Quantifying of Viscous Fingering Instability in Porous Media

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ABSTRACT

In this paper, nonlinear simulation of viscous fingering instability of miscible displacement involving nanofluid is investigated. Using vorticity and stream functions and the spectral method governing equation are obtained. Due to the fractality of fluid-fluid interface in instability phenomena, by using box counting method, its fractal dimension is calculated in different parameters such as deposition rate, mobility ratio and diffusion rates. The results show that increasing the deposition rate reduces the complexity of finger patterns and the diffusion rate of nanofluid has no effect on complexity of finger patterns, while increasing the diffusion rate of displaced fluid has significant effect on patterns and make it more complicated. The fractal analysis also shows that the effect of mobility ratio depends on the deposition rate. By considering deposition rate, although the mobility ratio has no effect on fractal dimension and effective time is constant and equal 275, start time of instability is delayed by 25 unit. It can be concluded that fractal analysis of viscous fingering phenomena can be considered as one of the instability characteristic.

KEYWORDS

Instability, Viscous Fingering, Nanoparticle, Fractal Analysis, Nonlinear Simulation

1. Introduction

Viscous fingering instability is a natural phenomenon and take place when a less viscous fluid is injected into a more viscous one leading to the formation of fingerlike patterns affect the sweep efficiency of the miscible displacement process [1]. Examples of these processes are secondary and tertiary oil recovery, fixed bed regeneration in chemical processing, soil remediation and filtration. Due to the complexity in the appearance of fingerlike patterns, fractal analysis can be conducted. Fractal, introduced by Mandelbrot in 1963 [2] is a branch of geometry that explains complex, rough and random shapes and is closely to several important geometrical concepts such as self-similarity, symmetry, periodicity and scale invariance.

Viscous fingering instability is a natural phenomenon and was first introduced by Hill [1] and thereafter many researchers have studied different aspects of instability. Injection of nanofluid in porous media is another aspect that has received very limited attention. Ghesmat et al [3]

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conducted linear analysis of nanoparticles on the dynamics of miscible Hele-Shaw flows and Dastvareh and Azaiez numerically simulated instabilities of nanofluid flow displacements in porous media [4]. Based on literature, quantity measuring parameters are limited to mixing length, contact area and sweep efficiency. In this paper, in order to characterize complexity of fingers and their patterns, fractal analysis of viscous fingering is conducted.

2. Methodology

Fig. 1 shows a horizontal plate with width H, length L used in this paper. It has been assumed that an incompressible fluid with viscosity μ_{a0} and initial concentration C_{a0} is injected from the left hand side along x axis with constant velocity U and is attempted to displace the second fluid with viscosity μ_{b0} and initial concentration C_{b0}. Fluid A contains nanoparticles with the concentration C_{n0} and viscosity μ_{n0}. The equation of motion and governing equations are shown as follows:

$$\nabla.\boldsymbol{u} = 0 \quad (1)$$

$$\nabla p = -\left(\frac{\mu}{K}\right)\boldsymbol{u} \quad (2)$$

$$\frac{DC_a}{Dt} = \frac{\partial C_a}{\partial t} + \boldsymbol{u}.\nabla C_a = \nabla.D_a \nabla C_a \quad (3)$$

$$\frac{DC_b}{Dt} = \frac{\partial C_b}{\partial t} + \boldsymbol{u}.\nabla C_b = \nabla.D_b \nabla C_b \quad (4)$$

$$\frac{DC_n}{Dt} = \frac{\partial C_n}{\partial t} + \boldsymbol{u}.\nabla C_n = \nabla.D_c \nabla C_n - k_{dep} C_n \quad (5)$$

In this paper, we follow the numerical scheme described in [4]. The equations are transformed in Hartley space using the Hartley transform. A random noise of very small magnitude in the initial condition is added to the concentration at the interface in the y direction, causes instability to start and fingers to grow.

