Nonlinear Electro-aero-elastic Vibration Analysis of a Nanocomposite Laminated Trapezoidal Actuator

Marziye Noroozi[1], Firooz Bakhtiari-Nejad[2*]

[1]PhD student, Mechanical Engineering Department, Amirkabir University of Technology, Tehran, Iran
[2]Professor, Mechanical Engineering Department, Amirkabir University of Technology, Tehran, Iran

ABSTRACT

The nonlinear vibration behaviors of a cantilevered piezoelectric carbon nanotube-reinforced composite trapezoidal plate as an actuator in micro air vehicles are considered in this article. The assumption of the uniformly distributed single-walled carbon nanotubes along the thickness is taken into consideration. The plate is exposed to subsonic airflow which is modeled by linear potential flow theory and subjected to combined parametric and external excitations. The large deflection von Karman plate assumptions are applied to derive the governing equations of the motion of the laminated trapezoidal plate by using Hamilton's principle. Galerkin's approach combined with proper transformation is formulated and utilized to transform the geometry of the trapezoidal plate into a rectangular computational domain. The nonlinear two-degrees-of-freedom ordinary differential equations with cubic nonlinearities in the case of 1:3 internal resonance and primary resonance are solved by using the multiple scales method. The frequency-response and time-response curves are obtained to analyze the nonlinear dynamic behavior of the plate and study the effects of different parameters such as the amplitudes and frequencies of the excitations and aerodynamic pressure on the nonlinear vibration and dynamic stability of the thin laminated plate. As a result, a complex softening nonlinearity is observed in frequency-response curves.

KEYWORDS

Trapezoidal actuator, combined parametric and external excitations, internal resonance, subsonic airflow, multiple scales method.

* Corresponding Author: Email: baktiari@aut.ac.ir, f_b_nejad@yahoo.com

1. Introduction

In the past decade, the high-performance piezoelectric nanocomposite laminated plates [1] have been applied to micro air vehicles. These plates can undergo large deflections which lead to nonlinear oscillations. Due to the existence of the modal interactions in high-dimensional nonlinear dynamic systems, several types of internal resonances are created which can lead to various forms of the nonlinear vibrations. Hence when a special internal resonant relationship between two linear natural frequencies is created, the large amplitude nonlinear responses may suddenly happen and cause serious damages to the piezoelectric nanocomposite laminated structures. So, research works on the nonlinear mechanical behavior of nanocomposite laminated piezoelectric plates with different practical shapes are playing a key role in such light-weight structures.

A survey of the literature shows that most of the research on quadrilateral and trapezoidal plates was devoted to bending, and free vibration analysis and studies are concerned about the dynamic response and nonlinear characteristics of trapezoidal plates are so limited. For the first time, Tian et al. [2] in 2019 used Rayleigh-Ritz approach combined with the affine transformation to investigate the nonlinear vibration characteristics of trapezoidal plates under a transverse harmonic excitation in the case of internal and primary resonances. This article attempts to fill the void by providing a geometrical nonlinear vibration and dynamic behaviors of a cantilever single-walled carbon nanotube-reinforced composite bonded with two piezoelectric surfaces trapezoidal plate subjected to the electrical and the transverse mechanical loads in subsonic airflow which have not conducted yet. The case of one-to-three internal resonance and primary resonance is considered and the method of multiple scales is utilized to solve the coupled nonlinear equations having cubic nonlinearities.

2. Problem definition

A cantilever laminated piezoelectric CNTRC trapezoidal plate subjected to the aerodynamic load ΔP and transverse load $Q_Z(t)$ is shown in Figure 1. It is assumed that the piezoelectric layers are very thin compared to the dimensions of the plate and the applied voltage $V_E(t) = V_D + V_A \cos(\Omega_1 t)$ is much larger than self-induced voltage, which is derived from the direct piezoelectric effect theory in the actuators and is simultaneously applied to both upper and lower piezoelectric layers.

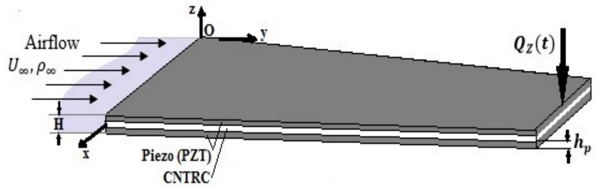

Figure 1. The cantilever laminated CNTRC trapezoidal plate with two surface-bonded piezoelectric actuator layers subjected to a subsonic airflow and external transverse load.

