Nonlinear analysis of hyperelastic plates using first-order shear deformation plate theory and a meshless method

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ABSTRACT

In this paper, the static analysis of hyperelastic plates under uniform and sinusoidal distributed loading is investigated. Right Cauchy-Green deformation tensor and Lagrange strains are used to derive the nonlinear strain relations. Also, the first-order shear deformation plate theory is considered. For the first time, the governing equations of hyperelastic plates using the neo-Hookean strain energy function is derived. The Lagrange equation is utilized to implement the variational method on potential energy function. The governing nonlinear differential equations are discretized using meshless collocation method and radial basis functions. Thin plate spline basis function is applied for deriving shape functions of the meshless method. The results are compared to the results of the finite element method (ABAQUS). The static analysis is investigated on hyperelastic plates for uniform and sinusoidal loading and various thicknesses. Additionally, the effect of thickness is studied on the deflection of the hyperelastic plates. The results show an acceptable accuracy for static analysis of hyperelastic plates under uniform and sinusoidal loading; also, the stress contour is the same in both methods. Consequently, the meshless collocation method using thin-plate spline basis function is an adequate method for analyzing FSDT hyperelastic plates due to no integration and imposing boundary conditions directly.

KEYWORDS

Hyperelastic plates, Neo-Hookean strain energy function, Static analysis, Meshless Method, Radial Basis Functions

INTRODUCTION

Hyperelastic plates are widely used in engineerings such as aerospace industry and pressure tanks. The stress-strain diagram of hyperelastic materials are nonlinear, and for this reason, the behaviour of these materials is represented using specific strain energy functions. Upadhyay et al. [1] represented a new model for the strain rate of visco-hyperelastic materials. Amabili et al. [2] studied nonlinear vibration and static analysis of hyperelastic plates. They used classical plate theory to describe the behaviour of hyperelastic plates. Breslavsky et al. [3] studied nonlinear vibration of thin hyperelastic plates. They considered physically and geometrically nonlinearity for determining the governing equations.

The governing equations of motion for hyperelastic plates are nonlinear, and it is not possible to solve them analytically.

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Consequently, a numerical method is required for analysis of hyperelastic plates. Wen et al. [4] studied Mindlin plates using the meshless method. They considered geometrical nonlinearity to obtain the governing equations. Singh et al. [5] studied laminated composite plates using a meshless method based on radial basis functions. They considered nonlinear terms in strains of the plates.

In this research, bending analysis of hyperelastic plates based on the first order shear deformation plate theory is studied. The neo-Hookean strain energy function is considered to describe the behaviour of hyperelastic plates. Also, physical and geometrical nonlinearity is considered. The meshless collocation method based on radial basis functions is used for discretizing the nonlinear governing equations. The discretized governing equations are solved using the arc-length continuation method.

METHODOLOGY

The right Cauchy-Green tensor on strain components is defined as:

$$\mathbf{C} = 2\mathbf{E} + \mathbf{I} = \begin{bmatrix} 2\varepsilon_{xx}+1 & \varepsilon_{xy} & \varepsilon_{xz} \\ \varepsilon_{xy} & 2\varepsilon_{yy}+1 & \varepsilon_{yz} \\ \varepsilon_{xz} & \varepsilon_{yz} & 2\varepsilon_{zz}+1 \end{bmatrix} \tag{1}$$

The neo-Hookean strain energy function is:

$$U = C_{10}\left(I_1 - 3\right) = 2C_{10}\left(\varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}\right) \tag{2}$$

where I_1 is the first invariant of the strain tensor. Also, the external work represents:

$$W = q(x,y)w(x,y) \tag{3}$$

Finally, by considering incompressibility and some simplification, the potential energy of hyperelastic plates is defined as:

$$\begin{aligned} \pi &= U - W \\ &= 2C_{10}\left(-8\varepsilon_{xx}^3 - 8\varepsilon_{xx}^2\varepsilon_{yy} - 4\varepsilon_{xx}\varepsilon_{xy}^2 - 2\varepsilon_{xx}\varepsilon_{xz}^2 - 8\varepsilon_{xx}\varepsilon_{yy}^2 \right. \\ &\quad -4\varepsilon_{xy}^2\varepsilon_{yy} - 2\varepsilon_{xy}\varepsilon_{xz}\varepsilon_{yz} - 8\varepsilon_{yy}^3 - 2\varepsilon_{yy}\varepsilon_{yz}^2 + 4\varepsilon_{xx}^2 \\ &\quad \left. +4\varepsilon_{xx}\varepsilon_{yy} + \varepsilon_{xy}^2 + \varepsilon_{xz}^2 + \varepsilon_{yz}^2 + 4\varepsilon_{yy}^2\right) - q.w \end{aligned} \tag{4}$$

By omitting in-plane displacements, the nonlinear strain relations are:

$$\begin{aligned} \varepsilon_{xx} &= z\,\varphi_{x,x} + \frac{1}{2}w_{,x}^2 \\ \varepsilon_{yy} &= z\,\varphi_{y,y} + \frac{1}{2}w_{,y}^2 \\ \varepsilon_{xy} &= z\,(\varphi_{x,y} + \varphi_{y,x}) + w_{,x}w_{,y} \\ \varepsilon_{xz} &= \varphi_x + w_{,x} \\ \varepsilon_{yz} &= \varphi_y + w_{,y} \end{aligned} \tag{5}$$

By substituting equation (5) in (4), the potential energy function is derived in term of the displacements and rotations. The governing equations of hyperelastic plate based on FSDT is derived using Lagrange equations.

