



## Frequency Domain Analysis of Water Hammer with Fluid-Structure Interaction in Viscoelastic pipe

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**ABSTRACT:** In this research, fluid-structure interaction including transient flow in a viscoelastic pipe has been studied in the frequency domain. The main purpose was to investigate the water hammer problem using extended transfer matrix method in a typical reservoir-viscoelastic pipe-valve system. One of the generally expected advantages of frequency domain analysis is that the integral form of equations in the time domain would be transformed into algebraic form. Here it would be more beneficial to utilize frequency domain methods since convolution integral which appears in viscoelastic models in time domain will also vanish. Transfer matrix method has been adopted to the transient flow in a viscoelastic pipe to derive field matrix where a non-oscillating valve is considered as a boundary condition. The generalized Kelvin-Voigt model was used to simulate the viscoelastic behavior of the pipe wall. The proposed model has been explored to solve two well-known case studies of fluid-structure interaction in the frequency domain. Results of both cases confirm the good agreement between analytical and experimental data. To investigate the simultaneous effects of viscoelasticity and fluid-structure interaction in the frequency domain a sample problem has been analyzed. Results for different conditions including interactional and non-interactional system together with both viscoelastic and elastic pipe material have been illustrated and compared. Also, a comparison among 3-element, 5-element, and higher order Kelvin-Voigt models has been performed based on which one may deduce the appropriateness of 3-element model.

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### 1- Introduction

For the two recent decades, modeling and analyzing transient flow in flexible pipes has been the subject of extensive research activities. Standard solution procedures in this field can be divided into two main groups including time and frequency domain analysis.

Although time domain solutions provide an in-depth knowledge about pressure fluctuations during transients, frequency response spectrum also contains valuable information such as standing waves (mode shapes), frequency dependent wave speed or friction and resonant frequencies of the system [1]. However, frequency domain solutions, though being less difficult in terms of implementation, received less attention. Many articles on frequency domain solutions of water hammer with Fluid-Structure Interaction (FSI) mechanisms are available in the literature, so are significant research works on Viscoelastic (VE) pipes is frequency domain. However, frequency domain investigations of FSI in VE pipes have not been reported until now. This paper aims at modeling water hammer with FSI including Poisson and junction coupling in a VE pipe in the frequency domain. Laplace transform of fluid and structural equations have been obtained to exploit Transfer Matrix Method (TMM).

Common boundary conditions, as well as Kelvin-Voigt mathematical model for VE pipes, have also been transformed

into Laplace domain which they ultimately allow for frequency domain solutions of the problem. The proposed model has been implemented in MATLAB software. Various results to assess VE and FSI effect separately and simultaneously and make comparisons with elastic or no-FSI conditions are presented.

### 2- Mathematical Model

#### 2- 1- Viscoelastic pipe model

According to the literature, two widely-established mechanical models including Kelvin-Voigt and Maxwell are provided to simulate viscoelastic materials [2]. Here a generalized Kelvin-Voigt model with a default of one KV cell is used. The general form of the KV transfer function is obtained from stress-strain relation [3] and is defined as follows:

$$[KV] = s\bar{J} = \sum_{K=0}^{N_{KV}} J_k - s \sum_{K=1}^{N_{KV}} \frac{J_k \tau_k}{1 + s\tau_k} = J_0 + \sum_{K=1}^{N_{KV}} \left( \frac{J_k}{1 + s\tau_k} \right) \quad (1)$$

#### 2- 2- Governing equations

The time domain set of governing equations includes six preliminary equations. They respectively include: continuity and momentum of fluid, the equilibrium of forces, axial strain definition and axial and circumferential stress-strain relations.

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These equations could be reduced to four equations by substituting and simplifications. Then to obtain an analytical solution for the problem and to adopt the equations for the TMM method, Laplace transform has been applied to this set of equations. The result could be presented in a typical matrix form below [4]:

$$AY + B \frac{\partial y}{\partial z} = \bar{r} \tag{27}$$

in which matrix Y, A, B, and r are as follows:

$$Y = [V, H, \dot{u}_z, \sigma_h]^T$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\rho_f g}{K} + \frac{D \rho_f g}{e} [KV] (1-\nu^2) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & \frac{g D \rho_f \nu}{2Ee} + \frac{g D \rho_f \nu}{2e} ([KV] - J_0) & 0 & \frac{-1}{\rho_f c_i^2} - [KV] + J_0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & g & 0 & 0 \\ 1 & 0 & -2\nu & 0 \\ 0 & 0 & 0 & \frac{-1}{\rho_f} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