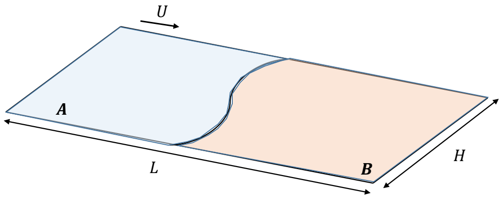

Figure 1. Schematic geometry of the problem

Fractal dimension describes the irregular or fragmented shape of complex objects. In order to use fractal analysis, the concentration contours should all be converted to fluid interface. The binary images are analyzed by implementation of the box-counting method, one of the most widely used fractal dimensions.

3. Discussion and Result

In order to validate the numerical simulation, mixing length is compared with [5]. Figure 2 demonstrates the variation of mixing length with time and the result shows good agreement.

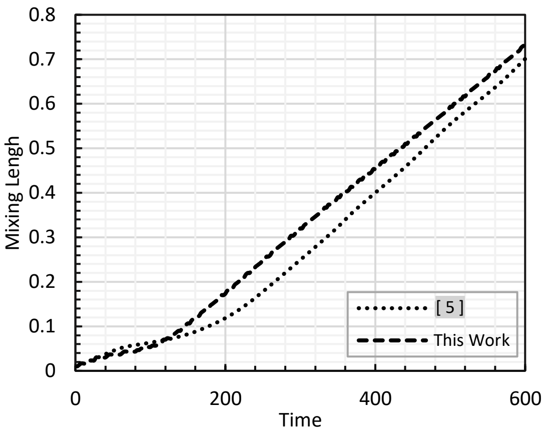

Figure 2. Mixing length for $A_r = 2$, $\delta_b = 1$, $\delta_n = 1$, $R_n = 1$, $R_b = 6$, $R_a = 2$, $Da_{dep} = 0.01$ in comparison with [5]

Concentration contour of displacement fluid is superposed with variation of fractal dimension with time and are plotted in figure 3. It can be concluded from the figure that the fractal dimension of the image is affected by the shape and growth of the fingers.

Figure 4 illustrated variation of fractal dimension with time for different deposition rate. It is clear that the fractal dimension increases and effective time range decreases as deposition rate is increased. It can be concluded that presence of nanoparticle deposition leads to simpler finger patterns.

