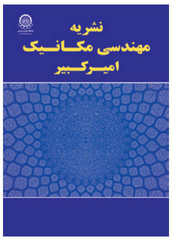

Amirkabir Journal of Mechanical Engineering

Amirkabir J. Mech. Eng., 52(5) (2020) 307-310
DOI: 10.22060/mej.2018.14885.5968

Derivation of Explicit Relationships in the Determination of the Natural Frequency of Euler-Bernoulli Cracked Beams on Elastic Foundation the Using Rayleigh Method

A. Alijani[1*] and M. Khomami Abadi[2]

[1] Department of Mechanical Engineering, Bandar Anzali Branch, Islamic Azad University, Bandar Anzali, Iran
[2] Department of Civil Engineering, University of Guilan, Rasht, Iran

ABSTRACT: In this paper, an approximate solution based on the Rayleigh's method is sought to analyze the vibration behavior of Euler-Bernoulli cracked beam resting on an elastic foundation. The modeling of the elastic foundation is implemented using the Winkler elastic spring theory and the stiffness factor of the elastic spring is specified corresponding to material characteristics of the elastic foundation. The Dirac's delta function is used to apply the crack opening mode in the equation of the Rayleigh in which the factor of this function can be identified in terms of the stiffness factor of an equivalent rotational spring by considering material and geometric parameters of the crack. In the present analysis, explicit relationships are originally established to obtain the natural frequency in three boundary conditions of simply supported-simply supported, clamped-free and clamped-clamped. In this method, the natural frequency of the first mode is determined as the ratio of the maximum enriched potential energy to the maximum kinetic energy. Based on these relationships, the effects of the crack depth, the crack location and the elastic foundation on the response of natural frequency of the beam are investigated. The results of the analysis demonstrate that increasing the crack depth decreases the natural frequency of the beam containing the crack; while the elastic foundation increases the natural frequency of the cracked beam. The comparison of the results of proposed relations with those of full modeling of the structure in ABAQUS software shows a reasonable accuracy of the present analysis.

Review History:

Received: 8/25/2018
Revised: 11/8/2018
Accepted: 12/3/2018
Available Online: 12/19/2018

Keywords:

Rayleigh method
Natural frequency
Cracked beams
Elastic foundation
Rotational spring

1. INTRODUCTION

The modeling of beams resting on the elastic foundation is used in the analysis of structures like building, highways and railroads. Since the presence of the crack in the structure increases the complexity of the analysis, accurate and simple models have been proposed by many researchers for investigating the mechanical behavior of the cracked beams. The crack was firstly modeled by using the rotational spring [1] and it was then completed by Irwin [2]. In the last paper, the crack has been modeled by using a linear rotational spring, while the concepts of the fracture mechanics are considered to determine the stress intensity factor. Relations between the stiffness factor of the rotational spring and material and geometric parameters of the cracked structures have been studied in Refs. [3-5]. The model of the linear elastic springs can properly simulate the behavior of the elastic foundation and the interaction between structures and soil. Based on the Winkler-Pasternak model, a free vibration analysis for clamped-free cracked beams was performed by Akbas [6]. Three analytical (exact solution), approximate (Galerkin) and numerical (finite element) approaches were used to investigate the bending behavior of the cracked beams resting on the elastic foundation in Ref. [7].

In this paper, new equations are proposed to determine the natural frequency of the cracked Euler-Bernoulli beams resting on the elastic foundation. The governing differential equation of the cracked beam is written based on the Dirac's delta function whose factor is obtained in terms of the material and geometric parameters. The elastic foundation is modeled by a uniformly distributed linear springs, which the foundation characteristics specify the spring stiffness factor. Two valuable achievements of derived explicit equations are the simplicity of those and the proper accuracy of obtained results. The comparison of obtained results is performed by a complete modeling in ABAQUS software. The effects of the crack depth and location and the elastic foundation on the natural frequency results are investigated in different boundary conditions.

