

# Dynamic Analysis of cylindrical sandwich shell with orthogonal stiffeners using high-order theory

R. Azar Afza<sup>1</sup>, K. MalekzadehFard<sup>2</sup>, M. Golaghapour Kami<sup>3</sup>, A. R. Pourmoayed<sup>4\*</sup>

<sup>1</sup> Department of Mechanical Engineering, Malek Ashtar University of Technology, Tehran, Iran

<sup>2</sup> Department of Aerospace Engineering, Malek Ashtar University of Technology, Tehran, Iran

<sup>3</sup> Department of Mechanical Engineering, Babol Noshirvani University of Technology, Tehran, Iran

<sup>4</sup> Department of Mechanical Engineering, University of Khatamul-Anbiya Air Defense, Tehran, Iran

## ABSTRACT

In this paper, free and forced vibrations of a cylindrical sandwich shell with orthogonal stiffeners using high-order theory were analyzed. The sandwich structure consists of two orthotropic composite face sheets and a flexible foam core and longitudinal and environmental stiffeners. The face sheets and core are perfectly glued together and there is no relative displacement between them at the interfaces. In order to analyze this shell under simply supported boundary conditions, the face sheets displacements are based on Kant's third-order theory and in the core, the second Frostig's model is used. The Rayleigh-Ritz method to solve free vibration and the assumed modes method to analysis forced vibration were used. In the forced vibrations section, sinusoidal loading is applied uniformly and radially to the shell. The vibrations of the stiffened sandwich shell are simulated with Abaqus software. Finally, the effect of various parameters as length to radius ratio, thickness to radius ratio, core thickness to total thickness, structure and material of face sheets and core on the vibration response of the structure has been investigated. To validate, the results of free and forced vibrations obtained from the present analysis were compared with the results of other references and Abaqus software.

## KEYWORDS

Free vibrations, forced vibrations, sandwich shell, orthogonal stiffeners, high-order theory

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\* Corresponding Author: Email: pourmoayed@mut.ac.ir

## 1. Introduction

The use of sandwich structures has been increasing in various industries such as aerospace, road transport, marine and construction structures in the last three decades. The main reason for this is the outstanding features of sandwich panels such as high strength to weight ratio, excellent corrosion and fatigue resistance, the possibility of using composite face sheets and core with directional properties and good performance at high temperatures. However, one of the ways to further increase the strength to weight ratio of these structures is to use longitudinal and environmental stiffeners. Among these, one of the most widely used sandwich structures is cylindrical panels with thin and thick composite face sheets that in the hulls of many industrial structures such as airplanes, missiles, shuttles, submarines, etc are used. A sandwich structure, whether beam, sheet or shell, is consist of two face sheets and a flexible soft core. Malekzadeh et al. [1] using first-order shear theory in the analysis of flexible core sandwich panels showed when the face sheets are thin, relatively good results were calculated. The buckling analysis of sandwich plates using first-order shear theory by Frostig are investigated [2]. Salimi et al. [3] vibration analysis of composite cylindrical shell reinforced with circumferential rib were studied.

According to the researches, it has been observed that so far no research has been done on the issue of free and forced vibrations of a cylindrical sandwich shell with a flexible core and orthogonal stiffeners with thick face sheets and all stress components. In this paper, for the face sheets improved high-order theory and for the core the second Frostig's model are used. The top and bottom face sheets are thick, and displacement field such as third order polynomials and all stress components were considered. Furthermore, the term  $(1 + z / R)$  is considered in the relationships related to the core stress resultants, which is a very important term in the analysis of cylindrical sandwich panels. Also, in this research the transient dynamic response of the stiffened composite sandwich cylindrical shell with simply support boundary conditions under the sinusoidal load applied to the shell is investigated.

## 2. Methodology

The Riley-Ritz method is used to solve the free vibration problem. This method is based on the minimum potential energy. According to the principle of minimum potential energy, From all the displacement fields that satisfy the boundary conditions of the problem, that the field in equilibrium equations is truth, potential energy of the entire system also are minimized. According to the Rayleigh-Ritz method, in

order that potential energy, which is a function of the coefficients  $\{d\}$ , to be minimal, the derivatives of the total energy potential respect to the coefficients used in the displacement field must be zero. Therefore, derivative is taken from the energy potential function of the whole system respect to the mentioned coefficients and equate it to zero.

To investigate the dynamic response, the results of the analysis of free vibrations (ie, natural frequencies and shape of modes) were used. Finally, the changes in shell displacement in the three directions  $x$ ,  $y$  and  $z$  in terms of time, using the Mathematica software based on the Assumed modes method were calculated.

Given that conventional theories are not able to predict the effects of precise shear deformations along the thickness of the face sheets and the core, it is necessary to provide a high-order analytical model that can take into account the effects of core flexibility. For this purpose, in this article, the higher order model of the sandwich panel has been used. The displacements fields in face sheets,  $u$ ,  $v$ ,  $w$  in the direction of  $x$ ,  $\theta$  and  $z$ , respectively are explained as follows:

$$\begin{aligned} u_i(x, \theta, z, t) &= u_0^i(x, \theta, t) + z_i u_1^i(x, \theta, t) + z_i^2 u_2^i(x, \theta, t) + z_i^3 u_3^i(x, \theta, t) \\ v_i(x, \theta, z, t) &= v_0^i(x, \theta, t) + z_i v_1^i(x, \theta, t) + z_i^2 v_2^i(x, \theta, t) + z_i^3 v_3^i(x, \theta, t) \\ w_i(x, \theta, z, t) &= w_0^i(x, \theta, t) + z_i w_1^i(x, \theta, t) + z_i^2 w_2^i(x, \theta, t) \end{aligned} \quad (1)$$