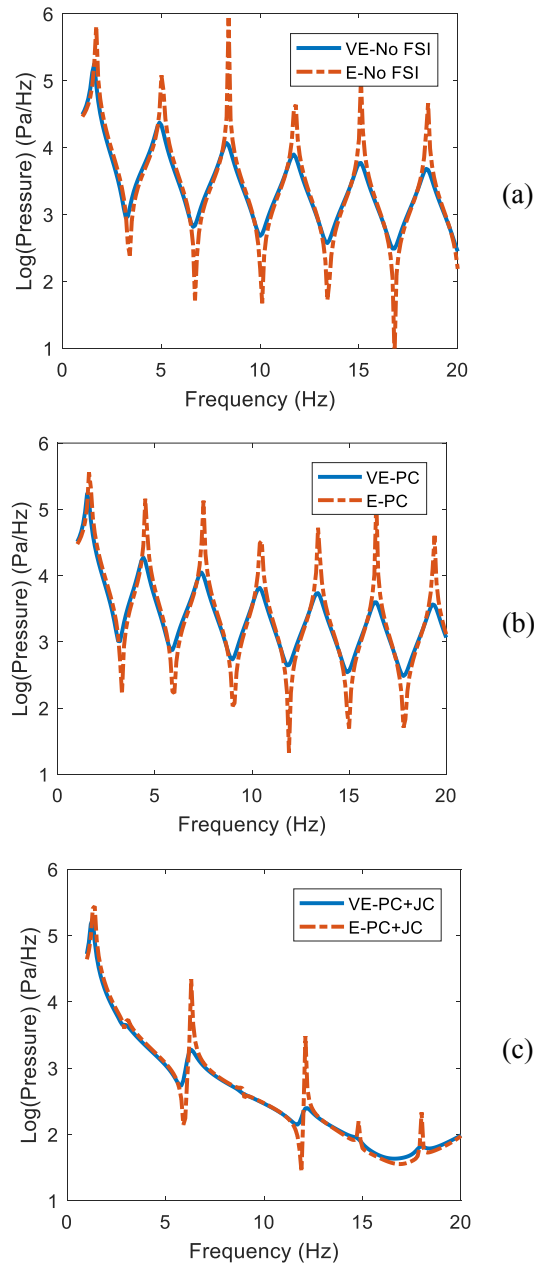
$$r = \begin{bmatrix} V(t=0) \\ \frac{g \rho_f}{K} H(t=0) + (1-\nu^2) \frac{g D \rho_f}{e} J_0 H(t=0) \\ 0 \\ \frac{-1}{\rho_f c_i^2} \sigma_z(t=0) + g \frac{D \rho_f \nu}{2Ee} H(t=0) \end{bmatrix} \tag{28}$$

After evaluating and verifying the application of the proposed model to water hammer problems, the model has been applied to a sample Reservoir-Pipe-Valve (RPV) system with viscoelastic Polyvinyl Chloride (PVC) pipe. This is discussed in the next section

### 3- Results and Discussion

In the case study, a PVC pipe with L=20m, R=398.5mm and wall thickness of 8mm is connected to an upstream reservoir with constant water level and a valve at its downstream end. The upstream end is considered to be immovable while the valve end is unrestrained and therefore providing Poisson and junction couplings. Other main mechanical properties of the system are: E=1.43 GPa, K=2.1 GPa, ν=0.46, ρ<sub>f</sub> = ρ<sub>s</sub> = 1000 kg/m<sup>3</sup>. Values of creep coefficients which will be used in the Kelvin-Voigt model are derived from sample values based on calibration done by Keramat and Haghghi [5] for the Imperial college experiment performed by Covas et al.[6]. For comparison purposes, some different cases are analyzed. Two types of elastic and viscoelastic pipe material, have been considered and each was analyzed using three different scenarios: No FSI analysis, FSI analysis including only Poisson coupling and FSI analysis including Poisson and junction couplings. Some of the resulting comparative diagrams are presented here.

As seen in Fig. 1 the VE properties particularly the retardation function put dissipation in the system and cause a lag dynamic, reduces the amplitude sparks and make the frequency response to be more smooth. Results also reveal



**Fig. 1. The amplitude of pressure oscillation at the downstream end for Elastic and VE pipe a: without FSI, b: considering Poisson coupling and c: considering Poisson and junction couplings**

that junction coupling could cause a significant influence of boundary conditions. It is observed from the presented graphs that odd harmonics are being more influenced and dissipated when junction coupling is taken into account.

### 4- Conclusions

A frequency domain model was developed to study transient flow in a viscoelastic pipe. The model was verified and then a typical case study was performed to obtain and evaluate the frequency response diagram of a PVC pipe

including Poisson and junction coupling. Here for evaluation purpose, several combinations of different physics have been studied through frequency response analysis. From the results, it can be concluded that: 1-Viscoelastic pipe behavior has considerable damping like effect on the response, especially in the peak regions. Therefore, neglecting the VE properties will cause the result to be overestimated. 2-Natural frequencies in both cases of VE and elastic pipe were almost identical, but the amplitude of oscillations is only deliverable by a correct VE model. 3-Neglecting Poisson coupling will lead to a considerable shift in frequency response. It means exact mechanisms of FSI shall be distinctly identified and be precisely introduced in the model in order to achieve reliable results.

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