*Corresponding author's email: alijani@iaubanz.ac.ir

2. FORMULATION

Equations in three sub-sections including the crack modeling, Winkler foundation model and Rayleigh method are represented to derive the explicit formulation of the natural frequency.

2.1. Crack modeling

Figs. 1b and 1c indicate two models for the crack illustrated in Fig. 1a. The discontinuous model shown in Fig.

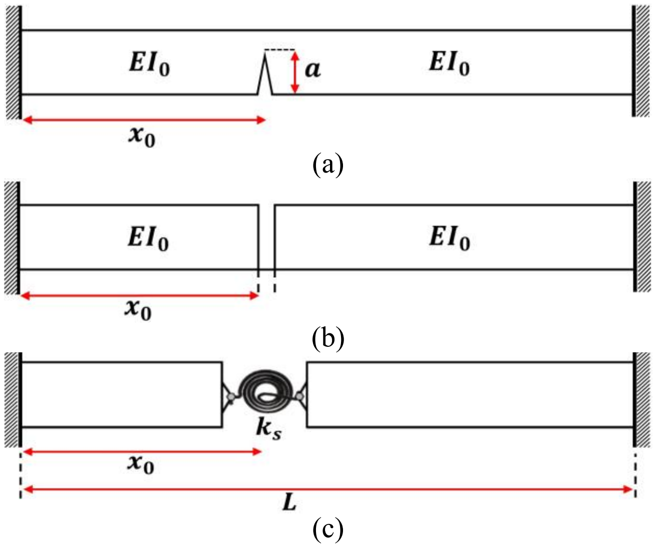

Fig. 1. a) Euler-Bernoulli cracked beam; b) Discontinuous model of flexural stiffness; c) Rotational spring model

1b is applied in the presented modal analysis, which the factor of γ used in Dirac's delta function is calculated in terms of the stiffness factor of equivalent spring (k_s).

$$EI(x) = EI_0\left(1-\gamma\delta(x-x_0)\right) \tag{1}$$

$$\gamma = \frac{EI_0}{k_s + \hat{A}EI_0} \quad , \quad \hat{A} = 2.013 \tag{2}$$

in which k_s is determined in terms of geometric and material properties as:

$$\frac{1}{k_s} = \frac{2(1-\nu^2)}{E}\int_0^a \left(\frac{1}{I_c} - \frac{1}{I_0}\right)da \tag{3}$$

Inserting Eq. (3) into Eq. (2) gives γ and then the flexural stiffness is obtained from Eq. (1).

2.2. Winkler elastic foundation model

One of common models in the analysis of the elastic foundation is to use the distributed linear elastic spring or Winkler model as shown in Fig. (2). In this model, soil is assumed as a homogenous and isotropic material into the linear elastic region [8].

The approximate solution of the presented problem has been carried out based on Rayleigh method. In this method, the natural frequency is determined by the ratio of the potential energy to the kinetic energy.

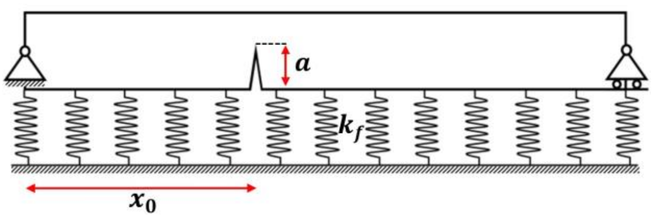

Fig. 2. Model of Winkler elastic foundation in cracked beam

$$\Pi_c^{st} = \Pi_c^B + \Pi_0^F \tag{4}$$

in which

$$\Pi_c^B = \frac{1}{2}\int_0^L EI_0\left[1-\gamma\delta(x-x_0)\right]\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \tag{5}$$

$$\Pi_0^F = \frac{1}{2}\int_0^L k_f w^2 dx \tag{6}$$

Also, the kinetic energy is formulated as:

$$K_c^{st} = K_c^B + K_0^F \tag{7}$$

$$K_c^B = \frac{1}{2}\int_0^L \rho A\left(\frac{\partial w}{\partial t}\right)^2 dx \quad , \quad K_0^F = 0 \tag{8}$$

