Stability and bifurcation of a rotating blade with varying speed

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ABSTRACT

In this paper, the nonlinear vibration of a rotating blade with varying rotating speed is investigated. The rotating blade is considered as a rotating cantilever Euler-Bernoulli beam without geometric nonlinearity. The angular velocity is assumed as a constant value which is fluctuated with small amplitude. The nonlinear partial differential equations of the rotating cantilevered beam are derived in three-dimensional using the Hamilton's principle. Then, the Galerkin discretization method is applied to the nonlinear partial differential equations to obtain three nonlinear ordinary differential equations. The method of multiple scales is utilized to derive six first order ordinary differential equations to describe the time variation of amplitudes and phases of interacting modes. The stability and bifurcation of fixed points are obtained by using the eigenvalues of the Jacobian matrix of the modulation equations. Numerical results demonstrated that near the primary resonance and internal resonance the fixed points lose the stability through the saddle node bifurcation. Moreover the transfer energy among the modes and jump in amplitude of modes occur in frequency response at the different cases of internal resonance.

KEYWORDS

Bifurcation diagram, rotating beam, varying rotating speed, internal and external resonance, fixed points

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1. Introduction

Rotating blades are used in wind and gas turbines and they are designed according to the flexibility of the blade, mathematical modeling of the blade under various conditions such as aerodynamic loads or high temperature. Some researchers have studied the nonlinear vibrations of the rotating blade[1-9]. Also, time varying rotational speed can change the steady state amplitudes of the rotating beam [2]. In this study, the axial, lateral and transverse vibrations of the rotating beam are investigated under the harmonic angular velocity. When the internal resonance conditions are imposed between the three modes, the energy is transformed from the excited mode to other modes. Moreover, the amplitude and frequency of harmonic term of angular velocity can influence the stability of the steady state solutions.

2. methodology

In order the study the nonlinear vibration of rotating beam, a rotating beam model is presented in Fig. 1. The basic assumptions are considered as follows: 1- Euler-Bernoulli beam theory is used in this analysis and there is no shear stress. 2- Rotational speed is composed of constant value (Ω_0) as well as small amplitude with harmonic term ($\Omega_2 \cos(\Omega_1 t)$). 3- The von-karman strain-displacement is employed.

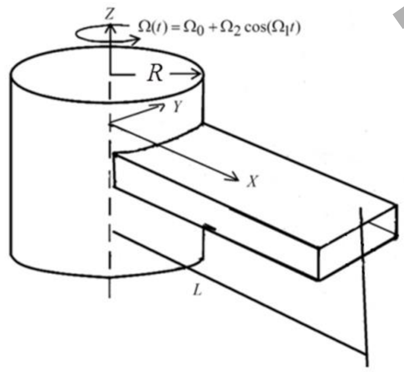

Fig. 1. Rotating Euler-Bernoulli beam with varying rotation speed Ω(t)

The dimensionless equations are derived using the extended Hamilton's principle as follows

$$\begin{aligned}&(\frac{\partial^2 u_d}{\partial t^2})-\Omega^2 u_d-(\frac{\partial^2 u_d}{\partial x^2})-(\frac{\partial^2 w}{\partial x^2})(\frac{\partial w}{\partial x})\\&-v(\frac{\partial \Omega}{\partial t})-2\Omega(\frac{\partial v}{\partial t})-(\frac{\partial^2 v}{\partial x^2})(\frac{\partial v}{\partial x})=0,\end{aligned} \tag{1}$$

$$\begin{aligned}&(\frac{\partial^2 v}{\partial t^2})-\Omega^2 v-\frac{3}{2}(\frac{\partial^2 v}{\partial x^2})(\frac{\partial v}{\partial x})^2-(\frac{\partial^2 u_d}{\partial x^2})(\frac{\partial v}{\partial x})\\&-(\frac{\partial u_d}{\partial x})(\frac{\partial^2 v}{\partial x^2})+D(\frac{\partial^4 v}{\partial x^4})+u_d(\frac{\partial \Omega}{\partial t})\\&+2\Omega(\frac{\partial u_d}{\partial t})-(\frac{\partial^2 u_s}{\partial x^2})(\frac{\partial v}{\partial x})-(\frac{\partial^2 v}{\partial x^2})(\frac{\partial u_s}{\partial x})\\&+(\frac{\partial \Omega}{\partial t})(\frac{R}{L}+x+u_s)-\frac{1}{2}(\frac{\partial^2 v}{\partial x^2})(\frac{\partial w}{\partial x})^2\\&-(\frac{\partial v}{\partial x})(\frac{\partial w}{\partial x})(\frac{\partial^2 w}{\partial x^2})=0,\end{aligned} \tag{2}$$

$$\begin{aligned}&(\frac{\partial^2 w}{\partial t^2})-\frac{3}{2}(\frac{\partial^2 w}{\partial x^2})(\frac{\partial w}{\partial x})^2-\frac{1}{2}(\frac{\partial^2 w}{\partial x^2})(\frac{\partial v}{\partial x})^2\\&-(\frac{\partial^2 v}{\partial x^2})(\frac{\partial v}{\partial x})(\frac{\partial w}{\partial x})-(\frac{\partial^2 u_d}{\partial x^2})(\frac{\partial w}{\partial x})\\&-(\frac{\partial^2 u_s}{\partial x^2})-(\frac{\partial u_d}{\partial x})(\frac{\partial^2 w}{\partial x^2})-\\&(\frac{\partial u_s}{\partial x})(\frac{\partial^2 w}{\partial x^2})+D(\frac{\partial^4 w}{\partial x^4})=0,\end{aligned} \tag{3}$$

