Kinematic Optimization of the Stirling engine for Maximum Output Work and Constraint of Occupied Space

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ABSTRACT

The Stirling engine has attracted researchers' attention in recent years due to some advantages such as low noise, external combustion, and the ability to use solar and other new energy sources. Moreover, these engines can also be used in applications with a low or high-temperature difference. The type of cylinders, their arrangement, and the transmission mechanism can affect this engine's performance. On the other hand, engineers and designers are always looking to increase the efficiency and effectiveness of mechanical systems, which in engines can lead to increasing the engine's work or power. In the current study, firstly, the dimensional analysis of different types of Stirling engines is done. Then, by defining the engine's geometric parameters as the design variables, the engine's output work will be maximized using optimization algorithms. Also, in order to prevent the increase of the dimensions of the engine and its occupied space, a new constraint in the problem will be used. Kinematic optimization is applied to four different types of Stirling engines. Three algorithms, namely genetic algorithm, particle swarm optimization, and imperialistic competition algorithm, have been used to solve the optimization problem. The results of kinematic optimization show that the output work of the engine with optimal dimensions has increased approximately 1.45 to 4.59 times.

KEYWORDS

Stirling engine, Kinematic, Thermodynamic, Optimization algorithms, Output work

1. Introduction

The Stirling engine is a closed-loop external combustion engine that its operating fluid never leaves the cylinders of the engine. This engine can be used in cases where a low-noise engine such as submarines is needed because it generates very low volume sound density [1, 2].

In this study, the four well-known layouts of the Stirling engine (Fig. (1)) are considered, and the thermodynamic relations, Schmidt's theory, and the kinematic relations are also regarded. As the outputs of the current study, pressure, volume, the output work of the engine, and the effect of the link's length on the output power of the engine are also investigated. All the geometric parameters of each layout of the Stirling engine are considered as the design variables. Also, in order to prevent the increase of the dimensions of the engine and its occupied space, a new constraint in the problem will be used. Kinematic optimization is applied to four different types of Stirling engines, where three algorithms, namely GA, PSO, and ICA, have been used to solve the optimization problem.

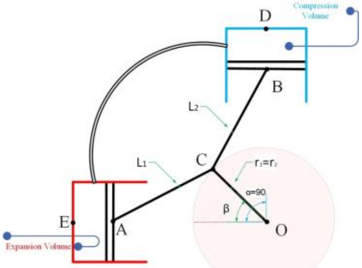
2. Kinematic Modeling

Using the governing kinematic relationships of each layout, the compression volume (V_c) and the expansion volume (V_e) can be calculated. If the values of pressure and volume relative to the rotation angle are known, the equation of fluid pressure variations as a function of volume changes would be yielded. Given this relation, the pressure diagram in terms of the volume of the engine and the output of the engine, which is actually the area of the enclosure, can be calculated. Fig.1 shows the geometry of the mechanisms for four different layouts of the Stirling engine, namely [3]:

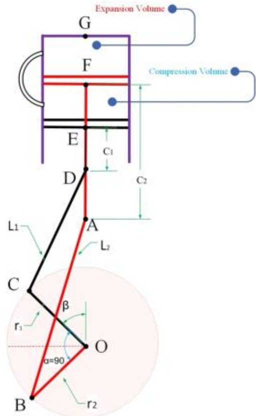
- α type with slider-crank linkage
- β type with slider-crank linkage
- γ type with slider-crank linkage
- α type with Ross-Yoke mechanism

3. Optimization

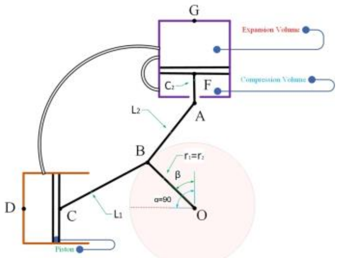
The output work for each layout depends on the pressure and volume. Indeed, the pressure and volume equations depend on the length of the links and the radius of the cylinders. Therefore, the variation in the output work is the function of the links length and the radius of the cylinders. The output can be maximized by considering the length of the links as well as the radius of each cylinder as design parameters and by performing optimization algorithms. Then the maximum output work can be simply achieved by using evolutionary algorithms that are used in many practical engineering problems.



