Numerical simulation of mixed convection of Bingham fluid between two coaxial cylinders

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ABSTRACT

In this paper, mixed convection of Bingham fluid between two coaxial cylinders has been studied numerically without using any regularization method. The temperature of the inner rotating cylinder is higher than the temperature of the outer stationary cylinder. The finite volume method and non-iterative PISO algorithm have been employed to solve the problem. One of OpenFOAM solver, icoFoam, has been modified for solving the exact Bingham model. After validating the modified solver, it has been used to solve the problem for the following ranges of conditions: Reynolds number, *Re*=10, Prandtl number, *Pr*=10, Grashof number, *Gr*=500, Bingham number, 0≤*Bn*≤1000, and aspect ratio (*AR*) of 0.1. The effects of the Bingham number on flow and heat transfer characteristics such as the shape and size of the unyielded regions, streamline contours, the local and mean Nusselt number, and the torque coefficient have been investigated. The mean Nusselt number and the torque coefficient decreases and increases, respectively, when the Bingham number increases. The variation range of the local Nusselt number and dimensionless tangential stress on the inner wall decrease with the Bingham number.

KEYWORDS

Numerical study, Heat transfer, Mixed convection, Bingham fluid, Two coaxial cylinders.

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1. Introduction

The Bingham fluid is a non-Newtonian fluid that behaves like a solid when the applied stress is less than a certain value, τ_y, and yields when the stress exceeds τ_y; After yielding, the relation between stress and strain is linear. Because of this behavior, the constitutive equation of a Bingham fluid is discontinuous. The main issue in the numerical simulation of a Bingham flow is the discontinuity of the constitutive equation. The regularization method is the most straightforward way to solve the issue. The discontinuous constitutive equation is replaced by a smooth equation in the regularization methods. The troublesome in convergence, the inaccuracy in the distinguishing of unyielded regions, and the high errors at large Bingham numbers are some of the weaknesses of the regularization methods [1-3]. Employing variational inequalities theory [4] is a way to study Bingham flow without any regularization. So far, almost all numerical simulations of the exact Bingham flow have been done by in-house code and for simple geometries. The first aim of the article is the creation of a solver that solves the flow and heat transfer of a Bingham fluid with the exact model regardless of geometric complexities. For this purpose, one of the OpenFOAM solvers has been modified.

After validation of the solver with [5], it has been employed to study mixed convection of Bingham fluid between two coaxial cylinders. The effect of Bingham number on the flow and heat transfer characteristics such as the size and shape of the unyielded regions, the streamlines contours, the velocity variations, the local and mean Nusselt numbers, and the torque coefficient have been investigated for the following conditions: Re=10, Pr=10, Gr=500, AR=0.1 and $0 \le Bn \le 1000$.

2. Problem statement and dimensionless governing equations

The space between two coaxial cylinders is filled with the Bingham fluid, as shown in Fig. 1. The hotter inner cylinder rotates with constant velocity, and the outer wall is at rest. The dimensionless form of the governing equations and the boundary conditions are written as follows:

$$\nabla . \mathbf{V}^* = 0 \quad (1)$$

$$\frac{D^*\mathbf{V}^*}{Dt^*} = -\nabla p^*_d + \frac{1}{Re}\nabla^* . \mathbf{A}^*(\mathbf{V}^*) + \frac{Bn}{Re}\nabla^* . \mathbf{\Lambda} - RiT^* \quad (2)$$

$$\mathbf{\Lambda} = P_M\left(\mathbf{\Lambda} + PrBn\mathbf{A}^*(\mathbf{V}^*)\right) \quad (3)$$

$$\frac{D^*T^*}{Dt^*} = \frac{1}{RePr}\nabla^{*2}T^* \quad (4)$$

$$\begin{array}{ll} V^*_\theta(r^*=0) = 1 & V^*_\theta(r^*=1) = 0 \\ V^*_r(r^*=0) = 0 & V^*_r(r^*=1) = 0 \\ T^*(r^*=0) = 1 & T^*(r^*=1) = 0 \end{array} \quad (5)$$

