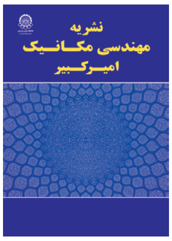

Amirkabir Journal of Mechanical Engineering

Amirkabir J. Mech. Eng., 52(5) (2020) 337-340
DOI: 10.22060/mej.2019.14832.5954

Optimal Control Based on Minimum-Energy Trajectory Planning of a Quadrotor

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ABSTRACT: Quadrotors have high energy consumption, hence minimizing their energy consumption plays a crucial role in terms of enhancing their operational range and flight time. In this paper, optimal control based on a minimum-energy trajectory planning algorithm has introduced between two positions to maximize the operation time. To do this, first, dynamic equations of a quadrotor and brushless motor are derived. Energy consumption of quadrotor is introduced as a cost function and the minimum energy path is determined using the optimal control theory. All constraints are combined with the Hamiltonian equation using Lagrange multiplier. Finally, simulation results are compared with results of conventional trapezoidal velocity profile which shows energy saving up to 4%. Also, results reveal that the influence of operation time is far more than path length on energy consumption. In order to verify the validity of the simulation results, they are compared with the results of an experimental model which is consisting of brushless motor, sensor, and control board. As well as using simulation results in different situations, a mathematical equation was extracted among path length, operation time and energy consumption which can be useful to estimate the maximum flight range or operation time considering the amount of energy of the battery.

Review History:

Received: 8/14/2018
Revised: 12/13/2018
Accepted: 12/27/2018
Available Online: 1/28/2019

Keywords:

Quadrotor
Minimum energy trajectory
Optimal control
Hamiltonian equations

1. INTRODUCTION

High energy consumption is one of the most important challenges about quadrotors [1]. The trapezoidal velocity profile is a common method which is used widely in all types of robots. The major problem of such an approach is a severe dependency on acceleration/deceleration rates that leads to a decrease in the energy efficiency of quadrotor [2]. For this reason, the minimum energy trajectory is the best strategy to enhance the energy efficiency [3].

The paper aimed at minimum energy path planning and propose an optimal controller so that quadrotor can reach the goal as well as satisfying its constraints. Considering the detailed model of motor and its ingredients, verifying the validity of simulation results by experimental data and also deriving the mathematical relation among duration flight, maximum flight range and energy consumption are contributions of the paper.

2. METHODOLOGY

The dynamic model of the quadrotor is as following [4]:

$$\begin{cases} \ddot{X} = \left(\sin\psi\sin\varphi + \cos\psi\sin\theta\cos\varphi\right)\dfrac{U_1}{m} \\ \ddot{Y} = \left(-\cos\psi\sin\varphi + \sin\psi\sin\theta\cos\varphi\right)\dfrac{U_1}{m} \\ \ddot{Z} = -g + \left(\cos\theta\cos\varphi\right)\dfrac{U_1}{m} \end{cases} \tag{1}$$

$$\begin{cases} \dot{p} = \dfrac{I_{yy} - I_{zz}}{I_{xx}} q\, r - \dfrac{J_{TP}}{I_{xx}} q\, \Omega + \dfrac{U_2}{I_{xx}} \\ \dot{q} = \dfrac{I_{zz} - I_{xx}}{I_{yy}} r\, p - \dfrac{J_{TP}}{I_{yy}} p\, \Omega + \dfrac{U_3}{I_{yy}} \\ \dot{r} = \dfrac{I_{xx} - I_{yy}}{I_{zz}} p\, q + \dfrac{U_4}{I_{zz}} \end{cases} \tag{1}$$

$$\begin{aligned} U_1 &= T_1 + T_2 + T_3 + T_4 = \mathrm{b}\left(\Omega_1^2 + \Omega_2^2 + \Omega_3^2 + \Omega_4^2\right) \\ U_2 &= \ell\left(T_4 - T_2\right) = \mathrm{b}\ \ell\left(\Omega_4^2 - \Omega_2^2\right) \\ U_3 &= \ell\left(T_3 - T_1\right) = \mathrm{b}\ \ell\left(\Omega_3^2 - \Omega_1^2\right) \\ U_4 &= \mathrm{d}\left(\Omega_2^2 + \Omega_4^2 - \Omega_3^2 - \Omega_1^2\right) \\ \Omega &= \Omega_1 + \Omega_2 + \Omega_3 + \Omega_4 \end{aligned} \tag{2}$$

