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Investigation of Power-Law Fluid Flow through a Two-Dimensional Microchannel Based on a Couple Stress Theory-Calculation of Characteristic Length

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ABSTRACT: The present paper aims to investigate the developed flow of Newtonian and non-Newtonian fluids in a two-dimensional microchannel based on the completely consistent couple stress theory and the characteristic length of the fluids. First, the velocity and volumetric flow rate profiles of Newtonian and power-law non-Newtonian fluids in the microchannel were obtained via analytical methods. After that, the characteristic material lengths of water as a Newtonian fluid and blood as a non-Newtonian fluid were obtained and then the results were compared with the experimental data of other papers. Comparing the characteristic lengths of water and blood indicated the dependence of characteristic material length scale on the fluid material. Calculating the characteristic length produced the blood velocity profile of the couple stress theory in microchannel which was in turn compared to the results of classical Navier-Stokes theory. According to the results, increasing the volumetric flow rate of the fluid also increases the difference between the results of couple stress theory and classical theory, indicating the increased influence of length on microchannel flow properties. Further, the velocity profile of water in the microchannel was compared with the experimental results, revealing a good consistency between them and the couple stress theory.

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1. INTRODUCTION

Microchannels are a type of microfluidic device used in different applications such as cellular biology, medical screening, chemical microreactors, and micro fuel cells. Many microfluidic devices are utilized in medical analysis, particularly blood analysis. Blood separation is one of the chief medical applications of microchannels [1]. Therefore, scholars are interested to study blood flow in microchannels. This study presents the first investigation of microchannel blood flow based on the completely consistent couple stress theory. The characteristic length was calculated using flow rate equations and laboratory data. Moreover, the accurate solution of developed Newtonian and non-Newtonian flows in the two-dimensional (2D) microchannel was obtained via the extended equations presented by Karami et al. [2] for studying power-law fluids.

2. METHODOLOGY

As shown in Fig. 1, there is a laminar, developed flow of a power-law, incompressible fluid between two flat, parallel planes. Based on the consistent couple stress theory and excluding gravity effects, continuity, momentum, and boundary condition equations are:

$$\nabla \cdot \boldsymbol{V} = 0 \tag{1}$$

$$\rho \frac{D\boldsymbol{V}}{Dt} = \nabla . \boldsymbol{A} \tag{2}$$

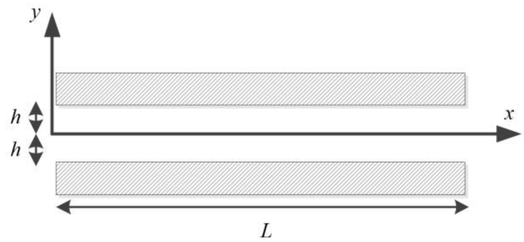

Fig. 1. Two-dimensional geometry of the microchannel

$$u = 0, M_x = 0, at\ y = h, -h, \tag{3}$$

$$\frac{du}{dy} = 0, at\ y = 0. \tag{4}$$

where, V, ρ, u and M_i are velocity vector, density, velocity in the x-direction and couple stress vector, respectively. $\boldsymbol{A}$ is stress tensor and given by [5]:

$$A_{ji} = -P\delta_{ij} + k\left(v_{j,i} + v_{i,j}\right)^{n-1}\left(v_{j,i} + v_{i,j}\right) + l^2 k \begin{bmatrix} (n-1)\left(v_{i,j} + v_{j,i}\right)^{n-2}\left(v_{i,jj} + v_{j,ij}\right) \\ \left(v_{k,ki} - v_{i,kk}\right) + \left(v_{i,j} + v_{j,i}\right)^{n-1} \\ \left(v_{k,kij} - v_{i,kkj}\right) - (n-1)\left(v_{i,j} + v_{j,i}\right)^{n-2} \\ \left(v_{j,ii} + v_{i,ji}\right)\left(v_{k,kj} - v_{j,kk}\right) \\ -\left(v_{i,j} + v_{j,i}\right)^{n-1}\left(v_{k,kji} - v_{j,kki}\right) \end{bmatrix} \tag{5}$$