The deflection of the beam can be considered as a separable form of space and time variables [9].

$$w(x,t) = X(x)\sin\omega t \tag{9}$$

Inserting Eq. (9) into Eqs. (5) to (8), the maximum potential and kinetic energies of the cracked beam resting on the elastic foundation is derived as:

$$\Pi_{c_{Max}}^{st} = \frac{1}{2}\int_0^L EI_0 (X'')^2 dx + \frac{1}{2}\int_0^L k_f X^2 dx \tag{10}$$

$$\begin{gathered} -\frac{1}{2}\int_0^L EI_0\gamma\delta(x-x_0)(X'')^2 dx \\ K_{c_{Max}}^{st} = \frac{\omega^2}{2}\int_0^L \rho A X^2 dx \end{gathered} \tag{11}$$

Since the total energy of an undamped vibration system based on Eq. (12a) is constant permanently,

$$\Pi_c^{st} + K_c^B = \text{Constant} \tag{12a}$$

An equality equation can be written by considering the maximum potential and kinetic energies as:

$$\Pi_{c_{Max}}^{st} = K_{c_{Max}}^{st} \tag{12b}$$

Therefore, the equation of the natural frequency can be determined as:

$$\omega = \sqrt{\frac{\Pi_{c_{Max}}^{st}}{\frac{1}{2}\int_0^L \rho A X^2 dx}} \tag{12c}$$

The natural frequency of the C-C cracked beams can be obtained from Eq. (12c) by considering the mode shapes as:

$$X = 1-\cos\left(\frac{2\pi x}{L}\right) \tag{13}$$

An explicit equation for C-C cracked beams is derived by inserting Eq. (13) into Eqs. (10) and (12c)

$$\omega_{CC}=\sqrt{\frac{\frac{4}{9}\zeta_1-\frac{2}{27}\zeta_2\cos\left(\frac{2\pi x_0}{L}\right)^2+k_f L}{\rho LA}} \tag{14}$$

in which parameters of ζ_1 and ζ_2. are defined as:

$$\zeta_1=\frac{Ebh^3\pi^4}{L^3} \tag{15a}$$

$$\zeta_2=\frac{Ebh^3\pi^4}{\left(\frac{\hat{A}}{12}+\frac{h^2-2ha+a^2}{12a^2\left(v^2-1\right)\left(2a-3h\right)}\right)L^4} \tag{15b}$$

2.3. Numerical modeling

In this research, the numerical modeling of the cracked beam resting the elastic foundation is performed in ABAQUS software based on the integral contour method.

3. Results and Discussion

In this section by using some case studies, the effects of the crack depth and location, and the elastic foundation in C-C boundary conditions on the natural frequency are investigated. Table 1 shows the geometric and material parameters used in the case studies, unless otherwise is mentioned.

Fig. 3 shows that if the crack sits at $\frac{x_0}{L}=0.2$ or 0.8, the effect of the crack depth on the natural frequency can be neglected.

Table 1. Geometric and material characteristics of the cracked beam on the elastic foundation

Geometric characteristics	Material characteristics
$L=3\text{m}$	$E=200\text{e}9\,\text{N/m}^2$
$h=0.3\text{m}$	$\nu=0.3$
$a/h=0.0-0.5$	$\rho_f=0$
$x_0/L=0-1$	$\rho=7860\,\text{N/m}^3$
$L_f=3\text{m}$	$E_f=4\text{e}7\,\text{N/m}^2$
$h_f=0.4\text{m}$	$k_f=1\text{e}8\,\text{N/m}^2$

