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Prediction and Control of Chaos in Nonlinear Rectangular Micro-Plate on the Elastic Foundation

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ABSTRACT: In this study, nonlinear dynamics of non-classical Kirchhoff microplate is analyzed and chaotic behavior is predicted and controlled by designing the robust adaptive fuzzy controller. Virtual displacement principle is employed to derive the governing equation of micro-plate resting on a nonlinear elastic foundation. In the governing equation, von-Karman geometric nonlinearity and couple stress theory are considered. Then eigenvalue governing equation is solved for fully simply supported boundary conditions and results are validated. In the next step, considering harmonic excitation of the first mode, the micro-plate forced vibration equation is derived using the Galerkin method. Regardless of modal interaction, the chaos threshold is then investigated. Homoclinic orbits of the unperturbed system are plotted and stable and unstable manifold transversely cut that is criteria to predict chaos according to Melnikov's method are studied. Using the maximum Lyapunov exponents numerical method, size-dependent chaos is also locally identified. Phase portrait, Poincare mapping and time response are plotted for different values of size ratios and the significant effect of size on the chaotic behavior of micro-plats is presented. Subsequently, designing the robust adaptive fuzzy controllers, chaotic vibrations are completely eliminated from the system and the robust adaptive fuzzy controller is introduced as an effective method for controlling chaos in these systems.

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1. INTRODUCTION

In the field of nanotechnology, mention to long-range applications of Micro-Nano-Electromechanical Systems (M-NEMS) there are massive engineering papers dedicated to studying these systems.

Reducing the size of M-NEMS in the scale of micro and nano causing high natural frequency makes more sensitivity and performance of M-NEMS in the role of sensors and other applications that is desired by the designer. The high frequency of systems hence needs a higher energy level in excitation yielding more nonlinear behavior such as frequency bending, dynamic jump and chaotic vibration. In conclusion, it is essential to study nonlinear dynamics and chaos in M-NEMS that is the aim of this paper.

The researchers have presented the chaotic behavior of the classical structure in numerous papers. Awrejcewicz et al. [1-3] studied the route of transition into chaos in plates and shells on triple articles.

The essence of heteroclinic loops and extremely complicated dynamical behavior of plates as chaotic vibration, symmetry breaking, and Smale horseshoes phenomena are also studied before [4, 5].

In the referred articles, classical elasticity theories are used to analysis of structures, whereas the classical theories don't valid on the scale of micro and nano illustrated by experimental research [6]. The modified couple stress theory as one of the none-classical elasticity theories is widely used to invest micro-structures [7, 8]. The forced vibration of Kirchhoff's nonlinear microplate was numerically studied based on The modified couple stress theory by Ghayesh and Farokhi [7]. In other paper, they hence did a similar search on imperfect microplate [8].

In this paper, the governing equations on the nonlinear dynamics of the microplate resting on the nonlinear elastic foundation are derived based on the coupling stress theory and the size-dependent chaotic behavior of the microplate will be studied using numerical and analytical methods. Then, the chaotic vibration of microplate will be controlled.

2. METHODOLOGY

The proposed system contains a rectangular microplate with dimensions $b \times a$ and cross-section height h resting on nonlinear elastic foundation with nonlinear stiffness $\tilde{k}_{nl}$, linear stiffness $\tilde{k}_l$ and shear stiffness $\tilde{k}_s$ that is made by epoxy ($E = 1.44\text{Gpa}$, $\nu = 0.38$) and l is length scale parameter. The nonclassical material constant in modified couple stress theory which has been measured by experimental test [6].

