Nonlinear Optimal Control of an Active Transfemoral Prosthesis Using State Dependent Riccati Equation Approach

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ABSTRACT

Nowadays, the scientific and technological advances have created the ability to replace prosthetic legs with amputated limbs, which the design of a suitable controller is still being discussed by researchers. Therefore, according to the importance of the subject, in this paper, a combination of a nonlinear optimal control method based on the state-dependent Riccati equation (SDRE) approach with the integral state control technique is proposed for an active prosthetic leg for transfemoral amputees. The main objective of this paper is to optimize the energy consumption of the robot/prosthesis system and desirable tracking of the vertical displacement in hip and thigh and knee angles. Also, due to the robustness properties of the suggested controller is investigated sensitivity analysis against ±30% parametric uncertainty and compared with a robust adaptive impedance control. The performance of the controller is assessed for both point-to-point motion and tracking modes by considering the saturation bounds of control signals. Finally, the simulation results show a decrease in control effort, desirable performance in tracking and relatively good robustness in the presence of parametric uncertainty and constant disturbance. Numerical results indicate a significant reduction in energy consumption and total cost in this method compared to the robust adaptive impedance control.

KEYWORDS

Tracking, Nonlinear optimal control, Integral state control, Robot/prosthesis system, Saturation bound

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1. Introduction

The innovation of this paper is as follows: using SDRE technique for an active transfemoral prosthesis to optimize energy consumption, which is one of the challenges in designing robotic prostheses. Using integral state control to improve tracking and eliminate constant disturbance in the study of prosthesis desirable performance in the track and repetition of healthy individuals walking.

2. Prosthetic leg

The proposed model for the three-rigid links prosthetic leg with three degrees of freedom is presented as Figure 1.

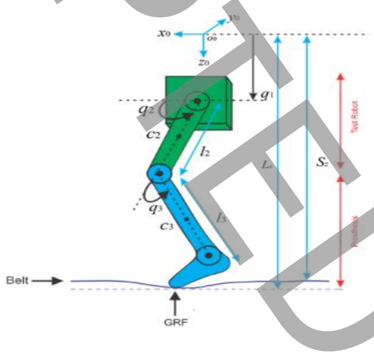

Figure 1. Prosthetic leg model with rigid ankle

The system's state space equation is expressed as (1):

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\\ \dot{x}_3(t)\\ \dot{x}_4(t)\\ \dot{x}_5(t)\\ \dot{x}_6(t)\end{bmatrix}=\begin{bmatrix}x_4(t)\\ x_5(t)\\ x_6(t)\\ [0\quad 0\quad 1]\mathbf{M}^{-1}(.)[\mathbf{u}(t)-\mathbf{T_e}-\mathbf{C_p}\dot{\mathbf{q}}-\mathbf{G_p}-\mathbf{R_p}]\\ [0\quad 0\quad 1]\mathbf{M}^{-1}(.)[\mathbf{u}(t)-\mathbf{T_e}-\mathbf{C_p}\dot{\mathbf{q}}-\mathbf{G_p}-\mathbf{R_p}]\\ [0\quad 0\quad 1]\mathbf{M}^{-1}(.)[\mathbf{u}(t)-\mathbf{T_e}-\mathbf{C_p}\dot{\mathbf{q}}-\mathbf{G_p}-\mathbf{R_p}]\end{bmatrix} \tag{1}$$

Additional information is provided in [1].

3. SDRE controller

Consider the state-dependent parameterized system with the following state space representation:

$$\begin{cases}\dot{\mathbf{x}}(t)=\mathbf{A}(\mathbf{x}(t))\,\mathbf{x}(t)+\mathbf{B}(\mathbf{x}(t))\,\mathbf{u}(t)\\ \mathbf{y}(t)=\mathbf{C}(\mathbf{x}(t))\,\mathbf{x}(t)\end{cases} \tag{2}$$

The performance index J_0 should be minimized in order to design optimal system control as equation (3):

$$J_0=\frac{1}{2}\int_0^\infty\left\{\mathbf{x}^T(t)\,\mathbf{C}^T\,\mathbf{Q}^{1/2}\mathbf{C}\,x(t)+\mathbf{u}^T(t)\mathbf{R}\,\mathbf{u}(t)\right\}dt \tag{3}$$

