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Vibration Analysis of Cable-Driven Parallel Robots to Define Critical Speeds

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ABSTRACT: Cable-driven manipulators are a generation of parallel cinematic chain robots which provide important features including wide workspace and cost-effective high speed operations. However, due to wideness of the workspace and the flexibility of the cables, they are susceptible to unwanted vibrations which reduce their precision. Therefore, determination of velocity limits in the operation workspace is of high importance. In this study, stability analysis and critical velocities of a four cable plane robot are considered. Governing equations of the system are extracted by use of finite element method and employing variable length element. The characteristic coefficients of the extracted equations are nonlinear and velocity dependent ones. To provide a stability analysis, the equations are linearized assuming that the end-effector experiences quasi-static movements and the system is subjected to low amplitude vibrations. Afterward, the corresponding eigenvalue problem is analyzed and critical speeds of the robot in whole workspace domain are calculated. Furthermore, vibration frequencies corresponding to the unstable eigenvalues are determined. It is observed that system critical speed reduces as the end-effector moves to the boundaries of the workspace. In contrast to this, the frequency of the corresponding unstable modes increases as the end-effector moves to the borders of the workspace.

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1- Introduction

Cable-Driven robots have a long history of industrial applications. Despite this, lots of their fundamental theoretical problems including kinematics, kinetics, and vibrations have been investigated in recent years. Due to longitudinal and bending flexibility of the cables, these robots are susceptible to unwanted vibrations in applications where large workspace, high velocity or considerable stiffness is demanded. The stiffness problem of cable robots has been investigated in many research works [1,2]. In this paper, the stability analysis of planar robot cables is studied to obtain the maximum allowable speeds. The governing equations of the system are obtained using variable length cable element. These equations are linearized considering the end-effector position, velocity, movement direction and the tensions of the stiffening cables. Then, the corresponding eigenvalue problem is solved to define natural frequencies and the critical velocity of the robot. According to our survey in the literature, the subject of the paper and the presented solution are novel.

2- Methodology

Fig. 1 shows a cable robot working in a vertical plane. The end effector is supported by four cables and its position and velocity are controlled through cables' tension control. The end effector is considered as a point mass and the variable length cable element and finite element method presented by Du et al. [3] is employed to extract the corresponding dynamic equations. In this method, each cable is divided into a specific number of variable length similar elements. The main advantage of this method is that facilitates the employment of conventional finite element method and matrix assembling technics. Fig. 2 illustrates a variable length element of cross-sectional area A, non-extended length $l(t)$, modulus of elasticity E and mass per unit length μ. Force vectors $\mathbf{f}_1=[f_{x_1}, f_{y_1}]^T$ and $\mathbf{f}_2=[f_{x_2}, f_{y_2}]^T$ stand for element nodal forces and positions of the end nodes are presented by position vectors $\mathbf{r}_1=[x_1, y_1]^T$ and $\mathbf{r}_2=[x_2, y_2]^T$. Moreover, global and local coordinates of a generic point P on the element are presented by $\mathbf{r}=[x, y]^T$ and s, respectively. Assuming that each element stays as a straight line during time, position vector of the generic point P is expressed in terms of end-node position vectors and matrix of shape functions, as follows:

$$\mathbf{r}(s,t)=\mathbf{N}\mathbf{r}_j \tag{1}$$

where $\mathbf{r}_j=[\mathbf{r}_1^T, \mathbf{r}_2^T]^T$ and $\mathbf{N}=[n_1\mathbf{I}, n_2\mathbf{I}]$ represent the vector of nodal positions and a matrix of shape functions, respectively. Here, $\mathbf{I}$ denotes identity matrix and shape functions are set to $n_1=1-s/l(t)$ and $n_2=s/l(t)$.

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Using the presented parameter definitions, the kinetic energy of the element, its potential energy (including strain gravitational parts) and virtual work of applied forces are determined, respectively, as follows:

$$T = \int_0^1 \frac{1}{2}\mu \dot{\mathbf{r}}^T \dot{\mathbf{r}}\, ds \tag{2}$$

$$U = \int_0^1 (\frac{1}{2} EA\varepsilon^2 - \mu g \mathbf{r}^T \mathbf{e}_3)\, ds \tag{3}$$

$$\delta H = \delta \mathbf{r}_j^T \mathbf{f}_j + \mu \dot{\mathbf{r}}_1^T \delta \mathbf{r}_1 v_1 + \mu \dot{\mathbf{r}}_2^T \delta \mathbf{r}_2 v_2 \tag{4}$$

where g is gravity constant, $\mathbf{e}_3$ indicates the vertical unity vector and ε represents the element's longitudinal strain which is equal to:

$$\varepsilon = \sqrt{\partial_s \mathbf{r}^T \partial_s \mathbf{r}} - 1 \approx \frac{1}{2}(\partial_s \mathbf{r}^T \partial_s \mathbf{r} - 1) \tag{5}$$