The governing equation of Kirchhoff's microplate with von-Karman geometric nonlinearity [9] is derived based on modified couple stress theory [10] via virtual displacement

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principle as the nondimensional equation:

$$\left(1+\frac{D^l}{D}\right)\nabla^4 w + k_l w - k_s \nabla^2 w + k_{nl} w^3 = \frac{\partial^2 w}{\partial t^2} + \frac{\tilde{F}}{D} + 12\left(\frac{w_{\max}}{h}\right)^2 \left(\left(\frac{1}{2}\left(\frac{\partial w}{\partial X}\right)^2\right)\frac{\partial^2 w}{\partial X^2} + \left(\frac{1}{2}\left(\frac{\partial w}{\partial Y}\right)^2\right)\frac{\partial^2 w}{\partial Y^2} + \frac{\partial w}{\partial X}\frac{\partial w}{\partial Y}\frac{\partial^2 w}{\partial Y \partial X} \right) \tag{1}$$

$$Y=\frac{y}{b}, X=\frac{x}{a}, t=\frac{ma^4}{D}\hat{t}, D=\frac{Eh^3}{12\left(1-\upsilon^2\right)}, \quad \frac{D^l}{D}=6\left(1-\nu\right)\left(\frac{l}{h}\right)^2 \tag{2}$$

Using variables separation, dynamic deflection of a microplate can be represented as $w(X,Y,t)=\varphi(X,Y)q(t)$, where $q(t)$ and $\varphi(X,Y)$ is time part and space mode shapes, respectively. Substituting dynamic deflection equation yield to eigenvalue equation. After solving this equation, natural frequency and linear mode shapes are obtained.

In order to analyze chaos in a microplate, Galerkin method is employed using the first linear mode shape of the microplate. This way, the nonlinear equation in the state space form can be expressed as:

$$\begin{aligned} \dot{q} &= v \\ \dot{v} &= \alpha q - \varepsilon\mu\dot{q} - \gamma q^3 + \varepsilon f \cos\left(\Omega t\right) \end{aligned} \tag{3}$$

where ε is the small scale parameter. The unperturbed system will be obtained by setting $\varepsilon=0$. The homoclinic orbits of the unperturbed system are used by Melnikov's method and size dependent chaos threshold function is then yielded as:

$$\frac{f_{cr}}{\mu} \le \frac{2\sqrt{2}\alpha^{1.5}}{3\sqrt{\gamma}\pi\Omega}\cosh\left(\frac{\pi\Omega}{2\sqrt{\alpha}}\right) \tag{4}$$

3. RESULTS AND DISCUSSION

The size-dependent chaos threshold is plotted for different values of size ratio (***h/l***) in Fig. 1. Accordingly, if the value of size ratio (***h/l***) changes to microscale beam, the chaos threshold will be increased dramatically.

In Figs. 2 and 3, Lyapunov exponents and phase portrait of the microplate is respectively depicted for tow values of size ratio (***h/l***). Mention to these figures, the microplate size has a significant effect on chaotic behavior.

Next, the chaotic vibration of the nonclassical microplate is controlled by designing robust adaptive fuzzy controller [11]. In Fig. 4, time response of microplate before and after activating controller is illustrated. It can be observed the robust adaptive fuzzy controller is an effective controlling method for nonclassical microplate.

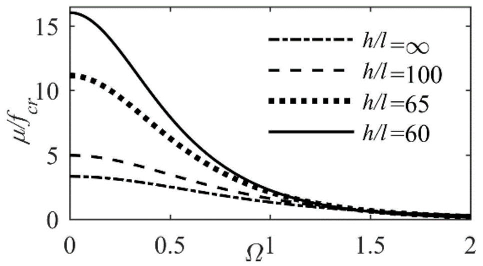

Fig. 1. Homoclinic bifurcation diagram and chaos threshold of the system for different values of size ratio (*h/l*)

