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Analytic Solution of Governing Equations of Heat and Moisture Transfer in a Capillary-Porous Body with Dirichlet Boundary Condition

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ABSTRACT: In this research, a new analytical solution of one dimensional coupled equations of moisture and heat transfer in a capillary-porous body is presented. These equations are known as the Luikov system of equations. These partial differential equations are coupled and non-homogenous, that could be considered linear with the assumption that the coefficients of the equations are independent of space, time, and (every) dependent variables. In the innovative method of this survey, at the first, it is assumed that the system of equations is independent of each other, it has been resulted in a general answer for equations by using the method of separation of variables. Next, the special answers will be obtained by considering coupled equations and using the Laplace transform method. In this survey, it has been studied the effect of dimensionless coefficients such as Luikov number, Fourier number, and phase change coefficient on the rate of heat and moisture transfer. The result shows the coupling important effect of Luikov number on the rate of heat and moisture transfer of capillary-porous body equations. It has also resulted that the phase change coefficient has a minor effect on moisture transfer which was also reported in the study of Luikov.

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1. INTRODUCTION

Heat and moisture transfer have an important role in new knowledge and industries. The equations of heat and mass transfer in capillary porous materials are presented by Luikov [1]. These equations are non-homogeneous coupled partial differential equations that by considering some assumptions can be written in the linear form. A solution of these equations has presented by Luikov [2]. Mikhailov and Özisik [3] have solved the coupled equations using integral transform method. Lobo et al. [4], have shown the existence of complex eigenvalue for the system of equations.

In the present study, in order to solve the unsteady one dimensional coupled system of equations, first, the general answers of separate uncoupled equations are obtained. In the novel method of the present study, separate eigenvalue for equations of the mass and the heat transfer are obtained. Using obtained different eigenvalues and applying the method of the separation of variables that leads to the flexibility of solutions, the general solution for the equations is obtained. Finally, considering obtained eigenvalues and eigenfunctions, the special solutions of the equations are obtained using the Laplace transform method.

2. GOVERNING EQUATIONS

The equations of the heat and mass transfer in capillary porous bodies with ignoring the pressure difference effect of filtration motion can be written as [2]:

$$\frac{\partial u(x,t)}{\partial t}=Lu\frac{\partial^2 u}{\partial x^2}-Lu\ Pn\frac{\partial^2 T}{\partial x^2} \tag{1}$$

$$\frac{\partial T(x,t)}{\partial t}=\frac{\partial^2 T}{\partial x^2}-Ko\,\varepsilon\frac{\partial u}{\partial t} \tag{2}$$

In the dimensionless form of the Eqs. (1) and (2), Lu is Luikov number, Pn is Possenove, Ko is Kossowich number and ε is phase change coefficient. The normalized temperature and moisture are defined as: $T=\frac{\theta(x,t)-\theta_\infty}{\theta_i-\theta_\infty}$ and $u=\frac{\mu(x,t)-\mu_\infty}{\mu_i-\mu_\infty}$.

The boundary conditions of these equations are assumed in Dirichlet form as:

$$\begin{cases} u(0,t)=0 & at\ (x=0) \\ u(1,t)=0 & at\ (x=1) \end{cases} \tag{3}$$

$$\begin{cases} T(0,t)=0 & at\ (x=0) \\ T(1,t)=0 & at\ (x=1) \end{cases} \tag{4}$$

With the initial conditions of:

$$\begin{aligned} u(x,0)&=1 \qquad 0<x<1 \\ T(x,0)&=1 \qquad 0<x<1 \end{aligned} \tag{5}$$

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3. SOLUTION METHOD

The uncoupled equations are assumed as:

$$\frac{\partial u(x,t)}{\partial t} = Lu\,\frac{\partial^2 u}{\partial x^2} \tag{6a}$$

$$\frac{\partial T(x,t)}{\partial t} = \frac{\partial^2 T}{\partial x^2} \tag{6b}$$

Applying the method of separation of variables with different eigenvalues the solution of the equation can be easily provided as:

$$u(x,t) = \sum_{n=1}^{\infty} c_n \sin(n\pi x)\psi_n(t) \tag{7a}$$

$$T(x,t) = \sum_{n=1}^{\infty} d_n \sin(n\pi x)\varphi_n(t) \tag{7b}$$

Now, by substituting these relations into Eqs. (1) and (2), two ordinary differential equations with respect to time are seen. Using the Laplace transform method for solving the obtained equations, and implementing boundary and initial conditions with considering orthogonality properties of equations, the solution of equations can be presented as:

$$u(x,t) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left(1-\left(-1^n\right)\right)\sin(n\pi x)\frac{\left(Ae^{s_1 t} + Be^{s_2 t}\right)}{\varphi_n(0)} \tag{8a}$$

$$T(x,t) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left(1-\left(-1^n\right)\right)\sin(n\pi x)\frac{\left(Ce^{s_1 t} + De^{s_2 t}\right)}{\varphi_n(0)} \tag{8b}$$

Where $\Phi_n(s)$ is written as:

$$\Phi_n(s) = \frac{\begin{vmatrix} (s + Lu\,\omega) & \varphi_n(0) \\ (s+\omega) & (1+Ko\,\varepsilon)\varphi_n(0) \end{vmatrix}}{\begin{vmatrix} (s + Lu\,\omega) & -(Lu\,Pn\,\omega) \\ (s+\omega) & (Ko\,\varepsilon s) \end{vmatrix}} \tag{9}$$

The s_1 and s_2 are derived from the roots of the denominator of Eq. (9).