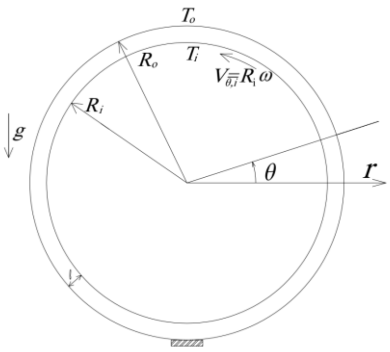

Figure. 1. The schematic diagram of the physical model.

3. Results and Discussion

Fig. 2 shows the distribution of the unyielded regions for different Bingham number. There are two kinds of unyielded regions: the dead zones in which the fluid is at rest, and the plug zones in which the velocity gradient is zero. At Bn=1, the small unyielded regions scatter throughout the domain. The regions merge and become larger as the Bingham increases. At Bn=1000, the buoyancy effect is negligible, and the dead area attached to the outer wall has a constant thickness.

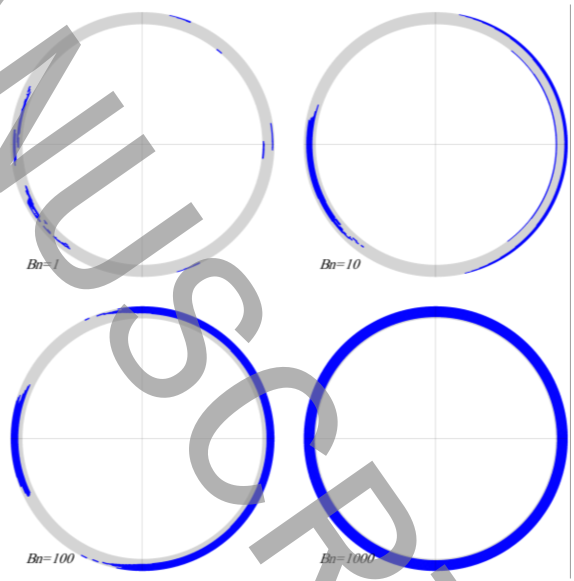

Figure. 2. The unyielded regions for *Gr*=500, *Re*=10, *Pr*=10, and different Values of Bingham number.

The dimensionless tangential velocity along θ=180° is plotted in Fig. 3. At *Bn*=0, the velocity magnitude first decreases then increases (due to buoyancy) and again decreases to reach zero as one moves from the outer to the inner wall. At the low Bingham numbers, the plug zones are formed in the middle of the annuls and velocity gradient approaches to zero. Fig. 3 is compatible with Fig. 4. For example, the plug zone on the left side of the annulus at *Bn*=100 does not stick to the outer wall and a thin layer of fluid yields in this region. The region is evident in Fig.3, where the velocity gradient of *Bn*=100 is not zero for the small interval near r^*=1.

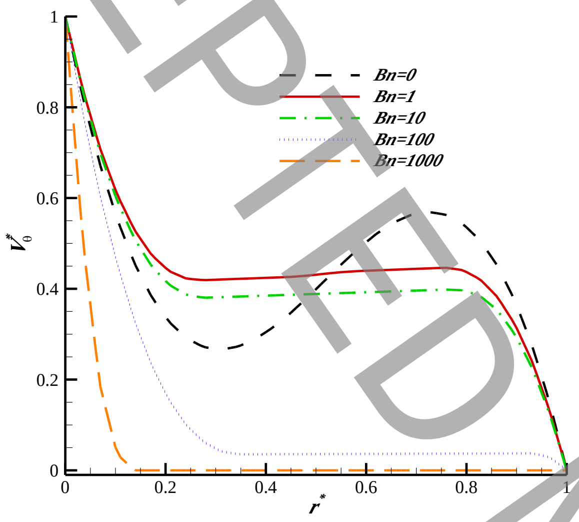

Figure. 3. The variation of dimensionless tangential velocity at θ=180° for *Gr*=500, *Re*=10, *Pr*=10, and different Bingham numbers.