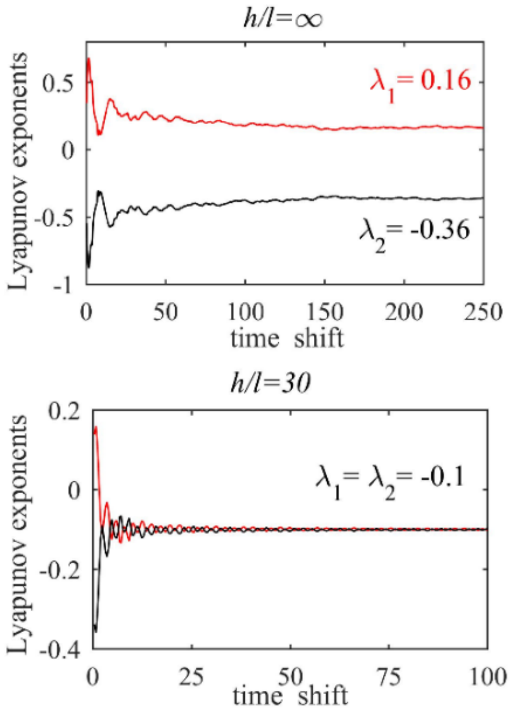

Fig. 2. Lyapunov exponents of the system for different values of size ratio (*h/l*)

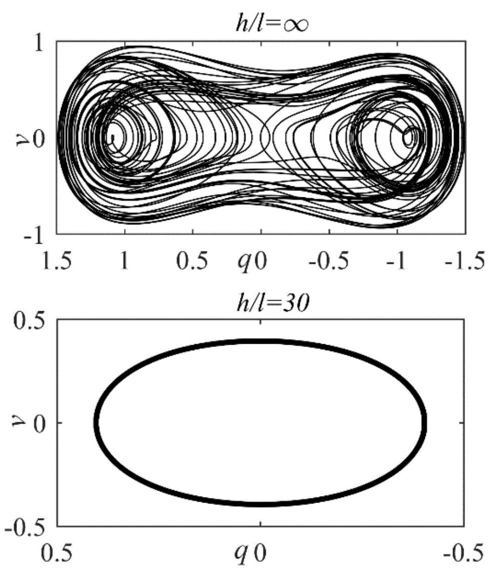

Fig. 3. Phase portrait of the microplate for different values of size ratio (*h/l*)

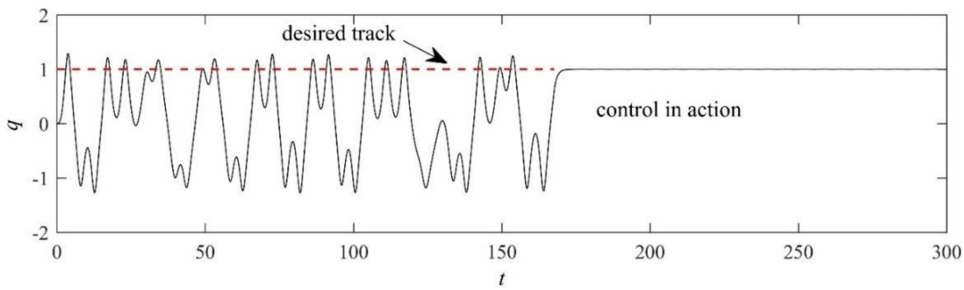

Fig. 4. Time history response of the microplate with the controller before and after activating controller at $t = 170$ s

4. CONCLUSIONS

In this paper, the size-dependent chaotic behavior of the microplate was studied based on the coupling stress theory using numerical and analytical methods. The homoclinic orbits equations of the unperturbed system were obtained. To investigate the homoclinic bifurcation of the system, the Melnikov method was used. The relationship between the critical force of the chaos threshold and microplate size has been predicted by Melnikov method. According to the Melnikov analysis, the scale of the microplate has a significant effect on the critical force of chaos threshold. Subsequently, the largest Lyapunov exponent as a numerical criterion was used to evaluate the sensitivity to the initial conditions and predictability of the system and the identification of chaos was performed for different size scale of the microplate. Results showed that in smaller size ratios, the largest Lyapunov exponent changed, thus the chaotic behavior of system also varied. Next, Phase portrait, Poincaré section and time response for different values of the size ratio were plotted and the impressive size effect has been displayed in the chaotic behavior of microplate.

After the chaos analysis, designing robust adaptive fuzzy controller, chaotic vibrations of the microplate are completely suppressed and the robust adaptive fuzzy controller is introduced as a powerful method of chaos controlling in nanoelectromechanical systems.

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