

# Control of Quadrotor by Using State Dependent Riccati Equation Method and Analyzing Its Dynamic Performance under Wind Field

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## ABSTRACT

In recent decades, quadrotors are considered, because of the special missions and reducing the cost of flight operation. In this paper, three flight missions are defined to the quadrotor for shooting the special area. Attitude control of Quadrotor is analyzed on basis of State-Dependent Riccati Equation. In first mission, an experimental sample is taken in order to find the Euler angles for implementation of routes. the sample quadrotor is on basis of proportional-derivative controller, for this purpose, results of simulation base on PD controller is conducted, so far, the results is validated by State-Dependent Riccati Equation controller method. In the second and the third missions, the quadrotor is given maneuver by State-Dependent Riccati Equation method and flies in more complex routes such as square and round to cover more surface. Considering external wind field is the important parameters for the mention missions. The feasibility of these missions related to quadrotor stability and guaranteed security in wind field, for this purpose, the influence of force and moment of wind field is applied to equations of motion of quadrotor. Dynamic performance of quadrotor is investigated for proportional-derivative, Linear Quadratic Regulator and State-Dependent Riccati Equation methods encountering wind field.

## KEYWORDS

Quadrotor, Attitude control, Altitude control, Nonlinear algorithm and wind field.

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## 1. Introduction

UAVs have become more popular in recent decades due to perform impossible missions and reduce operating costs [1]. So far, various linear and nonlinear control methods have been used for better performance. Consideration of external winds, turbulence, disturbances, as well as uncertain dynamic model, has expanded the scope of this research. Sydney et al. Used a nonlinear control method to evaluate the performance of a quadrotor in turbulent wind field [2]. Li et al. investigated multivariable finite time composite control strategy based on immersion and invariance for quadrotor under mismatched disturbances [3]. Tung Wan et al. investigated Quad-rotor Performance under Gust Wind and Heavy Rain Impacts [4]. Since the external wind field is an important factor in quad-rotor dynamic, using an applicable control method is essential to perform appropriate missions. In this study, firstly quadrotor equations of motion are presented, then deferent missions are defined to follow the routes by some control methods, then the external wind equations are added to quadrotor equations in order to evaluate SDRE (State Dependent Riccati Equation), LQR (linear quadratic regulator) algorithms and PD (proportional-derivative) controller.

## 2. Quadrotor Dynamic Performance Under Wind Effect

In this section, the wind field effect is applied to the quadrotor in all directions and its dynamic performance is investigated [5]. To consider the wind effect to quadrotor, the force and moment of wind should be applied to quad-rotor body as an external factor. In this study, discrete gust model has been used as wind model, then adding the wind to quad-rotor equations of motion is explained, eventually the result is analyzed. According to "Figure 1", to apply wind effect to the quadcopter, wind speed should first be converted to dynamic pressure and then to force and moment.

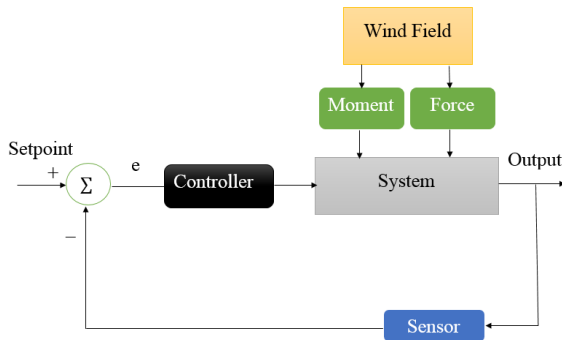


Figure 1. Diagram of the applied wind to the system

The applied force and moment to quadrotor due to wind field are added to main in accordance with Equation (1) and Equation (2):

$$\begin{aligned} \ddot{x} &= (\cos \psi \cos \theta) \frac{F_{w_x}}{m} + \\ &(-\sin \psi \cos \varphi + \cos \psi \sin \theta \sin \varphi) \frac{F_{w_y}}{m} + \\ &(\cos \varphi \sin \theta \cos \psi + \sin \varphi \sin \psi) \frac{1}{m} (u_1 + F_{w_z}) \\ \ddot{y} &= (\sin \psi \cos \theta) \frac{F_{w_x}}{m} + \\ &(-\cos \psi \cos \varphi + \sin \psi \sin \theta \sin \varphi) \frac{F_{w_y}}{m} + \\ &(\cos \varphi \sin \theta \sin \psi - \sin \varphi \cos \psi) \frac{1}{m} (u_1 + F_{w_z}) \\ \ddot{z} &= g - \frac{F_{w_x}}{m} \sin \theta + \frac{F_{w_y}}{m} \cos \theta \sin \varphi + \\ &\frac{1}{m} (u_1 + F_{w_z}) \cos \varphi \cos \theta \end{aligned} \quad (1)$$