In this paper, the state-dependent coefficient matrices are selected as proposed structure in [2]:

$$\mathbf{A}(\mathbf{x}(t))_{6\times6}=\begin{bmatrix}\mathbf{0}_{3\times3} & \mathbf{I}_{3\times3}\\ \mathbf{0}_{3\times3} & -\mathbf{M}_{3\times3}^{-1}(\mathbf{q}(t))\,\mathbf{C}_{3\times3}(\mathbf{q}(t),\dot{\mathbf{q}}(t))\end{bmatrix} \tag{4}$$

$$\mathbf{B}(\mathbf{x}(t))_{3\times6}=\begin{bmatrix}\mathbf{0}_{3\times3}\\ -\mathbf{M}^{-1}(\mathbf{q}(t))_{3\times3}\end{bmatrix} \tag{5}$$

According to the integral state control, the state-space equation of the augmented system is given by (6):

$$\begin{cases}\dot{\mathbf{x}}_a(t)=\overbrace{\begin{bmatrix}\mathbf{A}(\mathbf{x}(t)) & \mathbf{0}_{6\times3}\\ -\mathbf{C} & \mathbf{0}_{3\times6}\end{bmatrix}_{9\times9}}^{\mathbf{A}_a}\mathbf{x}_a(t)+\overbrace{\begin{bmatrix}\mathbf{B}(\mathbf{x}(t))\\ \mathbf{0}_{3\times3}\end{bmatrix}_{9\times3}}^{\mathbf{B}_a}\mathbf{u}(t)_{3\times1}+\begin{bmatrix}\mathbf{0} & \mathbf{I}\end{bmatrix}^T_{9\times9}\mathbf{r}(t)_{9\times1}\\ \mathbf{y}_a(t)=\underbrace{[\mathbf{C}\quad \mathbf{0}_{3\times6}]_{3\times9}}_{\mathbf{C}_a}\mathbf{x}_a(t)\end{cases} \tag{6}$$

Therefore, by applying the SDRE method for (6) the control law is obtained as follows:

$$\mathbf{u}_{(SDRE+CI)}(t)=-\mathbf{R}^{-1}\mathbf{B}_a^{\;T}(\mathbf{x}(t))\,\mathbf{K}(\mathbf{x}_a(t))\,\mathbf{x}_a(t) \tag{7}$$

where $\mathbf{K}(\mathbf{x}(t))$ is obtained by solving (8):

$$\begin{aligned}&\mathbf{A}_a^{\;T}(\mathbf{x}_a(t))\,\mathbf{K}(\mathbf{x}_a(t))+\mathbf{K}(\mathbf{x}_a(t))\mathbf{A}_a(\mathbf{x}_a(t))\\ &-\mathbf{K}(\mathbf{x}_a(t))\mathbf{B}_a(\mathbf{x}_a(t))\;\mathbf{R}^{-1}\mathbf{B}_a^{\;T}(\mathbf{x}_a(t))\,\mathbf{K}(\mathbf{x}_a(t))+\mathbf{C}_a^{\;T}\mathbf{Q}_a^{\;1/2}\mathbf{C}_a=0\end{aligned} \tag{8}$$

4. Result and Discussion

Actuator saturation are assumed for the amplitude of the control signals:

$$sat(u_i(t))=\begin{cases}u_{i,\max}(t) & , \quad if \quad u_i(t)>u_{i,\max}(t)\\ u_i(t) & , \quad if \quad u_{i,\min}(t)<u_i(t)<u_{i,\max}(t)\\ u_{i,\min}(t) & , \quad if \quad u_{i,\min}(t)>u_i(t)\end{cases}\;, i=1,2,3 \tag{9}$$

