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Hydrodynamic Behavior of Different No-Slip Condition on the Curved Boundaries in the Lattice Boltzmann Method

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ABSTRACT: This paper examines the various methods of applying no-slip boundary condition on a fixed and rotary cylinder in the lattice Boltzmann framework. For this purpose, five methods of bounce-back, linear and quadratic method of Yu and the linear and quadratic method of Bouzidi are chosen. The main challenge in all of these methods is how to calculate and interpolate the unknown distribution functions at the points around the boundary points. Results show that in the stable conditions (Re=20 and Re=40), the maximum error of calculation of the separation angle is 6.7 % and it is related to the bounce-back method, while in the stable conditions, a significant difference cannot be seen between the bounce-back and other methods. Also, the linear method of Bouzidi has the most error in calculating the separation length (6% for Re=20 and 8.82 % for Re=40). By increasing the Reynolds number and increasing the rotational velocity, a difference in the lift coefficient in the early times, t*> 7.78 grows for the conditions of k=0.2 and Re=200, between the bounce-back and other methods, however with increasing time, this difference reduces, whereas the three methods of linear Yu, linear Bouzidi and quadratic Bouzidi, continue to produce similar results.

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1. INTRODUCTION

In the field of numerical simulations, several attempts have been made to detect the curvilinear boundaries and preventing its step-like behavior. One is to map the curvilinear coordinates into the cartesian coordinate. However, the implementation of curved boundaries in the cartesian coordinate is more widely used because of the simplicity of the application and the lack of limitation to the specific coordinate system. Filippova and Hänel [1], were able to model curve boundaries in the cartesian system using the bounce-back rule and the idea of extrapolating the distribution functions. Mei et al. [2], developed the method of Filippova and Hänel and partially solved the problem of instability.

Since there is no comprehensive study on the comparison of different methods in applying the no-slip boundary conditions in curved boundaries, especially in rotating conditions, five methods of Bounce-Back (BB), linear Yu-Mei-Luo-Shyy (LYMLS), Quadratic Yu-Mei-Luo-Shyy (QYMLS), linear Bouzidi-Firdaouss-Lallemand (LBFL) and quadratic Bouzidi-Firdaouss-Lallemand (LBFL) are selected to compare and find their hydrodynamic behavior. Accordingly, a FORTRAN code is developed to evaluate the hydrodynamic parameters of the flow, such as the separation angle, separation length, and the drag and lift coefficients.

2. IMPLEMENTATION OF CURVED BOUNDARY

2.1. Bounce-back method

In this method, r_l stands for the position of the fluid near the boundary, r_p represents the point inside the cylinder near the boundary and r_w shows the point between them, located on the curved surface. According to the bounce-back rule, the collision step on the curved surface is as follows [3]:

$$f_{\bar{\alpha}}(r_l, t+\Delta t) = \tilde{f}_{\alpha}(r_l, t) - 2\rho w_{\alpha} \frac{\mathbf{u}_w \cdot \mathbf{c}_{\alpha}}{c_s^2} \tag{1}$$

where $\bar{\alpha}$ is opposite to α and α is a direction toward the curved boundary.

2.2. Method of Bouzidi

The exact location of the solid surface is obtained by the following ratio:

$$\Delta_w = \left| \left(r_l - r_w \right) / \left(r_l - r_p \right) \right| \tag{2}$$

The linear interpolation of BFL is defined as follows:
For $\Delta_w \le 0.5$:

$$f_{\bar{\alpha}}(r_l, t+\Delta t) = (1-2\Delta_w)\tilde{f}_{\alpha}(r_{l'}, t) + 2\Delta_w \tilde{f}_{\alpha}(r_l, t) - 2\rho w_{\alpha} \frac{\mathbf{u}_w \cdot \mathbf{c}_{\alpha}}{c_s^2} \tag{3}$$