In addition, v_1 and v_2 are lengths added to the element in its two ends and $\mathbf{f}_j = [\mathbf{f}_1^T, \mathbf{f}_2^T]^T$ is a vector of element nodal forces. Using Eqs. (2) to (5), the dynamic equations of the element are derived using Hamilton method [3]:

$$\mathbf{m}_j \ddot{\mathbf{r}}_j + \mathbf{c}_j \dot{\mathbf{r}}_j + \mathbf{k}_j \mathbf{r}_j = \mathbf{f}_j + \mathbf{f}_j^g \tag{6}$$

where $\mathbf{m}_j$, $\mathbf{c}_j$ and $\mathbf{k}_j$ are mass, energy transfer, and stiffness matrices, respectively, and $\mathbf{f}_j^g$ represents element's weight and they are functions of element length or length change rates. Using the conventional matrix assembling method and adding up the mass and the weight of the end effector to the corresponding node, the governing equation of the system is obtained:

$$\mathbf{M}\ddot{\mathbf{R}} + \mathbf{C}\dot{\mathbf{R}} + \mathbf{K}\mathbf{R} = \mathbf{F} + \mathbf{F}^g \tag{7}$$

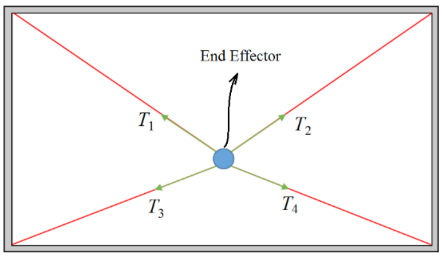

Fig. 1. The position and the velocity of the robot's end-effector controlled via cables' tension control

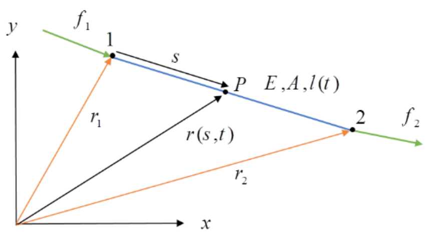

Fig. 2. Cable element

In the above equation, the stiffness matrix is a function of nodal positions vector and therefore, it's a nonlinear equation of nodal positions. We recall that mass, energy transfer and stiffness matrices are all functions of length change rates and therefore depend upon the end-effector velocity. Moreover, the end-effector is over-constrained. To solve this equation, we assume that the end-effector position is known and it moves with a specific constant velocity. In addition, the magnitude of cable tensions corresponding to cables no. 3 and 4 are predefined. Now, we follow the below steps:

1. Assuming that the system is in static equilibrium, Eq. (7) is reduced to a nonlinear static equation:

$$\mathbf{K}\mathbf{R}_{t,s} = \mathbf{F} + \mathbf{F}^g \tag{8}$$

where $\mathbf{R}_{t,s}$ defines the nodal positions at time t. This equation determines the position of cables intermediate nodes and the lengths of cable elements (l_t).

2. Afterward, we assume that the end-effector moves to its next position quasi-statically while the cables experience no vibrations. The same calculations are performed to determine the nodal positions and element lengths corresponding to the new position. These results along with movement time dt obtain nodal velocities and rates of element length changes:

$$\dot{\mathbf{R}}_{qs} = (\mathbf{R}_{t+dt,s} - \mathbf{R}_{t,s})/dt \tag{9}$$

$$\dot{l} = (l_{t+dt} - l_t)/dt \tag{10}$$

where, $\mathbf{R}_{qs} = \mathbf{R}_{t,s}$.

3. To determine the acceleration of nodal displacements, the second order time derivative of element lengths and the cable tensions, again we refer to Eq. (7):

$$\mathbf{M}\ddot{\mathbf{R}}_{qs} + \mathbf{C}\dot{\mathbf{R}}_{qs} + \mathbf{K}\mathbf{R}_{qs} = \mathbf{F} + \mathbf{F}^g \tag{11}$$