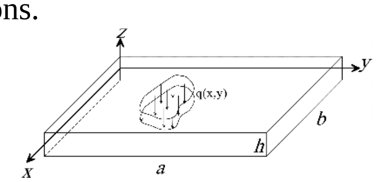

Fig. 1. The rectangular plate under distributed loading in the cartesian coordinate system

The variables of the plates and their derivatives are defined using radial basis functions as:

$$\begin{aligned} &\begin{Bmatrix} w \\ \varphi_x \\ \varphi_y \end{Bmatrix} = \sum_{i=1}^{N} \phi_i \begin{Bmatrix} a_i^w \\ a_i^{\varphi_x} \\ a_i^{\varphi_y} \end{Bmatrix}, \\ &\frac{\partial^k}{\partial X^k}\begin{Bmatrix} w \\ \varphi_x \\ \varphi_y \end{Bmatrix} = \sum_{i=1}^{N} \frac{\partial^k \phi_i}{\partial X^k} \begin{Bmatrix} a_i^w \\ a_i^{\varphi_x} \\ a_i^{\varphi_y} \end{Bmatrix} \end{aligned} \tag{6}$$

where shape functions are defined as:

$$\phi = [R^T(x)S_a + p^T(x)S_b] \tag{7}$$

and:

$$\begin{aligned} R^T(x,y) &= \left[R_1(x,y), R_2(x,y), ..., R_n(x,y)\right] \\ p^T(x,y) &= \left[p_1(x,y), p_2(x,y), ..., p_m(x,y)\right] \\ S_b &= \left[P_m^T R_Q^{-1} P_m\right]^{-1} P_m^T R_Q^{-1} \\ S_a &= R_Q^{-1}\left[1 - P_m S_b\right] \end{aligned}$$

$$\mathrm{R}_Q = \begin{bmatrix} R_1(x_1,y_1) & R_2(x_1,y_1) & \cdots & R_n(x_1,y_1) \\ R_1(x_2,y_2) & R_2(x_2,y_2) & \cdots & R_n(x_2,y_2) \\ \vdots & \vdots & \ddots & \vdots \\ R_1(x_n,y_n) & R_2(x_n,y_n) & \cdots & R_n(x_n,y_n) \end{bmatrix}$$

$$P_m = \begin{bmatrix} P_1(x_1,y_1) & P_2(x_1,y_1) & \cdots & P_m(x_1,y_1) \\ P_1(x_2,y_2) & P_2(x_2,y_2) & \cdots & P_m(x_2,y_2) \\ \vdots & \vdots & \ddots & \vdots \\ P_1(x_n,y_n) & P_2(x_n,y_n) & \cdots & P_m(x_n,y_n) \end{bmatrix} \quad (8)$$

$$p^T = [1, x, y, x^2, xy, y^2, \ldots]$$

Also, the thin-plate spline basis function is defined as:

$$R_i(x,y) = \left((x-x_i)^2 + (y-y_i)^2\right)^{\frac{\eta}{2}} \quad (9)$$

By substituting equation (6) into governing equations, the nonlinear algebraic system of equations is derived as:

$$\left(\mathrm{K_L} + \mathrm{K_{NL}}(\mathrm{a})\right)\mathrm{a} = \mathrm{F} \quad (10)$$

Where $\mathrm{K_L}$ and $\mathrm{K_{NL}}$ represent linear and nonlinear stiffness matrices. The nonlinear system of equations is solved using the arc-length continuation method.

Results and Discussion

The square hyperelastic plate made of silicon-rubber (C_{10}=207843.36 Pa) with unit length under distributed loading is investigated in this section. The results of the meshless method are compared to those of finite element Abaqus software. The load-deflection diagram of a square hyperelastic plate with clamped edges is shown in Fig. 2. Also, geometrical nonlinearity and a combination of geometrical and physical nonlinearity is considered in this figure.

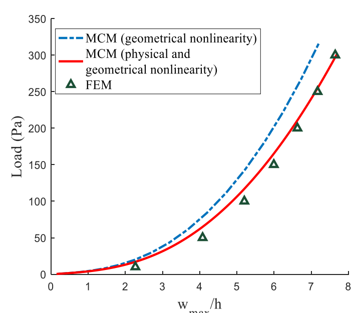

Fig. 2. Load-deflection diagram of a square hyperelastic plate with clamped edges using the meshless method and finite element method

The contour of stress along x-direction for a square plate with clamped boundary condition under uniformly distributed loading is shown in Fig. 3.

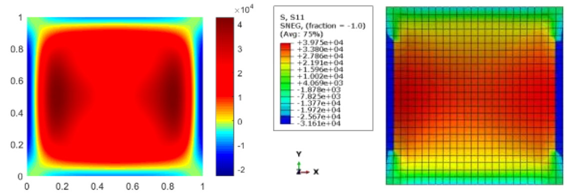

Fig. 3. The contour of stress along x-direction for a square plate with clamped edges and *h/a=0.01* under distributed loading *q=300*Pa using the meshless method (left) and finite element method (right)

Conclusion

In this research, the governing equations of hyperelastic plates investigated using meshless collocation method for the first time. The results are shown that the meshless collocation method is accurate in comparison to the finite element method. Also, clamped and simply supported boundary conditions are considered, and the results show that the meshless collocation method is stable with various boundary conditions.

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