Governing equations are derived using the classical laminated plate theory and von Karman type equations for the geometric nonlinearity with Hamilton’s principle. The trapezoidal plate geometry is mapped into a rectangular computational domain. The nonlinear partial differential equations of motion for the cantilever laminated trapezoidal plate are transformed into a two-degree-of-freedom nonlinear system under the electromechanical loads and the aerodynamic pressure by using Galerkin’s approach. The method of multiple scales is employed to solve the coupled nonlinear equations having cubic nonlinearities in the case of one-to-three internal resonance and primary resonance.

3. Results and Discussion

The influence of the amplitude of external transverse excitation on frequency-response curves of the trapezoidal plate obtained by considering two-mode analysis in the presence of aerodynamic load and parametric excitation is depicted in Figure 2. It can be seen at the same excitation to linear frequency ratio the higher amplitude of external excitation leads to a higher amplitude of vibration. Besides, it can be observed that the nonlinearity increases by increasing the amplitude of external transverse excitation. Also, it can be noticed in Figure 2(b) that there is a complex softening nonlinearity with two peaks in higher coupled mode. This type of nonlinearity happens due to the presence of the 1:3 internal resonance between two modes which resulting in the exchange of energy among two different modes.

The variation of the amplitudes of the responses as a function of the amplitude of the external excitation for several frequency ratios in the presence of aerodynamic load, parametric and external excitations is shown in Figure 3(a, b). It is clear that depending on the value of frequency ratio, some curved are single-valued while others are multivalued which leads to jumping phenomena. In addition, the presence of internal resonance and complex nonlinearity causes the different behavior of the curves in Figure 3(a) and Figure 3(b).

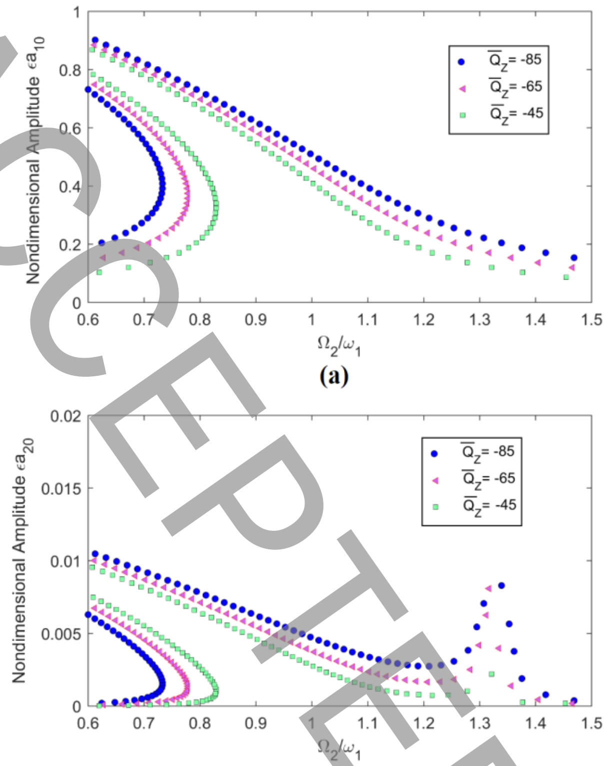

Figure 2. Effect of amplitude of external excitation on frequency-response curves of the plate in the presence of aerodynamic load, parametric and external excitations, (a) mode (1, 1), (b) mode (2, 2)