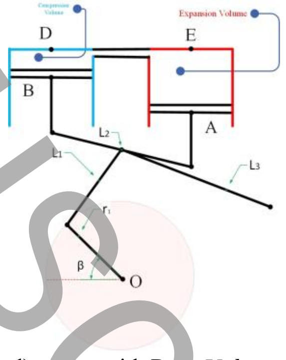
a) α type with slider-crank linkage



b) β type with slider-crank linkage



c) γ type with slider-crank linkage



d) α type with Ross-Yoke mechanism

Fig.1. Geometry of the mechanisms for different layouts [3]

Moreover, a constraint is used to prevent the engine size is increased. The optimization problem will be summarized as follows:

$$\begin{aligned} & Max\ \ F = W \\ & s \cdot t \cdot :\ \ P_{\min} \le P \le P_{\max} \\ & \qquad OS \le OS_{\max} \\ & \qquad x_{min} < x_i < x_{max} \qquad x = [x_1, x_2, \cdots, x_n] \end{aligned} \tag{1}$$

Where W is the output work and x is the vector of the design variables. The list of the design variables and their permissible variations are taken from Ref [3]. OS is the occupied space for each layout, considered based on the following relations:

$$OS_{\alpha\_SC} = \left(2\pi r_1^2 .min(E_r, C_r)\right) + \left(\pi E_r^2 (OE - r_1)\right) + \left(\pi C_r^2 (OD - r_1)\right) \tag{2}$$

$$OS_{\beta\_SC} = \left(2\pi r_1^2 E_r\right) + \left(\pi E_r^2 (OG - r_1)\right) + \left(\pi D_r^2 (OD - r_1)\right) \tag{3}$$

$$OS_{\gamma\_SC} = \left(2\pi r_1^2 .min(E_r, D_r)\right) + \left(\pi E_r^2 (OG - r_1)\right) + \left(\pi D_r^2 (OG - r_1)\right) \tag{4}$$

$$\begin{aligned} OS_{\alpha\_RY} = & \left(\pi r_1^2 .min(E_r, C_r)\right) + \left(2.min(E_r, C_r).max\,(2r_1, 2l_2)(r_1 + l_1)\right) \\ & + \left(\pi E_r^2 (OE - l_1 + r_1)\right) + \left(\pi C_r^2 (OE - l_1 + r_1)\right) \end{aligned} \tag{5}$$

Moreover, the values of the $P_{\min}$, $P_{\max}$, and $OS_{\max}$ are represented in Table 1.

Table 1. The permissible ranges of the problem constraints

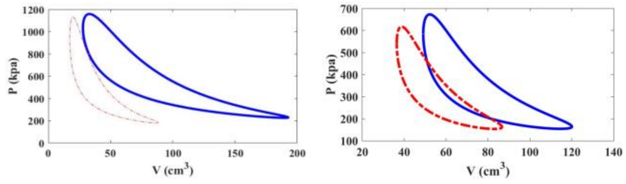
Engine type	$P_{\min}$ (kPa)	$P_{\max}$ (kPa)	$OS_{\max}$ (cm^3)
α _SC	180	1200	301.59
β _SC	150	700	257.61
γ _SC	250	600	383.27
α _RY	150	3000	1097.82

4. Results

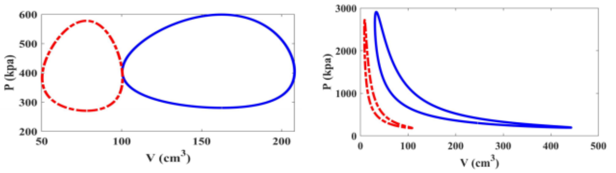
The optimization results of the different layouts are demonstrated in Table 2. In this table, the value of W_0 in each case is extracted from Ref [2]. Furthermore, the P-V diagram for the original and optimal engines, using different algorithms are shown in Figs. 2-4, respectively.

