



# Analytical Solution of the Heat Transfer in Heterogeneous Composite Conical Shells with Temperature Dependent Conduction Coefficients

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**ABSTRACT:** This paper presents an analytical solution for heat transfer in heterogeneous composite conical shells with temperature dependent conduction coefficients for the first time. The geometry of the shell is completely conical shaped and the fibers are wound around the laminate in the desired direction. In order to achieve the most general solution, the general boundary condition is considered at the basis of shell and the effect of heat convection resulted from flow motion around the body and different kinds of non-axisymmetric radiative heat flux at the outer side of the shell is modeled. The heterogeneous effect in this case is the results of the dependency in conduction heat transfer coefficient on temperature. Therefore, the heat transfer equation should first be transformed using the Kirchhoff transform to a solvable equation using integral transformation, then, the partial differential equation becomes an ordinary differential equation Fourier transformation. Finally, the transformed differential equation can be solved Green's functions. In the end, the reversal integral transformation and reversal Kirchhoff conversion are applied to obtain heterogeneous temperature distribution. Validation of this analytical solution is performed by comparing the analytical results with the solution of second-order finite difference method and some applied cases are considered to investigate the capability of current solution for solving the industrial problems in the production of composite conical pressure vessels.

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## 1- Introduction

In different industrial branches, conical geometry is very applicable, but because of the complex geometry that cone has, there are just few analytical solutions on heat transfer in composite conical shells [1, 2]. On the other hand, the study on the heat transfer in composite materials provides valuable knowledge in analyzing fiber placement in the production process and also preventing thermal fracture [3, 4].

In this study, we focus on the two-dimensional heat transfer in  $x$  and  $\varphi$  directions in composite conical shells with the linear dependency of conductivity on temperature. The shell is considered to have a full conical shape and the fibers are wound around it. The geometry and boundary conditions of conical shell are presented in Fig. 1. The general boundary conditions at the base of the cone are considered to cover the entire heat transfer mechanisms. For validation of the present analytical solution, the analytical results compared with the solution of second-order finite difference method. In order to investigate the capability of this analytical solution for solving the industrial problems, some applied cases in production of composite conical pressure vessels are considered.

## 2- Methodology

By applying the energy balance in a conical element, one can derive the heat conduction partial differential equations as presented by Norouzi and Rahmani [1]:

$$\bar{k}_{11}(T) \frac{\partial^2 T}{\partial x^2} + \frac{\bar{k}_{22}(T)}{x^2 \sin^2 \gamma} \frac{\partial^2 T}{\partial \varphi^2} + \frac{2\bar{k}_{12}(T)}{x \sin \gamma} \frac{\partial^2 T}{\partial x \partial \varphi} + \frac{\bar{k}_{11}(T)}{x} \frac{\partial T}{\partial x} - \frac{h}{\delta} (T - T_\infty) = -\frac{u''' \delta + q''}{\delta} \quad (1)$$

where  $T$  represents temperature,  $u'''$  the rate of heat generation,  $q''$  the external heat flux,  $T_\infty$  the ambient temperature,  $h$  the heat convection coefficient,  $\delta$  the thickness of the conical shell,  $\gamma$  the half of cone apex angle and  $\bar{k}_{ij}$  the conduction coefficient in the off-axis coordinate. Also,  $\varphi$  and  $x$  are spatial variables.

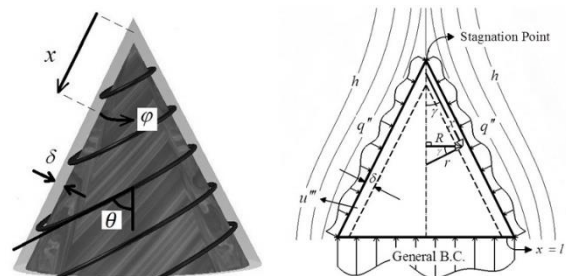


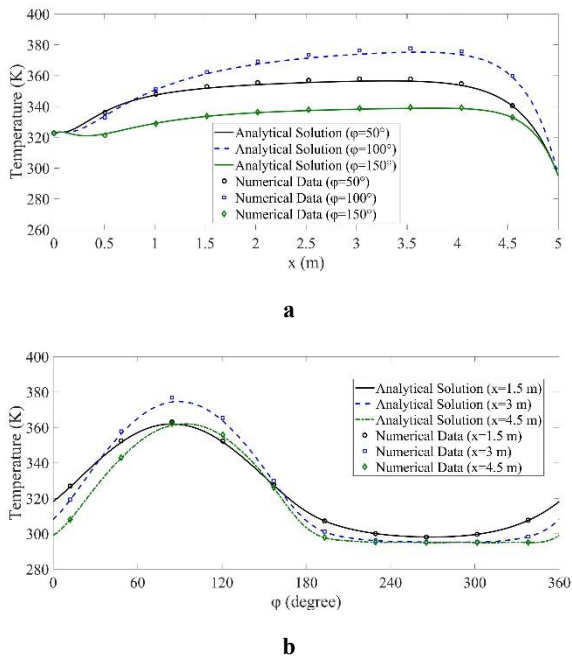
Fig. 1. Direction of fibers plus coordinate system and boundary conditions of composite conical shell [1]