For $\Delta_w > 0.5$:

$$f_{\bar{\alpha}}(r_l, t+\Delta t) = \frac{2\Delta_w - 1}{2\Delta_w}\tilde{f}_{\bar{\alpha}}(r_l, t) + \frac{1}{2\Delta_w}\tilde{f}_{\alpha}(r_l, t) - \frac{1}{2\Delta_w} 2\rho w_{\alpha} \frac{\mathbf{u}_w \cdot \mathbf{c}_{\alpha}}{c_s^2} \tag{4}$$

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Table 1. Comparison of the C_d, *L/D*, and θ with respect to the FEM for *Re*=20 in a fixed condition

Method	C_d	error%	L/D	error%	θ	error%
BB	2.070	1.2	0.97	3	40.36	7.6
QYMLS1	×	---	×	---	×	---
QYMLS2	2.068	1.12	0.96	2.13	42.18	2.06
LYMLS	2.065	1	0.95	1.1	42.3	3
QBFL1	×	---	×	---	×	---
QBFL2	2.071	1.27	0.93	1.06	43.1	1.37
LBFL	2.075	1.4	1	6	43.86	0.3
FEM [7]	2.045	---	0.94	---	43.7	---

Instead of using the linear interpolation method, we can use the second-order interpolation to find the distribution functions [4].

2.3. Method of Yu

Linear interpolation for finding distribution functions in the YMLS method is as follows [5]:

$$f_{\bar{\alpha}}(r_l, t+\Delta t) = \frac{\Delta_w}{1+\Delta_w} f_{\bar{\alpha}}(r_{l'}, t+\Delta t) + \frac{1}{1+\Delta_w} f_{\bar{\alpha}}(r_w, t+\Delta t) \tag{5}$$

where the amount of $f_{\bar{\alpha}}(r_l, t+\Delta t)$ could be evaluated by the second-degree interpolation as follows:

$$f_{\bar{\alpha}}(r_l, t+\Delta t) = \frac{2}{(2+\Delta_w)(1+\Delta_w)} f_{\bar{\alpha}}(r_w, t+\Delta t) + \frac{2\Delta_w}{1+\Delta_w} f_{\bar{\alpha}}(r_{l'}, t+\Delta t) - \frac{\Delta_w}{2+\Delta_w} f_{\bar{\alpha}}(r_{l''}, t+\Delta t) \tag{6}$$

Also, the amount of force which acts on the cylinder can be calculated as follows [6]:

$$F = \sum_{l} \sum_{\alpha_l^{cyl}} F_{\alpha_l}^{cyl} \tag{7}$$

where:

$$F_{\alpha_l}^{cyl} = c_{\alpha_l}^{cyl} \tilde{f}_{\alpha_l}^{cyl}(r_l^{cyl}, t)\Delta x\, \Delta y + c_{\alpha_l}^{cyl} \tilde{f}_{\bar{\alpha}_l}^{cyl}(r_l^{cyl}, t+\Delta t)\Delta x\, \Delta y \tag{8}$$

3. RESULTS AND DISCUSSION

3.1. Fixed cylinder

The comparison between values of the drag coefficient C_d, the ratio of the vortex length to the diameter L/D, and the angle of separation θ in *Re*=20 are reported and compared with the available numerical results in Table 1. According to Table 1, the BB method in calculating the angle of separation θ and the LBFL method in calculating the L/D values lead to 7.6% and 6% error respect to the Finite Element Method (FEM), respectively. Table 1 shows the conditions that led to the divergence of QBFL and QYMLS with the QBFL1 and QYMLS1 expressions. These two methods converge with larger computational domain and an increase in the single release time. In this situation, the two QBFL2 and QYMLS2 methods converge to the values given in Table 1.