$$\begin{aligned} \ddot{\varphi} &= \dot{\theta} \dot{\psi} \frac{I_y - I_z}{I_x} - \frac{J_r}{I_x} \dot{\theta} \dot{\Omega} + \\ &\frac{(F_{w_{z2}} - F_{w_{z4}})}{I_x} l + \frac{l}{I_x} u_2 \\ \ddot{\theta} &= \dot{\varphi} \dot{\psi} \frac{I_z - I_x}{I_y} + \frac{J_r}{I_y} \dot{\varphi} \dot{\Omega} + \\ &\frac{(F_{w_{z1}} - F_{w_{z3}})}{I_y} l + \frac{l}{I_y} u_3 \\ \ddot{\psi} &= \dot{\varphi} \dot{\theta} \frac{I_x - I_y}{I_z} + \frac{(F_{w_{x2}} - F_{w_{x4}})}{I_z} l + \\ &\frac{(F_{w_{y1}} - F_{w_{y3}})}{I_z} l + \frac{l}{I_z} u_4 \end{aligned} \quad (2)$$

$F_{w_x}$ ,  $F_{w_y}$ ,  $F_{w_z}$  are the forces due to wind in x, y and z directions.  $l$  is the quadrotor body's link length from C.G. to rotor.

## 3. Quadrotor Attitude and Altitude Control

In this section, firstly attitude control of the quadrotor is investigated. Then position and altitude of the quadrotor is controlled. SDRE algorithm is used for attitude control of the quad-rotor, then PD controller is used for

x and y position control, finally, in order to control the altitude of the quadrotor, feedback linearization method is selected.

#### 4. Simulation of Quadrotor Under Wind Effect

The external wind is applied to the quadrotor with 45 and 30 degrees relative to the x and z axis respectively. SDRE algorithm is used for attitude control of the quadrotor when encountering wind field. Then the results are compared to LQR algorithm and PD controller in similar condition. Comparison of quadrotor position and altitude for mention control methods are illustrated in “Figure 2”, “Figure 3” and “Figure 4”. In PD controller and LQR algorithm method, the quadrotor deviates from its desired flight path in x and y direction because of the external wind effect, but SDRE algorithm method follow the desired route. SDRE and LQR algorithm provide more performance than PD controller in altitude control.

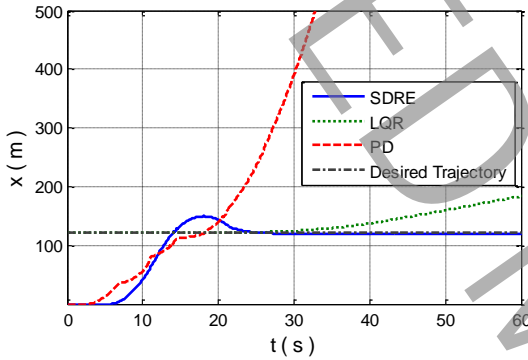


Figure 2. quadrotor position in x direction for three control methods

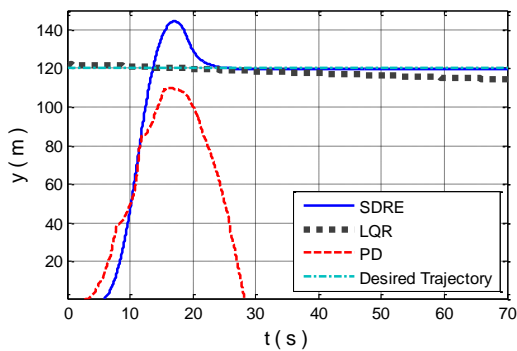


Figure 3. quadrotor position in y direction for three control methods

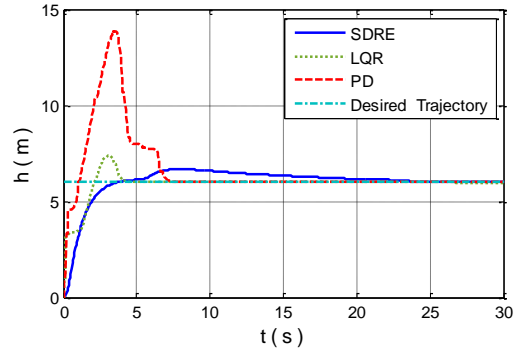


Figure 4. quadrotor altitude in for three control methods

#### 5. Conclusions

In this study, three flight scenarios have been proposed. SDRE algorithm have been implemented to system. The advantage of this method over linear methods is that the quadrotor is controlled to a certain point without operator's intervention to change of Euler angles. it is a closed-loop control system and possible to control when encountering wind field. The quadrotor is stable vertically due to the four rotors, but is weak along the horizontal. So, SDRE algorithm can maintain its stability. In open-loop control system, operators cannot predict the instantaneous forces and moments due to external wind. Therefore, they cannot preserve the quadrotor stability. Another advantage of SDRE algorithm method is complex flight path tracking.

#### 6. References

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