The distribution of the local Nusselt Number on the inner wall is shown in Fig.4 for different Bingham number. The warm fluid moves upward due to the buoyancy force, so the minimum value of the temperature gradient or Nuesselt number occurs at the top of the annulus. Vice versa, the maximum value of the Nusselt number occurs at the bottom. As the Bingham number increases and the unyielded regions are formed, heat-transfer occurs more uniformly, and the variation of the Nusselt number decreases. The mean Nusselt number decreases with the Bingham number.

The distribution of dimensionless tangential stress, $\tau_{r\theta}^*$, on the inner wall is plotted in Fig. 5 for different Bingham number. The maxim $\tau_{r\theta}^*$ occurs at θ=180° due to the exitance of opposite buoyancy. By comparing Figs 5 and 2, one can figure out that the zig-zagging behavior of $\tau_{r\theta}^*$ at *Bn*=100 is due to the formation of the unyielded layer there. As the Bigham number increases, *Bn*=100 and 1000, the direction of $\tau_{r\theta}^*$ does not change, and its variation decreases.

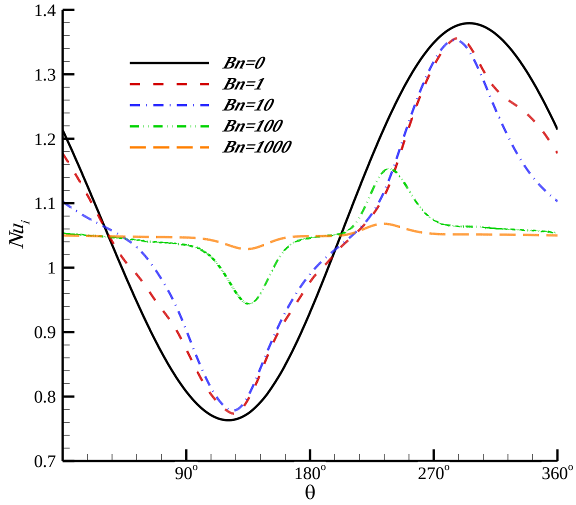

Figure 4. The variation of the local Nusselt number on the inner wall for *Gr*=500, *Re*=10, *Pr*=10, and different values of Bingham number.

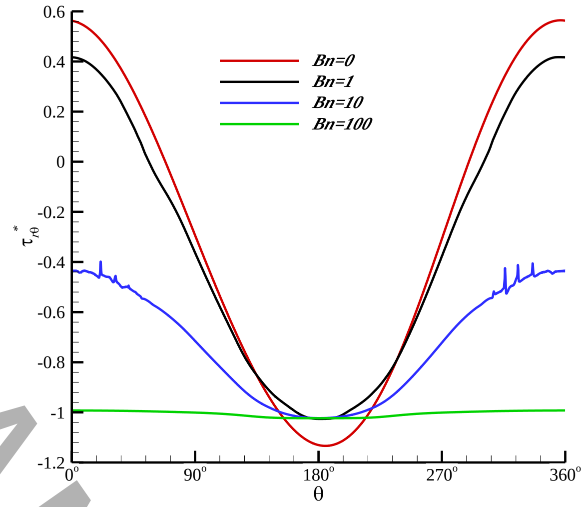

Figure 5. The distribution of the dimensionless shear stress on the inner wall for *Gr*=500, *Re*=10, *Pr*=10, and different values of Bingham number

4. Conclusion

In this article, a solver, which solves the flow and heat transfer of Bingham fluid with the exact model, has been obtained. As a numerical experiment, mixed convection of Bingham fluid between two coaxial cylinders has been studied. The flow and heat transfer characteristics have been presented and discussed.

5. References

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