T_i ,Ω_i represent thrust and rotational velocity of the *i*-th propeller, respectively. As well as *b, d* represent thrust and drag coefficient of the propeller and also ℓ addresses arm length of the system. Angular acceleration in body frame $(\dot{p}, \dot{q}, \dot{r})$ and inertial frame $(\ddot{\varphi}, \ddot{\theta}, \ddot{\psi})$ are not equal to each other. For less variation of Euler angles from zero position, it can be assumed that transmission function (T_θ) is close to the identity matrix then: $(\dot{\varphi}, \dot{\theta}, \dot{\psi}) \approx (p, q, r)$

The motor model should be added to the quadrotor

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dynamics. So, the final form of the motor model is as [5]:

$$\left(J_P + \eta N^2 J_M\right)\dot{w}_P = -\frac{K_E K_M}{R}\eta N^2 w_P - d w_P^2 + \frac{K_M}{R}\eta N v \quad (3)$$

Based on the equation of energy consumption of brushless motors, the cost function is considered as:

$$E_W = \int (\mathrm{k}_1 u^T u - \mathrm{k}_2 w_P^T u)\ dt \quad (4)$$

$$\left[\dot{\aleph}_1, \dot{\aleph}_2, \dot{\aleph}_3\right]^T = f_{16\times 1}\left(\aleph_1, \aleph_2, \aleph_3, u\right) \quad (5)$$

$$\aleph_{1,2,3} = \left[w_P, X, Y, Z, \varphi, \theta, \psi, \dot{X}, \dot{Y}, \dot{Z}, \dot{\varphi}, \dot{\theta}, \dot{\psi}\right] \quad (6)$$

$$\left| u^i \right| \le 1\ ,\ \left(i = 1, 2, 3, 4\right) \quad (7)$$

where $k_1 = {V_s}^2 / R$, $k_2 = \frac{k_E N V}{R}$, u and w_P are 4×1 vectors. To extract the optimal control inputs which guarantee the minimum energy trajectory and satisfy the constraints, the Hamiltonian is constructed as:

$$\mathrm{H} = \mathrm{k}_1 u^T u - \mathrm{k}_2 w_P^T u + \lambda^T \left[f\left(\aleph_1, \aleph_2, \aleph_3, u\right)\right] \quad (8)$$

3. LABORATORY PROTOTYPE

To validate results and to compare it with experimental data, a prototype was designed and used. Sensors data and control commands are conveyed through an ATMega32A microcontroller with sample time 0.02 seconds. The produced thrust by the propeller is measured according to the Pulse-Width Modulation (PWM) signal applied to the motor. Because of the lack of encoder, the rotational velocity of the propeller was determined based on the mathematical model of motor-propeller. Extractive experimental data of motor has been presented in Table 1.

4. RESULTS AND DISCUSSION

In order to extract the consumption energy of minimum energy trajectory and evaluation and comparison with common trapezoidal velocity profile, simulations were performed in different time and positional conditions which the results have been presented in Table 2. It illustrates that if the distance between start and end points increases in constant duration flight, the amount of saving energy of optimal trajectory in comparison with trapezoidal velocity profile will decrease.

Also, Table 2 shows that the difference in energy consumption between optimal and trapezoidal profiles rise with prolonging duration flight. Results demonstrate that the amount of saving energy is almost 4%. Fig. 1 shows the optimal energy consumption for one of a more used condition.

To validate simulation results and compare with actual status, the PWM signals corresponding with obtained voltages from simulation were applied to the motor. The experimental data including motors voltages and currents is shown in Fig 2. The amount of energy consumption was determined using voltage and current experimental data which is about 304 j. It