4. Now, we assume that the system undergoes a small-amplitude vibrational displacement in addition to its quasi-static movement:

$$\mathbf{R} = \mathbf{R}_{qs} + \mathbf{R}_d \tag{12}$$

Then, Using Eqs. (7) and (11), we get

$$\mathbf{M}\ddot{\mathbf{R}}_d + \mathbf{C}\dot{\mathbf{R}}_d + \mathbf{K}'\mathbf{R}_d = \mathbf{0} \tag{13}$$

where $\mathbf{K}'$ is determined using the below equation:

$$\mathbf{K}'\mathbf{R}_d = \mathbf{K}\big|_{\mathbf{R}=\mathbf{R}_{qs}+\mathbf{R}_d} \left(\mathbf{R}_{qs} + \mathbf{R}_d\right) - \mathbf{K}\big|_{\mathbf{R}=\mathbf{R}_{qs}} \mathbf{R}_{qs} \tag{14}$$

In general, $\mathbf{K}'$ is a nonlinear function of $\mathbf{R}_d$. However, with the assumption of small amplitude vibrations and neglecting higher-order terms, it reduces to a constant matrix. This way, the linearization process is completed. More details and discussions about linearization and solution of Eq. (14) for a similar problem, are presented in Ref. [4]. Here, assuming that element elongations in lateral vibrations are negligible, we simplify Eq. (14), as follows:

$$\mathbf{K}' = \mathbf{K}\big|_{\mathbf{R}=\mathbf{R}_{qs}} \tag{15}$$

5. Finally, a proportional damping is added to the energy transfer matrix and characteristic equation is solved to determine system's eigenvalues. The system is stable whenever all the real parts of the eigenvalues are negative. Otherwise, it becomes unstable.

3- Results and Discussion

A robot cable with a rectangular workspace of 20m length and 10m width is considered. The center of the coordinates is set to the center of the rectangle. The end effector mass is 2kg and cable parameters are set to $E = 125\text{MPa}$, $A = 0.5\,\text{cm}^2$ and $\mu = 0.4\,\text{kg/m}$. The end-effector solely displaces horizontally and its velocity is increased in steps of 0.1 m/s until it crosses the stability border for the first time. Since not all arbitrary values of cable tensions provide quasi-static movement conditions, we use the following equations to adjust the tensions of cables no. 3 and 4:

$$T_3 = \sqrt{100\left(5 + x_c/2\right)^2 + 45^2}$$

$$T_4 = \sqrt{100\left(5 - x_c/2\right)^2 + 45^2}$$

In the presented results, each cable is divided into 10 elements which guaranty the convergence of the numerical calculations. To assess the validity of the presented results and the precision of numerical calculations, the critical velocity of a horizontally moving single cable is calculated and compared to that presented by Dehadrai et al. [5]. According to their numerical calculations, for all values of cable tensions, the critical velocity of a horizontally moving cable is equal to its wave speed ($V_{Inst.} = \sqrt{T/\rho A}$). The instability velocity calculated here is $1.055\sqrt{T/\rho A}$ which is close to the mentioned value.

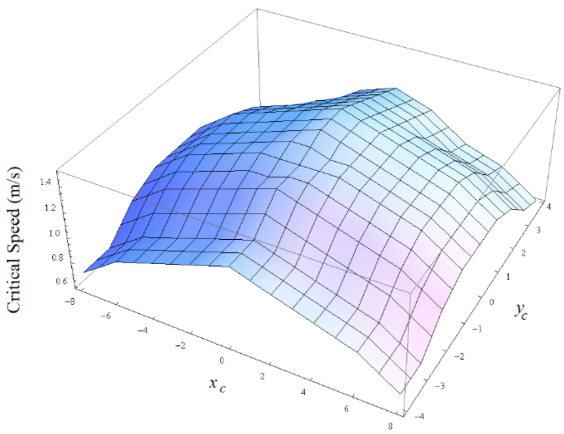

Fig. 3. Variations of critical speed versus end-effector's position

Fig. 3 presents a critical velocity of the cable robot versus its end-effector position. It shows critical velocity is maximum when the end-effector is positioned in its workspace center and reduces as it moves to the borders. This is because some of the cable lengths increase as the end effector moves toward the borders and corresponding stiffness reduces considerably. These cables are more susceptible to unwanted vibrations and reduce system's stability. Despite instability velocities, the value of instability frequencies increases as the end-effector moves to the borders of its workspace.

4- Conclusion

Stability analysis of planar robot cables is considered. The nonlinear governing equations of the system are extracted using variable length cable element method. These equations are linearized by separating the movement parameters into quasi-static and small amplitude vibrational components. Using the quasi-static movements, the cables nodal positions, element lengths and the time derivatives of element lengths are estimated. Then, adding up the small amplitude vibrational movements to the quasi-static ones, the linearized dynamic equations of the system are derived and the instability velocities and frequencies are calculated. Numerical calculations show that system instability velocity reduces as the end effector approaches to its workspace borders. In contrast, instability frequency increases as the end effector move toward the borders and the corners of the workspace.

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