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where n is the power-law index. Therefore, considering the developmental conditions and the dimensionless parameters below we have:

$$\bar{y}=\frac{y}{h},\bar{x}=\frac{x}{L},\bar{u}=\frac{u}{U},L^{*}=\frac{l}{h},\bar{P}=\frac{h^{2}P}{L\mu_{0}U},\quad \mu_{0}=\frac{kU^{n-1}}{h^{n-1}}. \tag{6}$$

Eqs. (1) and (2) are simplified as:

$$\frac{d\bar{u}}{d\bar{x}}=0,\bar{v}=0,\frac{\partial\bar{P}}{\partial\bar{y}}=0 \tag{7}$$

$$-\frac{\partial\bar{P}}{\partial\bar{x}}+\frac{d}{d\bar{y}}\left\{-\left(-\frac{d\bar{u}}{d\bar{y}}\right)^{n}\right\} -L^{*2}\frac{d}{d\bar{y}}\left\{\begin{array}{l}-(n-1)\left(-\frac{d\bar{u}}{d\bar{y}}\right)^{n-2}\left(\frac{d^{2}\bar{u}}{d\bar{y}^{2}}\right)^{2}\\+\left(-\frac{d\bar{u}}{d\bar{y}}\right)^{n-1}\frac{d^{3}\bar{u}}{d\bar{y}^{3}}\end{array}\right\}=0 \tag{8}$$

$$\bar{u}=0,M_{x}=\frac{d^{2}\bar{u}}{d\bar{y}^{2}}=0,at\ \bar{y}=1,-1, \tag{9}$$

$$\frac{d\bar{u}}{d\bar{y}}=0,at\ \bar{y}=0. \tag{10}$$

By selecting an appropriate variable change, the $\bar{u}$ value is obtained analytically using Maple 2018:

$$\bar{u}=\left(-\frac{d\bar{P}}{d\bar{x}}\right)^{1/n}\left\{\begin{array}{l}-\int_{0}^{\bar{y}}\left\{\begin{array}{l}\frac{L^{*}}{\sqrt{n}}\left(\frac{e^{-\sqrt{n}\bar{y}/L^{*}}-e^{\sqrt{n}\bar{y}/L^{*}}}{e^{-\sqrt{n}/L^{*}}+e^{\sqrt{n}/L^{*}}}\right)\\+\bar{y}\end{array}\right\}^{1/n}d\bar{y}\\+\int_{0}^{1}\left\{\begin{array}{l}\frac{L^{*}}{\sqrt{n}}\left(\frac{e^{-\sqrt{n}\bar{y}/L^{*}}-e^{\sqrt{n}\bar{y}/L^{*}}}{e^{-\sqrt{n}/L^{*}}+e^{\sqrt{n}/L^{*}}}\right)\\+\bar{y}\end{array}\right\}^{1/n}d\bar{y}\end{array}\right\} \tag{11}$$

The volumetric flow rate is obtained using Eq. (11):

$$Q=\frac{2h^{1/n+2}w}{k^{1/n}}\left(-\frac{dP}{dx}\right)^{1/n}\times\int_{0}^{1}\left\{\begin{array}{l}-\int_{0}^{\bar{y}}\left\{\frac{L^{*}}{\sqrt{n}}\left(\frac{e^{-\sqrt{n}\bar{y}/L^{*}}-e^{\sqrt{n}\bar{y}/L^{*}}}{e^{-\sqrt{n}/L^{*}}+e^{\sqrt{n}/L^{*}}}\right)+\bar{y}\right\}^{1/n}d\bar{y}\\+\int_{0}^{1}\left\{\frac{L^{*}}{\sqrt{n}}\left(\frac{e^{-\sqrt{n}\bar{y}/L^{*}}-e^{\sqrt{n}\bar{y}/L^{*}}}{e^{-\sqrt{n}/L^{*}}+e^{\sqrt{n}/L^{*}}}\right)+\bar{y}\right\}^{1/n}d\bar{y}\end{array}\right\}d\bar{y} \tag{12}$$

In Eqs. (11) and (12) for $L^*=0$, the velocity profile and volumetric flow rate are obtained using the classical theory.