The permissible bounds for hip displacement force, thigh and knee torque are (-1200 - 1200) N, (-900 - 900) N.m, and (-400 - 400) N.m respectively. Figure 2 show good performance in positions tracking. Examination of the figures show that after an initial transient peak, which is due to the difference between the initial values of the desired trajectories, tracking in the nominal mode and in presence of uncertainty is similar, this indicates that the proposed controller robust performance is satisfactory. As seen in Figure 3, at the moment of starting due to the difference between the initial conditions and the desired trajectories, the control signals have reached their saturation values, which is not very effective in this analysis because after almost 0.2s as the error disappears, the values are reduced and remain within the appropriate range. On the other hand, over time, the range of control signals in the nominal mode and in the presence of uncertainties remains almost constant, which indicates the robust performance of the proposed controller in this case. Numerical indicators in Table 1 have also shown a significant reduction in energy consumption and total cost in this method, proper tracking and good robustness compared to [3].

Table 1. $Cost_E$, $Cost_U$, **and Cost**

Controller	Nominal	-30% uncertainty	+30% uncertainty
SDRE + Integral state control	$Cost_E = 1.19$ $Cost_U = 0.29$ $Cost = 1.47$	$Cost_E = 1.19$ $Cost_U = 0.22$ $Cost = 1.40$	$Cost_E = 1.25$ $Cost_U = 0.43$ $Cost = 1.67$

5. Conclusion

Examination of the results showed a significant reduction in energy consumption, desirable tracking of positions according to the data from Motion Studied Laboratory of the Cleveland State University and appropriate robustness to parametric uncertainty.

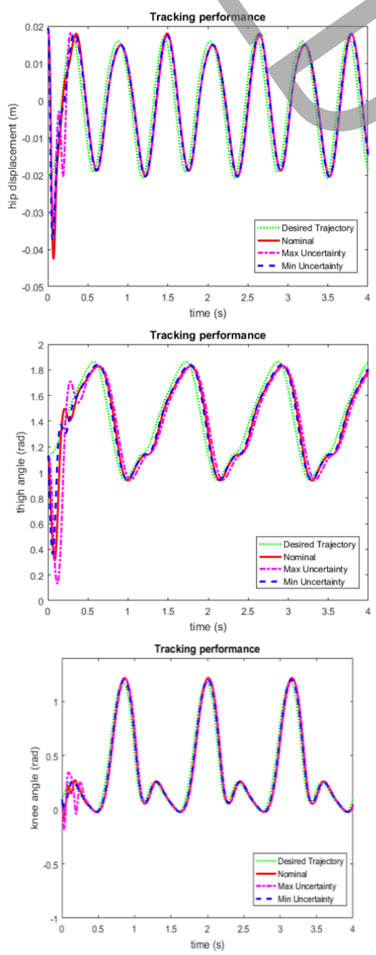

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Figure 2 - Tracking performance in nominal mode and with ±30% uncertainty in presence of the saturation bound

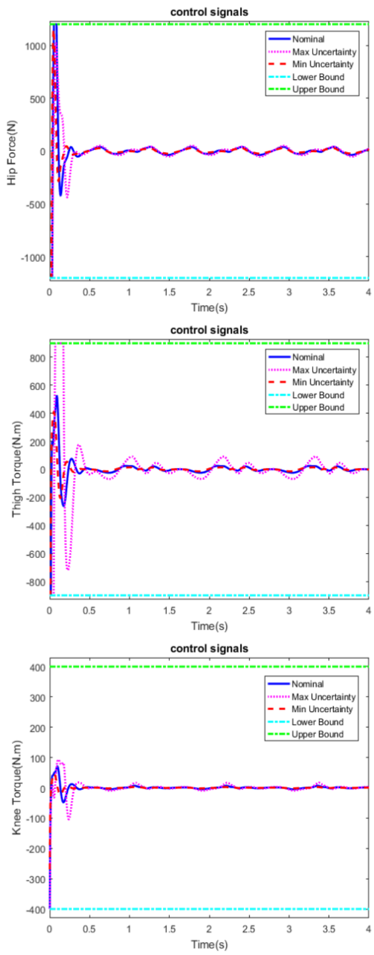

Figure 3- Control signals in nominal mode and with ±30% uncertainty in presence of the saturation bound

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