSTRACT

In this research, an approach based on isogeometric method is developed to study free vibration behavior of functionally graded carbon nanotubes reinforced composite skew folded plates. In this method, Non-Uniform Rational B-Splines (NURBS) basis functions are used for approximation of the geometry as well as the displacement field. The plates are reinforced by single-walled carbon nanotubes which are assumed to be graded through the thickness direction with different distribution patterns. The effective mechanical properties of composite skew folded plates are captured by the modified rule of mixtures approach. Modeling of the skew folded plate is accomplished by two NURBS patches which is one of the strengths of the research. The equations of motion of each patch are derived based on classical plate theory and then are discretized using NURBS basis functions. The final form of the discretized equations is generated after transformation of the element matrices of each patch and then applying the continuity conditions along the boundary of the patches with the aid of bending strip method. Afterward, several numerical examples are provided to prove the accuracy and reliability of the proposed formulation. The results exhibit that the present approach can precisely predict the natural frequencies of skew folded plates with a low computational cost. Eventually, a set of new results are presented for different geometrical and material parameters of the skew folded plate.

KEYWORDS

Isogeometric analysis, Bending strip method, Skew plates, Folded plates, Carbon nanotubes.

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1. Introduction

Nowadays, skew plates are widely used as structural element of many modern structures. Moreover, folded plates are an appropriate candidate for using in different engineering applications. Meanwhile, several researches have been performed on the improvement of reinforced composite materials during recent years. Due to excellent mechanical, thermal and electrical properties of carbon nanotubes (CNTs), they are a suitable choice as a reinforcement phase in a polymer matrix. Distribution of CNTs in matrix may be uniform or functionally graded (FG) in the thickness direction of the plate. The latter case is referred to as functionally graded carbon nanotube reinforced composite (FG-CNTRC) skew folded plates.

Numerous studies have been executed to investigate free vibration behavior of rectangular folded plates. Most of these studies have been accomplished in the last two decades, along with the development of high-speed computers. In this regard, various numerical techniques have been proposed to study mechanical behaviors of folded plates. These methods are finite strip method [1], combined boundary element-transfer matrix method [2], mesh-free Galerkin method [3], finite element method (FEM) [4] and isogeometric method [5].

The isogeometric analysis (IGA) is a powerful numerical technique which was firstly proposed by Hughes and his co-workers [6]. In this method, the exact geometric description is used to approximate the solution field. Since introduction of the IGA, it has been extensively employed to analyze FG-CNTRC structures.

2. Solution Method

In this study, it is assumed that each patch of the skew folded plate is fabricated from a polymer matrix reinforced by single walled carbon nanotubes (SWCNTs) with uniform distribution (UD) as well as three FG distributions defined as FG-X, FG-O, and FG-V. The effective mechanical properties of FG-CNTRC skew folded plates are estimated via modified rule of mixture.

In order to derive the equations of motion of each patch, the local coordinate system is placed in the midplane corner of the patch. The displacement field of each patch is approximated based on the classical plate theory (CPT). The weak form for free vibration analysis is obtained using the principle of virtual work. Then the field equations are discretized using NURBS basis functions. Afterwards, the element matrices which are evaluated in the local coordinate system of the patch, are transferred to the global coordinate system. Moreover, the bending strip method [7] is used to define the continuity conditions along the intersection of the patches.

3. Results and Discussion

As a part of validation study, the non-dimensional fundamental frequency parameter ($\bar{\omega}_1$) of simply supported FG-CNTRC square plates with $L/h = 50$ are listed in Table 1. The results are prepared for various distribution of CNTs as well as different volume fractions. The presented data are compared with FSDT based solution conducted by Zhu et al. [8]. They employed FEM to extract the results. According to provided data in Table 1, it can be observed that the convergence behavior is very good. Moreover, both set of results exhibit a very good agreement.

Figure 1 illustrates variation of the fundamental frequency ratio ($\beta_1 = \omega_1^{\text{skew}} / \omega_1^{\text{rectangular}}$) with respect to the skew angle (θ) for simply supported and fully clamped FG-X CNTRC skew folded plates. it can be easily deduced that the fundamental frequency ratio is always greater than unity, which means that skew folded plates have greater fundamental frequency compared to the rectangular ones.

4. Conclusions

In this paper, the IGA is employed to study free vibration behavior of FG-CNTRC skew folded plates. The equivalent mechanical properties of the plate are approximated according to the modified rule of mixture. The skew folded plate is modeled by two patches. The governing equations of each patch are derived with the aid of principle of virtual work based on the CPT. After an appropriate coordinate transformation, the bending strip method is applied to fulfill the continuity conditions. Several numerical examples are presented to show the efficacy of the proposed formulation and to discuss the effect of related parameters. It is observed that, for all distribution patterns, with increasing CNTs volume fraction, the fundamental frequency of skew folded plate increases. Moreover, among all considered distribution patterns, FG-X and FG-O shapes give highest and lowest frequencies. Furthermore, it is concluded that the skew angle has a pronounced effect on the computed results.

Table 1. Convergence and comparison study of non-dimensional fundamental frequency parameter $\left(\bar{\omega}_1 = \omega_1\left(L^2/h\right)\sqrt{\rho^{\mathrm{m}}/E^{\mathrm{m}}}\right)$ for various types of simply supported FG-CNTRC square plates with different CNTs volume fractions, $\left(\alpha = 180^{\mathrm{o}},\ \theta = 0^{\mathrm{o}},\ L/h = 50\right)$.

V^*_{CNT}	Method	$N_\xi = N_\eta$	UD	FG-V	FG-O	FG-X
0.11	Present (CPT)	1	19.5826	16.4483	14.2494	23.6461
		3	19.5813	16.4471	14.2484	23.6446
		5	19.5813	16.4471	14.2484	23.6446
		7	19.5813	16.4471	14.2484	23.6446
		9	19.5813	16.4471	14.2484	23.6446
	FEM (FSDT) [8]		19.223	16.252	14.302	22.984
0.14		1	21.8974	18.3017	16.0070	26.5209
		3	21.8959	18.3004	16.0059	26.5192
		5	21.8959	18.3004	16.0059	26.5192
		7	21.8959	18.3004	16.0059	26.5192
		9	21.8959	18.3004	16.0059	26.5192
	FEM (FSDT) [8]		21.354	17.995	15.801	25.555
0.17		1	24.1173	20.2095	17.6958	29.1809
		3	24.1157	20.2080	17.6945	29.1791
		5	24.1157	20.2080	17.6945	29.1791
		7	24.1157	20.2080	17.6945	29.1791
		9	24.1157	20.2080	17.6945	29.1791
	FEM (FSDT) [8]		23.697	19.982	17.544	28.413

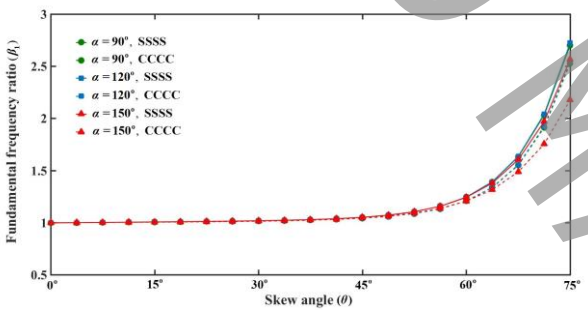

Figure 1. Variation of fundamental frequency ratio (β_1) with respect to skew angle (θ) for simply supported and fully clamped FG-X CNTRC skew folded plates with three different crank angles, $\left(V^*_{CNT} = 0.17,\ L/h = 50\right)$.

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