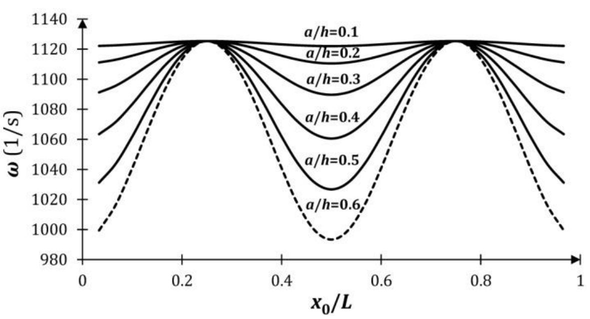

Fig. 3. The response of the natural frequencies of cracked beam on the elastic foundation in terms of different depths and locations of the crack in clamped-clamped boundary condition

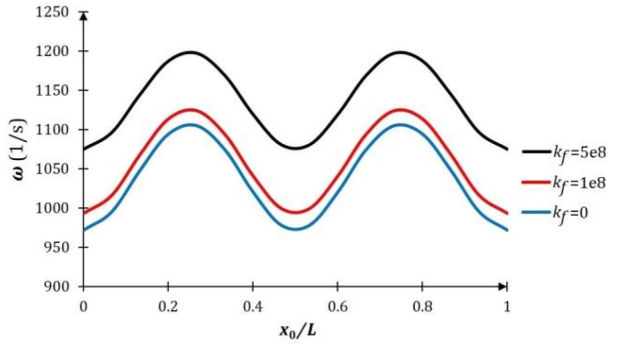

Fig. 4. Effect of stiffness factor of elastic foundation on natural frequency in Simply-supported-simply supported

Also, Fig. 4 illustrates the effect of the stiffness factor of the elastic foundation on the natural frequency. As shown in Fig. 4, increasing the stiffness factor of the elastic foundation increases the natural frequency in which increasing 5 times of the stiffness factor increases the natural frequency almost 8%.

4. CONCLUSION

In this paper, novel equations for the modal analysis of the Euler-Bernoulli cracked beam resting on the elastic foundation were proposed. The effects of the geometric and material parameters of the beam, the crack depth, the crack location and the stiffness factor of the elastic foundation are explicitly observed in the equations. ABAQUS was used to model the cracked beam and to compare the results of the proposed equations. The results show that increasing the crack depth decreases the natural frequency of the beam, while the elastic foundation increases the natural frequency.

REFERENCES

[1] G. R. Irwin, J. A. Kies, Critical energy rate analysis of fracture strength, *Journal of Welding*, 33(1) (1954) 193-198.

[2] G. R. Irwin, Analysis of stresses and strains near the end of a crack traversing a plate, *Journal of Applied Mechanics*, 24(1) (1957) 361-364.

[3] P. Ricci, E. Viola, Stress intensity factors for cracked T-section and dynamic behaviour of T-beams, *Engineering Fracture Mechanics*, 73 (2006) 91-111.

[4] T. Yokoyama, M.C. Chen, Vibration analysis of edge-cracked beams using a line-spring model, *Engineering Fracture Mechanics*, 59(3) (1998) 403-409.

[5] A.D. Dimarogonas, Vibration of cracked structures: A state of the art review, *Engineering Fracture Mechanics*, 55(5) (1996) 831-857.

[6] S. D. Akbas, Free Vibration Analysis Of Edge Cracked Functionally Graded Beams Resting On Winkler-Pasternak Foundation, *International Journal of Engineering & Applied Sciences*, 7(3) (2015) 1-15.

[7] A. Alijani, M. Mastan Abadi, A. Darvizeh, M. Kh. Abadi, Theoretical approaches for bending analysis of founded Euler–Bernoulli cracked beams, *Archive of Applied Mechanics*, 88(6) (2018) 875–895.

[8] K. V. Terzaghi, Evaluation of coefficient of subgrade reaction, *Geotechnique*, 5(4) (1995) 297-326.

[9] A. W. Leissa, M. S. Qatu, *Vibrations of Continuous Systems*, First edition, McGraw-Hill United States of America, 2011.