Table 2. The optimization results

Engine	Optimal W	GA	PSO	ICA
α SC	$W\left(\text{kg}(\text{cm/s})^2\right)$	31022	31754	31227
	W/W_0	2.04	2.09	2.05
β SC	$W\left(\text{kg}(\text{cm/s})^2\right)$	11024	11702	10652
	W/W_0	1.5	1.59	1.45
γ SC	$W\left(\text{kg}(\text{cm/s})^2\right)$	29658	30974	27036
	W/W_0	2.48	2.59	2.26
α RY	$W\left(\text{kg}(\text{cm/s})^2\right)$	102071	100996	101249
	W/W_0	4.59	4.54	4.55

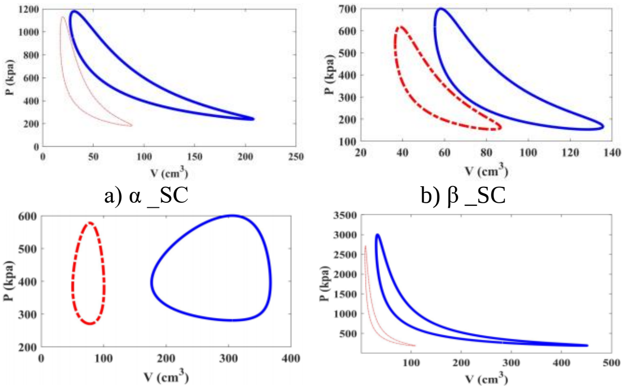


a) α _SC b) β _SC



c) γ _SC d) α _RY

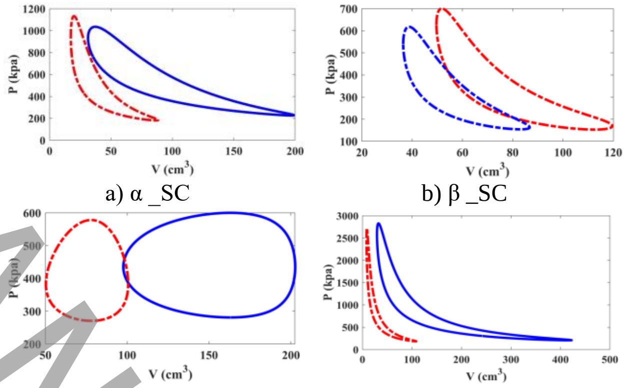
Fig. 2. Result of the GA algorithm for different layouts



a) α _SC b) β _SC

c) γ _SC d) α _RY

Fig. 3. Result of the PSO algorithm for different layouts



a) α _SC b) β _SC

c) γ _SC d) α _RY

Fig. 4. Result of the ICA algorithm for different layouts

A comparison between the three optimization methods for the four different Stirling engines is shown in Fig.5.

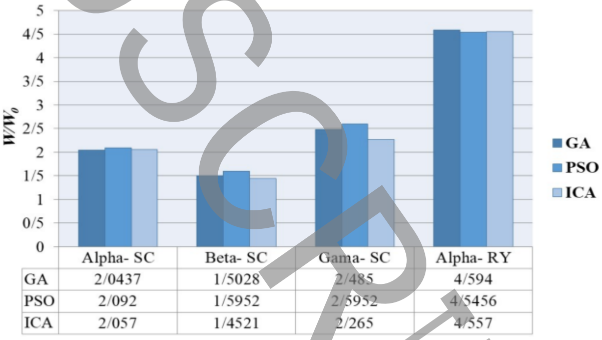


Fig. 5. Comparison between three optimization algorithms

5. Conclusion

In the current study, the dimensional synthesis of the Stirling engine is considered based on four different layouts. This optimization problem's design variables included the mechanism's geometric parameters, and the engine's occupied space is considered the main constraint. The optimization aimed to increase the output work of the engine. This optimization problem is solved for different layouts using three optimization algorithms GA, PSO, and ICA. The results show:

- Irrespective of other parameters, increasing the length of the crankshaft leads to higher output work.
- Regardless of the effect of other parameters, increasing the connecting rod's length leads to lower output work.
- Alpha type Stirling engine with Ross-Yoke mechanism has the best result compared to other layouts; for maximizing the output work based on optimizing the geometric parameters.

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- In comparison between optimization techniques, the PSO method had the best results in three cases and in one case, the GA method had the best results.

6. References

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[2] J. Egas, D.M. Clucas, Stirling engine configuration selection, Energies, 11(3) (2018) 584.
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