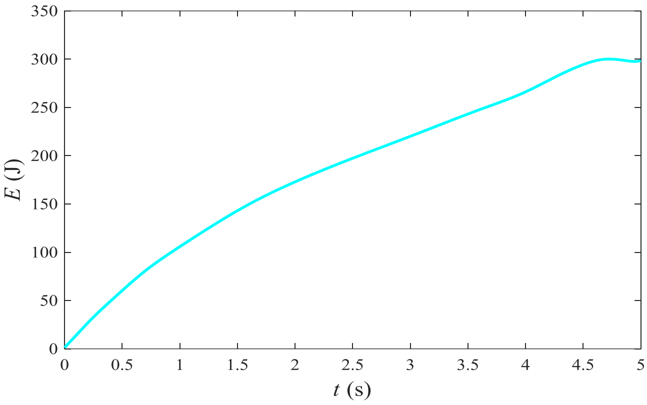

Fig 1. The energy consumption profile along the minimum energy path

Table 1. The results of experimental data of motor

PWM	40	60	80	100	120	140
Trust(gr)	15.31	31.04	51.76	80.34	107.71	145.11
W(rad)	531.5	756.8	977.2	1217.5	1409.7	1636.2
PWM	160	180	200	220	240	255
Trust(gr)	182.97	226.80	274.56	325.17	378.94	421.94
W(rad)	1836.4	2045.6	2250.7	2449.4	2644.1	2790.1

Table 2. The consumption energy in various conditions

t_f (s)	X_f	Y_f	Z_f	Min energy (j)	Trapezoidal velocity profile (j)
10	10	0	0	532.86	556.80
10	20	0	0	531.71	554.56
10	0	15	0	527.01	549.08
10	0	0	20	537.78	558.44
20	25	0	0	1062.7	1097.26
20	0	10	0	1066.3	1102.69

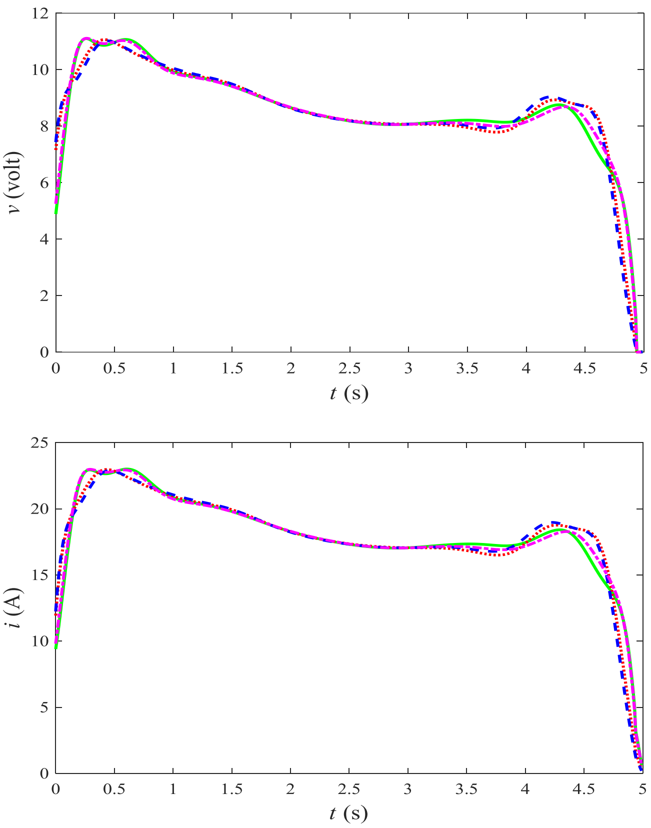

Fig 2. Voltage and current obtained from sensors

is compatible with the simulation result which shows energy consumption 298 j.

There is no explicit relation among flight parameters. Having a defined function which links these parameters to each other could be profitable. Using simulation results for a wide range of different times and distances and also using fitting methods an equation was derived as:

$$E\left(t,L\right) = 1063t + 19.38L + 26.95t^2 - 19.13tL + 1761 \qquad (9)$$

where t, E and L represent duration flight, energy consumption in optimal mode and distance between start and end points, respectively.

5. CONCLUSION

Results demonstrate that the most effective parameter in energy consumption of a quadrotor is duration flight. They show that the relative difference between the energy consumption of optimal and trapezoidal profile decreases with increasing duration flight. Experimental results verify the validation of simulation results. Moreover, the presented mathematical relation reveals strict conformity with experimental data and therefore it can be used and cited for similar researches. Eventually, in a short time, the influence of the optimal path is more while Trapezoidal velocity profile is recommended for middle and long time.

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