3.2. Rotating cylinder

In this section, the values of the drag C_d and the lift C_l *coefficients in the Re*=200 have been reported to examine the behavior of the boundary conditions of BB, LYMLS, LBFL and QBFL in the fixed and rotating cylinders. It should be noted that the results of the QYMLS method have not been presented due to non-convergence in the problem conditions (selection of parameters *U*, *D* and τ). Table 2 shows the results for $k = R\Omega/U = 0$. Since for *k*=0 the coefficients of the drag and lift are oscillating, hence, the maximum and minimum values of these values are reported. In order to compare the behavior of boundary conditions, the LYMLS method was selected as the base method and the error of each methods is measured and reported compared to this method.

Now, changes in the drag coefficients for the conditions of *k*=0.2 and *Re*=200 are shown in Figs. 1 and 2, respectively. By comparing the results, it is seen that in the early times, $t^*<7.78$, differences between BB results with the other three methods are clear, but with increasing time, this difference is reduced.

4. CONCLUSIONS

Based on the simulations, the following results can be expressed:

- In the case where the cylinder is fixed and the flow has a small Reynolds number, using the bounce-back method, in spite of the simplicity of the operation, has an acceptable accuracy in calculating and predicting the drag coefficient; so that the need to apply methods with higher precision or

Table 2. maximum and minimum values for C_d and C_l for a fixed cylinder, *k*=0 at *Re*=200

	BB	LYMLS	LBFL	QBFL
$C_{d,\,max}$	1.464	1.329	1.329	1.315
$E_{Cd,\,max}$ (%)	10.16	0.0	0.0	1.05
$C_{d,\,min}$	1.24	1.116	1.12	1.102
$E_{Cd,\,min}$ (%)	11.11	0.0	0.36	1.25
$C_{l,\,max}$	0.716	0.591	0.585	0.578
$E_{Cl,\,max}$ (%)	21.15	0.0	1.02	2.2
$C_{l,\,min}$	-0.712	-0.565	-0.576	-0.562
$E_{Cl,\,min}$ (%)	26.06	0.0	1.95	0.53

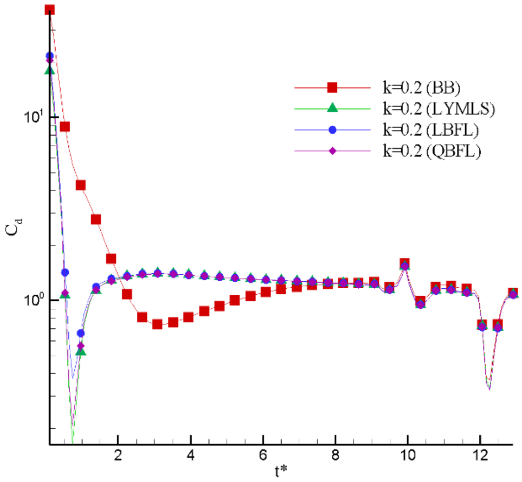

Fig. 1: Variation of drag coefficient for a rotating cylinder at *k*=0.2 and *Re*=200

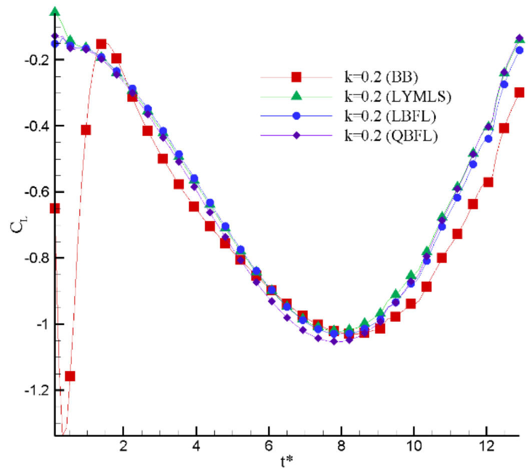

Fig. 2: Variation of lift coefficient for a rotating cylinder at *k*=0.2 and *Re*=200

methods with curvature boundary capability is neglected.

• The BB method has a big error in calculating the angle of separation θ and the LBFL method overshoots in calculating the L/D ratio. So, in the same conditions they become divergent compared to the other three methods.

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