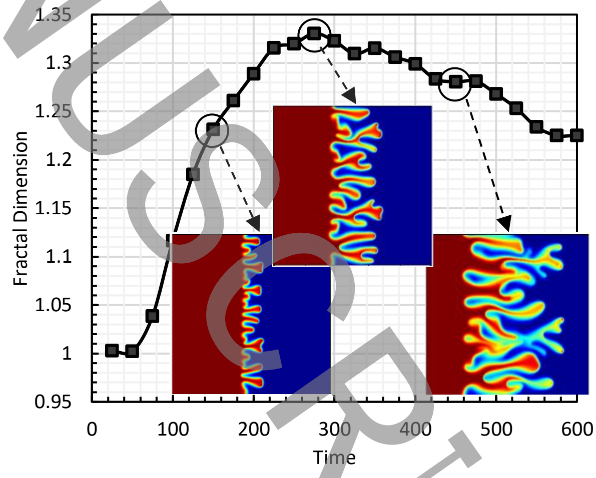

Figure 3. Variation of fractal dimension with time

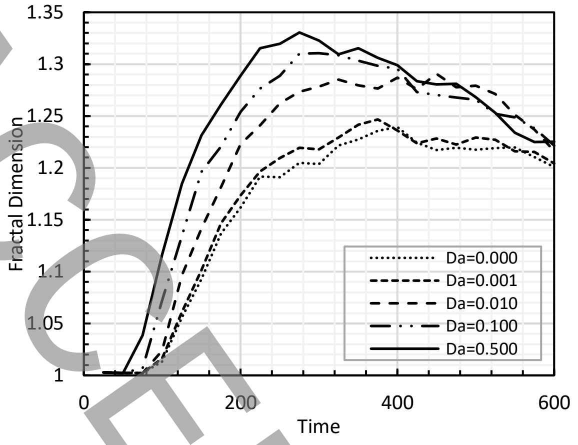

Figure 4. Fractal dimension for $R_n = 1$, $\delta_b = 1$ and $\delta_n = 1$

Figure 5 shows the effect of nanofluids viscosity ratio. According to this figure, the increase of R_n has no significant on fractal dimension and the effective time range remain constant. In other words, the decrease of nanofluids viscosity ratio only cause finger patterns grows at earlier times. Further analysis for special case $Da_{dep} = 0$ shows that fractal dimension decreases as R_n is increased. It means that the presence of nanoparticle deposition cause the effect of R_n decrease with time.

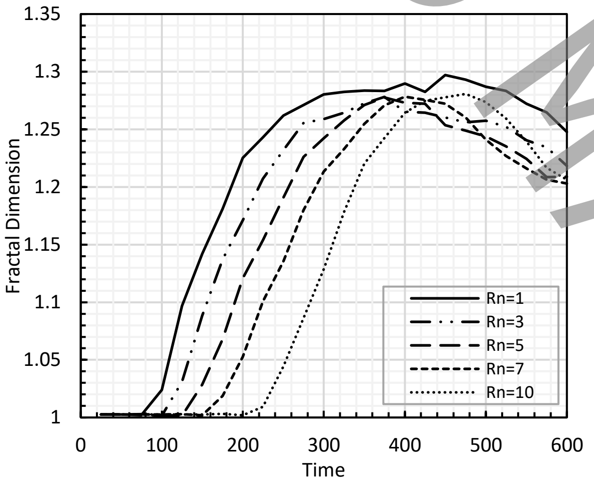

Figure 5. Fractal dimension for $\delta_b = 1$, $\delta_n = 1$ and $Da_{dep} = 0.01$

Figure 6 depicts the variation of fractal dimension with time for different nanoparticle diffusion rate. It can be seen δ_n has slight effect on fractal dimension.

4. Conclusion

In this study, nonlinear simulation of the viscous fingering instability of a nanofluid displacement through a homogenous porous medium is conducted.

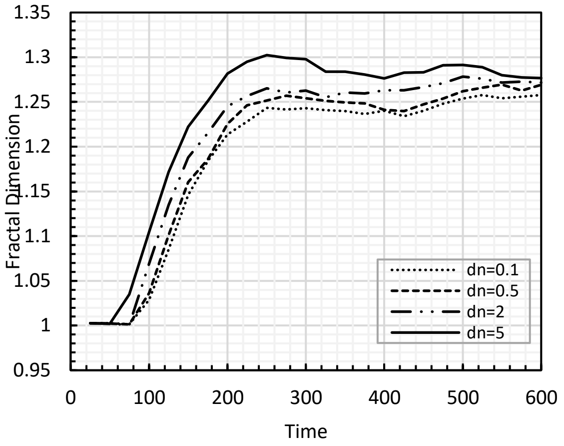

Figure 6. Fractal dimension for $\delta_b = 1$, $R_n = 1$ and $Da_{dep} = 0.01$

The study has focused on the fractal analysis of the mentioned instability. In addition, effect of different parameters on fractal dimension were investigated. The results show that by an increment in value of the deposition rate, the fractal dimension and also effective time range increases. The variation of nanofluids viscosity ratio, in presence of nanoparticle deposition, has no effect on fractal dimension. It also should be noted that the growth of fingers are delayed 25 units of time as nanofluid viscosity ratio increases 2 units. In the absence of nanoparticle deposition, fractal dimension decreases as R_n is increased. Also fractal analysis of viscous fingering instability in different values of nanoparticle and displaced fluid diffusion rate shows that although δ_n has slight effect on fractal dimension, δ_b has significant effects. Increases in δ_b cause a decrease in fractal dimension which means less complicated finger patterns.

5. References

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