4. RESULTS AND DISCUSSIONS

To validate the result and in consequence, the method presented in this research, our results are compared with Kulasiri and Woodhead [5] analytical solution. In the research of reference [5], a long wall with the thickness of x=0.1m is assumed to be in expose of constant ambient temperature and moisture content. The thermo-physical properties of the wall are taken as: r=2400 kJ/kg, a_m=3.0×10^{-6} m^2/h, c=1284 kJ/kg^{o}C, k=0.12 W/m^{o}C, the boundary and initial temperature and moisture content are also assumed: T_i=10^{o}C, $T_{ambient}$=80^{o}C, u_i=0.5, $u_{ambient}$=0.12. Using 20 terms of the series of Eq. (8), the temperature of the porous media at x=0.05 with respect to time is shown in Fig. 1 and is compared with Kulasiri and Woodhead [5].

In the Fig. 2, it is shown that the phase change coefficient has lesser significance in the dynamics of heat and moisture profiles as also is mentioned in the reference [5]. The temperature profiles versus Fourier number (time) by considering the effect of Luikov number that represents the mass diffusion to heat transfer is depicted

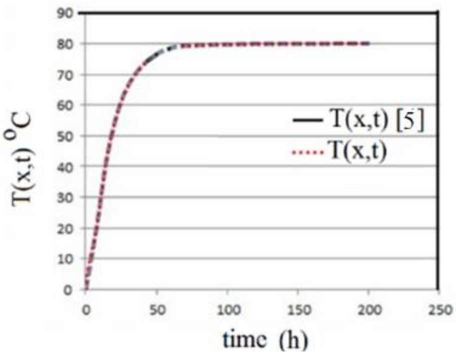

Fig. 1. Temperature verstime at x=0.05

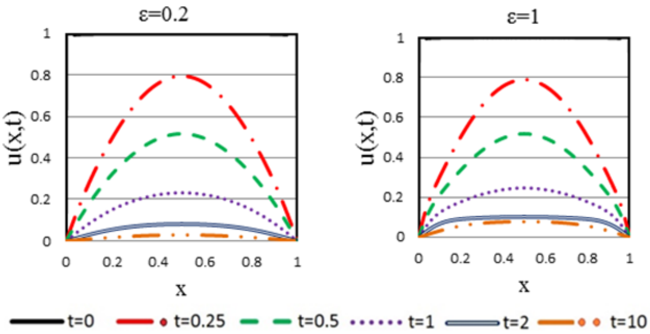

Fig. 2. The transient moisture content profile versus time for different phase change coefficient

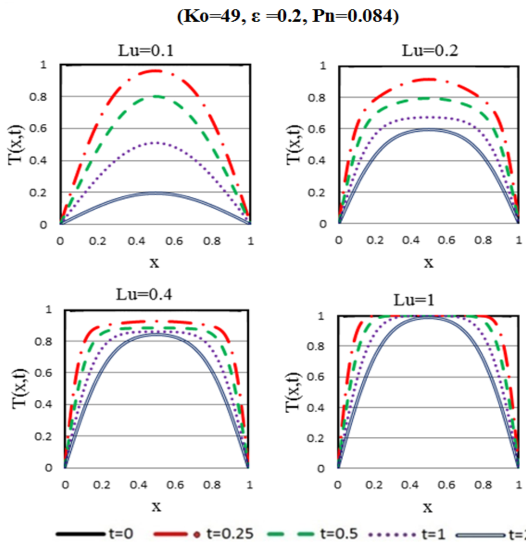

Fig. 3. Temperature distribution with respect to time for different Luikov number

in Fig. 3. In the high *Lu* number that the moisture diffusion is dominated, the temperature distribution variation seems to be more uniform due to the presence of the moisture. However, for the lower *Lu* number, i.e. $Lu<0.001$, the coupling between the equations of heat and mass can be neglected according to the study of Mendes and Philippi [6].

5. CONCLUSIONS

The governing equations of the drying the porous materials have been solved analytically. The solutions were explored to show that diffusion effects cannot be ignored. The phase change has less importance in the vapor diffusion relative to liquid transfer when filtration phenomena due to pressure difference are ignored.

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