Where, u_s, u_d and v, w are respectively the static and dynamic axial displacements and lateral and transverse displacements. By applying the Galerkin method

$$\begin{aligned}&u_d(x,t)=G(x)p(t), w(x,t)=H(x)q(t),\\&v(x,t)=S(x)r(t),\end{aligned} \tag{4}$$

The ordinary differential equations (ODE's) are obtained in three directions. $G(x), S(x), H(x)$ are the mode shapes of cantilever beam in axial, lateral and transverse directions. The multiple scales method is applied to the equations by assuming the straight forward solution as

$$\begin{aligned}&q(t,\varepsilon)=q_0(T_0,T_1)+\varepsilon q_1(T_0,T_1)+...\\&q(t,\varepsilon)=q_0(T_0,T_1)+\varepsilon q_1(T_0,T_1)+...\\&r(t,\varepsilon)=r_0(T_0,T_1)+\varepsilon r_1(T_0,T_1)+...\end{aligned} \tag{5}$$

By substituting Eq. (4) in ordinary differential equations, we get

$$\begin{aligned}&D_0^2 p_0+\omega_u^2 p_0=0,\ D_0^2 r_0+\omega_v^2 r_0=0,\\&D_0^2 q_0+\omega_w^2 q_0=0,\end{aligned} \tag{6}$$

$$\begin{aligned}&D_0^2 p_1+\omega_u^2 p_1=Nonlinearterms,\\&D_0^2 r_1+\omega_v^2 r_1=Nonlinearterms,\\&D_0^2 q_1+\omega_w^2 q_1=Nonlinearterms,\end{aligned} \tag{7}$$

By determining the secular terms and applying the solvability conditions, the modulation equations are obtained for different cases. The fixed points of equations are obtained and the stability of them is shown in numerical simulations.

3. Results and Discussion

The stability of fixed points is investigated for the rotating blade with time varying angular velocity. The dynamic behavior of the rotating beam is studied in axial, lateral and transverse directions. The fixed points or steady state amplitudes of rotating beam lose stability through the saddle-node or Hopf bifurcations. Modal interaction is occurred between the axial, lateral and transverse modes in the presence of internal resonance condition and external resonance. Fig. 2 shows the steady state amplitudes versus the Ω_2 for $\Omega_1=\omega_v+\varepsilon\sigma$ and internal resonance condition $\omega_v=2\omega_u+\varepsilon\delta$ for $\sigma=\delta=-1$. As seen in this figure, the axial mode (a_u) is increased at $\Omega_2=0/5,2/5$ due to modal interaction. Moreover, Jumps in lateral motions or a_v are seen at $\Omega_2=0/52,2/5$. In Fig 3. $\Omega_1=\omega_w+\varepsilon\sigma$ and $\omega_u=2\omega_w+\varepsilon\delta$, the axial and transverse modes are excited and jumps occur at SN points.

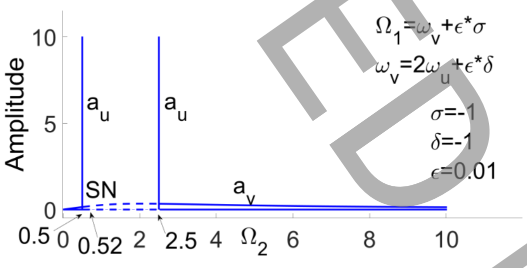

Fig. 2. amplitude of axial, lateral and transverse modes of beam versus Ω_2 amplitude

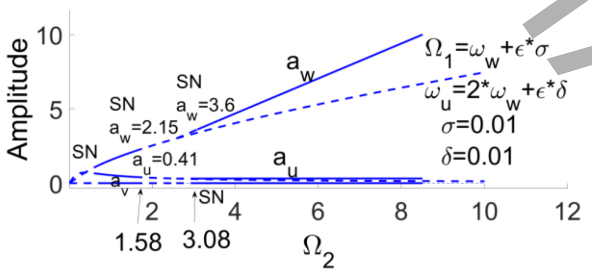

Fig. 3. amplitude of axial, lateral and transverse modes of beam versus Ω_2 amplitude

4. conclusions

In this study, the nonlinear vibration of rotating beam is analyzed for time varying spinning speed. The nonlinearity is caused from the von karman strain-displacement relations and harmonic term of angular velocity. The governing equations are derived in axial, lateral and transverse directions using the Hamilton's principle. The steady state amplitudes in three dimensions are presented in numerical simulations. Results show the modal interaction and jump in amplitudes in the presence of 1:1 and 2:1 internal resonances.

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