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**Table 1. Specifications of polymeric composite materials [6]**

Material	Density (g/cm <sup>3</sup> )	Heat Capacity (J/gk)	k <sub>011</sub> (W/mk)	k <sub>022</sub> (W/mk)
1	1.770	0.687	54.85	5.193
2	1.547	0.690	24.66	4.795



**Fig. 2. Temperature variations of composite conical shell which the fiber's angle is 45° and composite material is material 1; a) in**

For the sake of simplicity, the modified temperature is defined as:

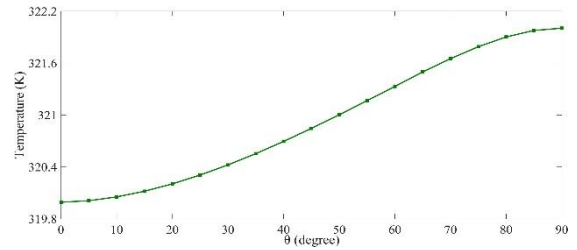
$$\hat{\theta}(x, \varphi) = T(x, \varphi) - T_{\infty} \tag{2}$$

With using of Eq. (2), The modified heat conduction Partial Differential Equation (PDE) can be written as:

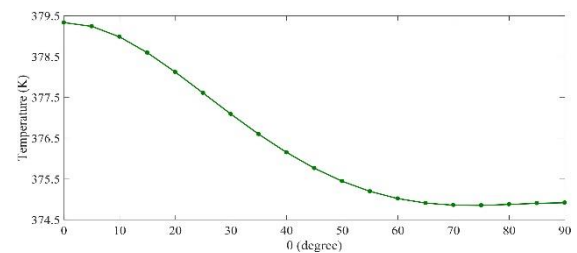
$$\begin{aligned} \bar{k}_{11}(\hat{\theta}) \frac{\partial^2 \hat{\theta}}{\partial x^2} + \frac{\bar{k}_{22}(\hat{\theta})}{x^2 \sin^2 \gamma} \frac{\partial^2 \hat{\theta}}{\partial \varphi^2} + \frac{2\bar{k}_{12}(\hat{\theta})}{x \sin \gamma} \frac{\partial^2 \hat{\theta}}{\partial x \partial \varphi} \\ + \frac{\bar{k}_{11}(\hat{\theta})}{x} \frac{\partial \hat{\theta}}{\partial x} - \frac{h}{\delta} \hat{\theta} = -\frac{u''' \delta + q''}{\delta} \end{aligned} \tag{3}$$

The general boundary condition is applied at the base of cone in order to cover wide range of thermal conditions. The below relation is considered for this linear boundary condition:

$$\sigma \hat{\theta}(l, \varphi) + \omega \frac{\partial \hat{\theta}}{\partial x}(l, \varphi) = f(\varphi) \tag{4}$$



**Fig. 3. Mean temperature distribution versus fiber's angle variations for Material 1**



**Fig. 4. Maximum temperature distribution versus fiber's angle variations for Material 1**

Herein we consider solar radiation distribution for external heat flux and solar radiation on the composite conical shell is defined as [5]:

$$q'' = \begin{cases} q'' \sin \varphi & 0 \leq \varphi \leq \pi \\ 0 & \pi \leq \varphi \leq 2\pi \end{cases} \tag{5}$$

In order to solve the heat conduction PDE presented in Eq. (3), first, we use Kirchhoff transformation and then the integral transform method with respect to angular direction  $\varphi$  is applied. By using this transformation, the heat conduction PDE becomes an Ordinary Differential Equation (ODE). The resulted ODE by using green functions method can be solved. Finally, by applying the inverse integral transform and by means of an inverse Kirchhoff transform the temperature distribution in heterogeneous composite conical shell is obtained.

### 3- Results and Discussion

This section is focused on the capability of present analytical solution for solving the industrial problems in the production process of composite conical pressure vessels. Table 1 presents the properties of two typical types of carbon-carbon composite materials which are used commonly in industry. Herein we consider reference temperature as 295 K and also thermal coefficient, known as  $\beta$  is equal to 0.001 1/K.

In order to validate the present analytical solution, a numerical calculation based on the second-order finite difference is provided. Fig. 2 depicted the validation results. It shows the temperature variations of composite conical shell versus  $x$  and  $\varphi$  directions for material 1. The results show

that the analytical solution is in good agreement with the numerical data.

In Fig. 3 the mean temperature versus fiber angle values is depicted for material 1. It shows that with an increase in fiber angle values, the mean temperature is increased.

In Fig. 4 the maximum temperature versus different values of the fiber angle is depicted for material 1 and results show that the lowest values of the maximum temperature occurs at the higher values of the fiber angle.

#### 4- Conclusions

In this paper, the effect of the fiber angle was specifically examined. The results show that by increasing the fiber angle, the mean temperature in the composite conical pressure vessel increases, and the zero value for fiber angle is recommended as the most efficient choice. However, by increasing the fiber angle, the maximum temperature in the composite conical pressure vessel is reduced and the higher value of fiber angle was proposed as the most efficient choice. Therefore, we recommend that in designing these types of pressure vessel, both the mean temperature and maximum temperature parameters should be considered to choose the most efficient value of the fiber angle.

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