Based on the second Frostig's model, the displacement field components of the core layer are derived as:

$$\begin{aligned} u_c(x, \theta, z, t) &= u_0^c(x, \theta, t) + z_c u_1^c(x, \theta, t) + z_c^2 u_2^c(x, \theta, t) + z_c^3 u_3^c(x, \theta, t) \\ v_c(x, \theta, z, t) &= (1 + \frac{z_c}{R}) v_0^c(x, \theta, t) + z_c v_1^c(x, \theta, t) + z_c^2 v_2^c(x, \theta, t) + z_c^3 v_3^c(x, \theta, t) \\ w_c(x, \theta, z, t) &= w_0^c(x, \theta, t) + z_c w_1^c(x, \theta, t) + z_c^2 w_2^c(x, \theta, t) \end{aligned} \quad (2)$$

The displacement field of external Stiffeners assuming to follow the external face sheet with Eqs. (3) are calculated [4].

$$\begin{aligned} u_s &= u_i(x, \theta, 0, t) - z \frac{\partial w_i(x, \theta, 0, t)}{\partial x} \\ v_s &= v_i(x, \theta, 0, t) (1 - \frac{z_s}{R_i}) \frac{z_s}{R_i} \frac{\partial w_i(x, \theta, 0, t)}{\partial \theta} \\ w_s &= w_i(x, \theta, \frac{h_i}{2}, t) \quad , \quad z_s = -\frac{d_s + h_i}{2} \end{aligned} \quad (3)$$

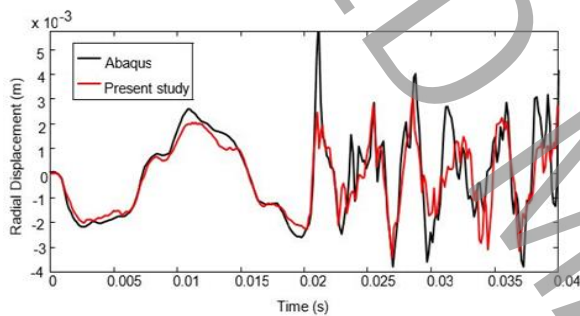
## 3. Results and Discussion

The number of modes ( $m \times n$ ) intended for obtaining the answer, for a sandwich shell with simply support boundary conditions,  $(21 \times 31)$  are considered. The considered numbers  $m$  and  $n$  must be such that we converge to the answer. The excitation duration is assumed to be 0.02 seconds, then the structure enters the free vibrations with certain initial conditions. The

analyzed shell has 6 rings and 8 stringers that are evenly distributed throughout the shell. The loading in this paper, sinusoidally ( $12 \sin(400t)$ ) on the outer surface of the sandwich shell on a rectangular surface from an angle of  $0$  to  $\pi/4$  in the circumferential direction and also from  $8L/9$  to  $L$  along the cylinder is applied. In Table 1, the results of the present analysis with the reference [5] were compared. Maximum error with Ref [5] is about percent 4. Figure 1, the radial displacement resulting from the harmonic loading in the middle of the cylinder for both cases the analytical and abaqus software showed. As can be seen, the results of the analytical code and the Abaqus software are well matched.

**Table 1. Comparison of sandwich shell results with thick face sheets without stiffeners**

m	n	Ref [5]	Peresent Study	Error %
1	1	465.02	483.90	4.06
1	2	375.06	382.71	2.04
1	3	496.95	516.02	3.84
2	3	680.6	671.27	1.37



**Figure 1. Comparison of time response (radial displacement) in the middle of the cylinder**

#### 4. Conclusions

In this study, the response to free and forced vibrations of composite sandwich shell under sinusoidal loading were investigated. Sandwich shell high-order theory was used to derive the relationships. Riley-Ritz method to solve free vibrations and Assumed Modes Method for forced vibrations is used. In order to validate the results, in addition to comparing the results with references, the stiffened shell is modeled in finite element software and the Abaqus results with the analytical results in different cases are compared. The following are some of the important findings of this study:

- With increasing of length to radius ratio ( $L/R$ ), the natural frequencies decreased. Changing  $L/R$  at smaller values of  $n$ , makes a greater difference in natural frequencies.

- Whatever the height-to-width ratio of the stringer ( $d/b$ ) furthered, the natural frequencies of the shell is higher. Of course, this ratio should not be so large that the stringer becomes a thin plate.
- Assuming the thickness of the face sheets is constant, with increasing of the core thickness, the values of the natural frequencies are increased. This process is reasonable due to the increase in the stiffness of the sandwich shell with increasing core thickness.
- By increasing the angle of the fibers in the face sheets from  $0$  to  $90$  degrees, the values of base natural frequencies initially increased to a certain angle value and then decreased. In fact, at angles of  $0$  and  $90$  degrees, we see the lowest values of natural frequencies. This results in forced vibrations under sinusoidal excitation of the amplitudes in the structure are greater when the angle of the fibers in the face sheets is  $0$  or  $90$  degrees.
- With increasing number of longitudinal and environmental stiffeners, the amount of displacement and consequently the amount of stress decreases regularly.
- In response to the forced vibrations of the stiffened sandwich shell under sinusoidal stimulation, the phenomenon of beating is observed. hence, the response amplitude alternately first increased and then decreased.

#### 5. References

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