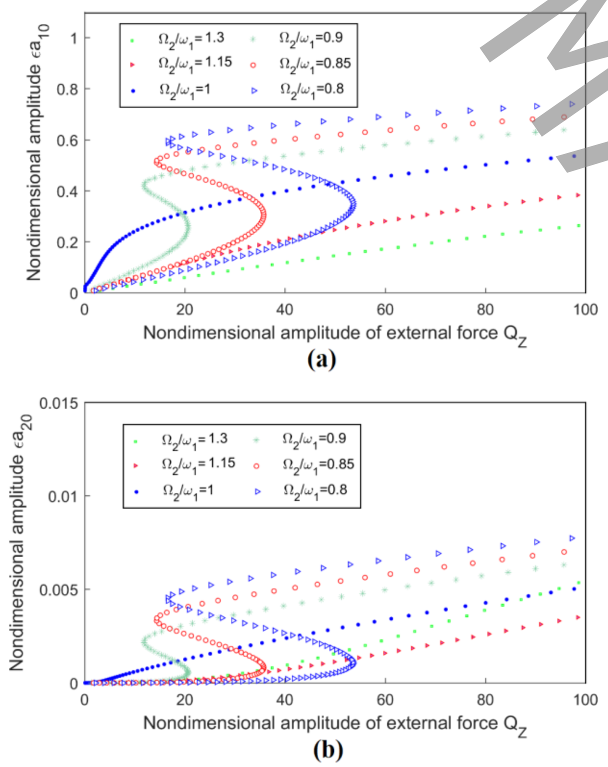

Figure 3. Amplitude of response of (a) mode (1, 1), (b) mode (2, 2), versus amplitude of the external excitation for several frequency ratios in the presence of aerodynamic load, parametric excitation

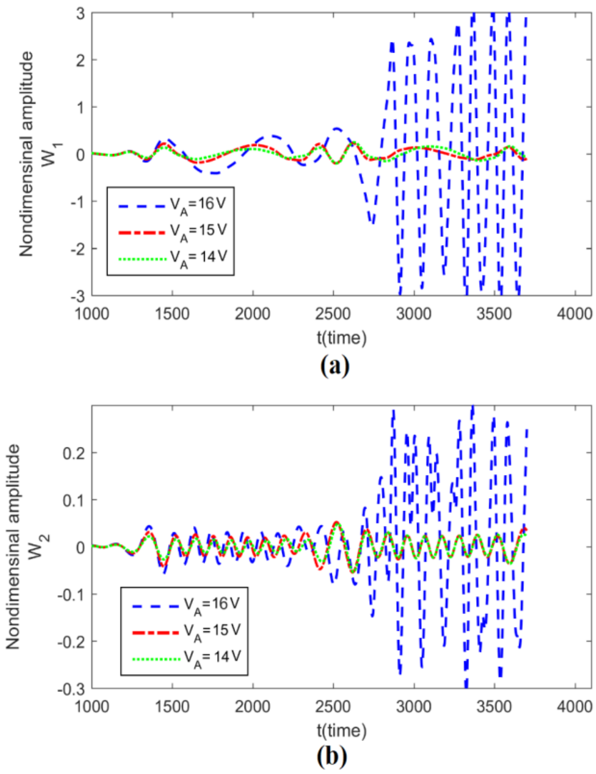

Figure 4. Effect of the amplitude of electrical excitation on time response of (a) mode (1, 1), (b) mode (2, 2).

The influence of amplitude of parametric excitation on the time history responses of the two modes of the trapezoidal plate is studied by using numerical integration. In Figure 4(a, b) when $V_A = 16\ \text{V}$, the amplitude of mode responses grows exponentially and unbounded oscillatory happens so the dynamic instability can be observed.

4. Conclusions

In this paper, the nonlinear theoretical model is used to study the nonlinear vibration of the cantilever laminated piezoelectric CNTRC trapezoidal actuator under combined parametric and external excitations and exposed to the subsonic airflow. The resonance case considered here is 1:3 internal resonance and primary resonance. The following important results were obtained

(1) Internal resonance provides a mechanism of transferring energy between the two modes, so a complex softening nonlinearity with two peaks in the higher mode is observed in frequency-response curves which depend on the airflow velocity and amplitude of external excitation.

(2) The stability analysis of steady-state solutions shows that depending on the value of the detuning parameter, some response curves are multivalued which leads to the jumping phenomena in the resonance region.

(3) The amplitude and frequency of electrical excitation have significant effects on dynamic instability.

5. References

[1] M. Rafiee, X. He, K. Liew, Non-linear dynamic stability of piezoelectric functionally graded carbon nanotube-reinforced composite plates with initial geometric imperfection, International Journal of Non-Linear Mechanics, 59 (2014) 37-51.

[2] W. Tian, Z. Yang, T. Zhao, Analysis of Nonlinear Vibrations and Dynamic Responses in a Trapezoidal Cantilever Plate Using the Rayleigh-Ritz Approach Combined with the Affine Transformation, Mathematical Problems in Engineering, 1 (2019) 1–23.