3. RESULTS AND DISCUSSION

Substituting the experimental data obtained by Sampaio et al. [3] in Eq. (12) and trial and error, the dimensionless characteristic length for three values of volumetric flow rate (10, 20, and 30 ml/min) and for n=1 in C_1 microchannel are calculated (Table 1). The viscosity and density of water at 25ºC are respectively assumed 0.000889 Pa. s and 997 kg/m^3. According to Table 2, the characteristic length of water as a Newtonian fluid in the microchannel with the defined geometry and boundary conditions is not very dependent on the flow rate and pressure gradient.

Table 3 shows the dimensionless characteristic length of a dog's blood in the C_2 microchannel for three different values of pressure gradient and volumetric flow rates of 10, 20, and 30 µl/min. these values were obtained via trial and error using Eq. (12).

The Q_0 volumetric flow rate was obtained based on the classic theory and Eq. (12) for three different values of the pressure gradient in the microchannel (Table 3). Comparing this data with the results in Table 2 indicates the dependence of characteristic length for the non-Newtonian fluid of blood on the fluid material, showing a slight decrease with the increased in the flow rate.

Fig. 2 compares the velocity profile of the water Newtonian fluid in a rectangular microchannel (height ×width=222×694 µm) with the experimental and numerical data of Qu et al. [4]. The velocity profile obtained from Eq. (11) is based on the completely consistent couple stress theory for Newtonian fluids and for a flow rate of 46431.6 µl/min, the Reynolds number 1895, and pressure gradient -1473.843 kPa/m. The dimensionless characteristic length is 0.372 (Eq. (12)). The difference between theoretical results and experimental data is due to simplification presumptions such as the 2D

Table 1. Geometrical dimensions of the microchannel

microchannel	height (µm)	weight (µm)
C_1	38	160
C_2	222	694

Table 2. The dimensionless characteristic material length scale of water for three different flow rate values

Volumetric flow rate (µl/min)	L^*
10	0.29
20	0.288
25	0.29

Table 3. The dimensionless characteristic material length scale of dag blood for three different flow rate values

dP/dx (kPa/m) [3]	Experimental flow rate (µl/min) [3]	L^*	Q_0 (µl/min)
796.563	10	0.273	8.16
1138.06	15	0.26	12.42
1495.04	20	0.230	17.16

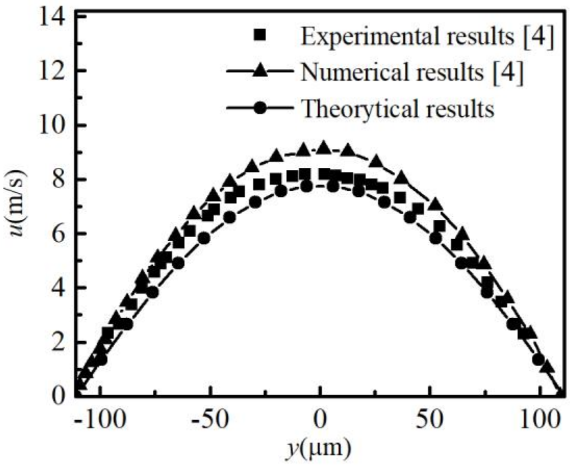

Fig. 2. Comparison of the theoretical water velocity profile L^*=0.372 in microchannel C_2 and experimental results of Qu et al. [4]

shape of the microchannel; while in the numerical analysis, microchannel geometry is modeled in three-dimensional (3D). Nevertheless, the minimum and maximum differences between the numerical and experimental results are 1.20 and 11.48 percent, respectively. As a result, it can be said that the analytical results of couple stress theory are acceptable when compared to laboratory results.

4. CONCLUSION

The velocity profiles for the power-law Newtonian and non-Newtonian fluids in a 2D microchannel were obtained using the completely consistent couple theory (Eq. (12)). The volumetric flow rate was obtained using the analytical velocity relations. This study indicates the dependence of characteristic length on the fluid material. Further, the characteristic material length of water as a Newtonian fluid is not dependent on the volumetric flow rate and pressure gradient of the flow. However, the characteristic length of dog blood slightly increases with the increase in volumetric flow rate. The comparison of the velocity profile of water with the experimental and numerical data presented by Qu et al. [4]; which indicates the accuracy of completely consistent couple stress theory.

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